It’s smallness is not a pity, time will reveal its importance !

Investing
in your future

European Union
European Regional
Development Fund

Gravitational Wave lensing in General Relativity and beyond

It’s smallness is not a pity, time will reveal its importance !

PhD Seminar

Supervisor : **Prof.dr.hab. Marek Biesiada**

Sreekanth Harikumar
Department of Astrophysics (BP4)
National Centre for Nuclear Research

Some historical developments

Around 1789 **Henry Cavendish** calculated the Newtonian deflection of light(did not publish his results)

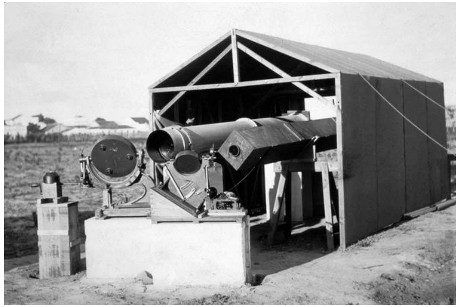
Johann Georg von Soldner (1804) first published light deflection based on Corpuscular theory of light

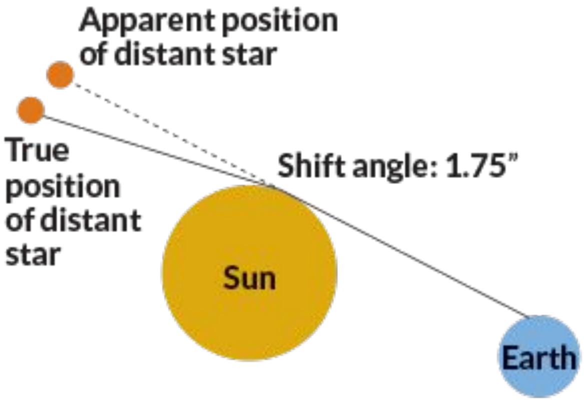
$$\hat{\alpha} \approx \frac{2GM}{c^2R} \approx 0.875''$$

A.Eddington

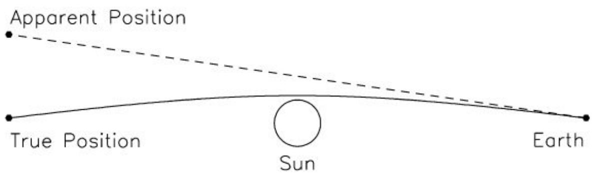
Einstein's deflection angle

$$\hat{\alpha} = \frac{4GM}{c^2R} = 1.75''$$





https://subratachak.wordpress.com/2017/12/22/newtonian-gravity-vs-general-relativity/



❖ Eddington(1920) : possibility of multiply images
❖ Chwolson (1924) : creation of fictitious double stars

Including Einstein, all concluded there is a little chance of observing them.

Fritz Zwicky

❖ Zwicky (1937a,b) : galaxies could acts as lenses and split images, he also calculated the probabilities.

They also act as a magnifier

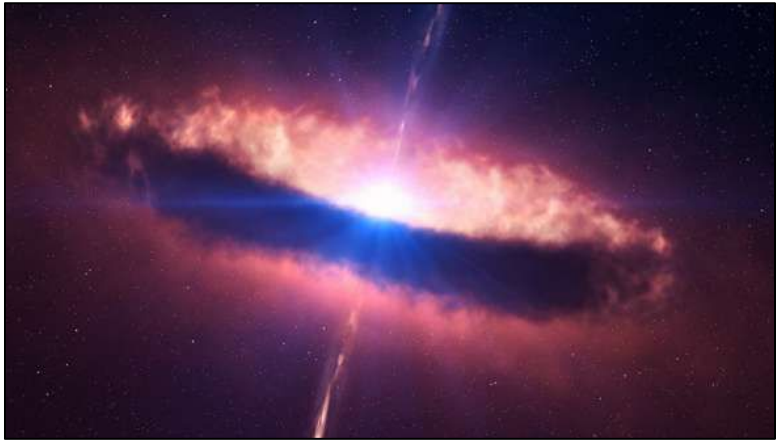
S. Refsdal

❖ Refsdal (1964) : Determination of Hubble constant from gravitational lensing.

Eddington, A.S. 1920, Space, Time and Gravitation (Cambridge: University Press)
Chwolson, O. 1924, Astron. Nachr., 221, 329
Zwicky, F. 1937a, Phys. Rev. Lett., 51, 290
Zwicky, F. 1937b, Phys. Rev. Lett., 51, 679
Refsdal, S. 1964, MNRAS, 128, 307

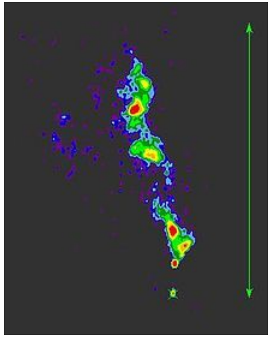
Quasars (Quasi-stellar Radio source)

They found me in **1950's**

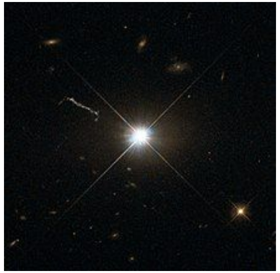


Active **G**alactic **N**ucleus(**AGN**)

It is a **Supermassive Black Hole** feeding the gas at the centre of a distant galaxy.



3C 48(Radio)



3C 273(Optical)

They are the most Luminous, Powerful and Energetic objects known in the universe

Schmidt, M. 1963, Nat, 197, 1040

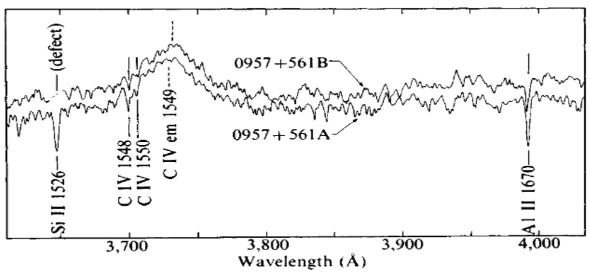
https://commons.wikimedia.org/wiki/File:Radio_map_of_quasar_3C48.jpg

https://esahubble.org/images/potw1346a/

Twin quasars

0957+561 A, B: twin quasistellar objects or gravitational lens?

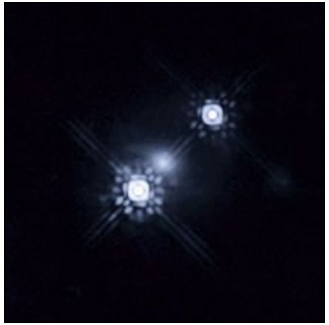
D. Walsh
University of Manchester, Nuffield Radio Astronomy Laboratories, Jodrell Bank, Macclesfield, Cheshire, UK
R. F. Carswell
Institute of Astronomy, Cambridge, UK
R. J. Weymann
Steward Observatory, University of Arizona, Tucson, Arizona 85721



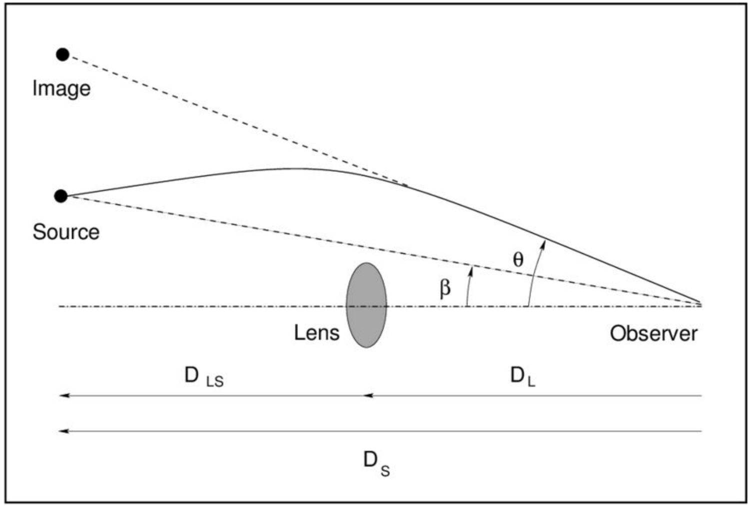
(Separated by 6”)

First gravitationally lensed object!!

Currently there are 220 known lensed quasars



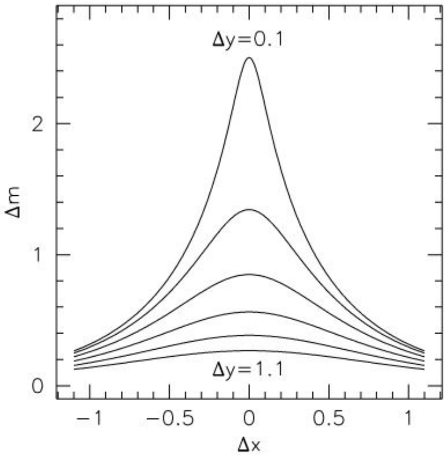
Gravitationally lensed quasar HE 1104-1805



https://research.ast.cam.ac.uk/lensedquasars/index.html
https://esahubble.org/images/heic1116a/

Lensing of Stars by Stars

Paczynski (1986b)



Paczynski curve


Bohdan Paczyński

MACHO, OGLE, EROS, DUO

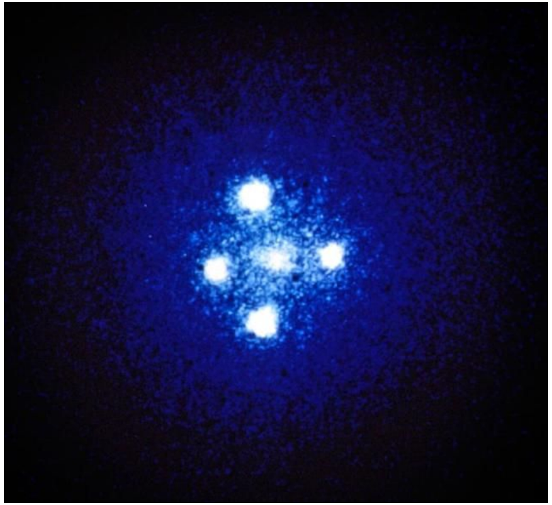
Large Magellanic Cloud

Microlensing of Stars in LMC by the stars in our galactic halo

$$\tau = \begin{cases} (3.3 \pm 1.2) \times 10^{-6} & \text{(Paczyński et al. 1994)} \\ (3.9^{+1.8}_{-1.2}) \times 10^{-6} & \text{(Alcock et al. 1997)} \end{cases}$$

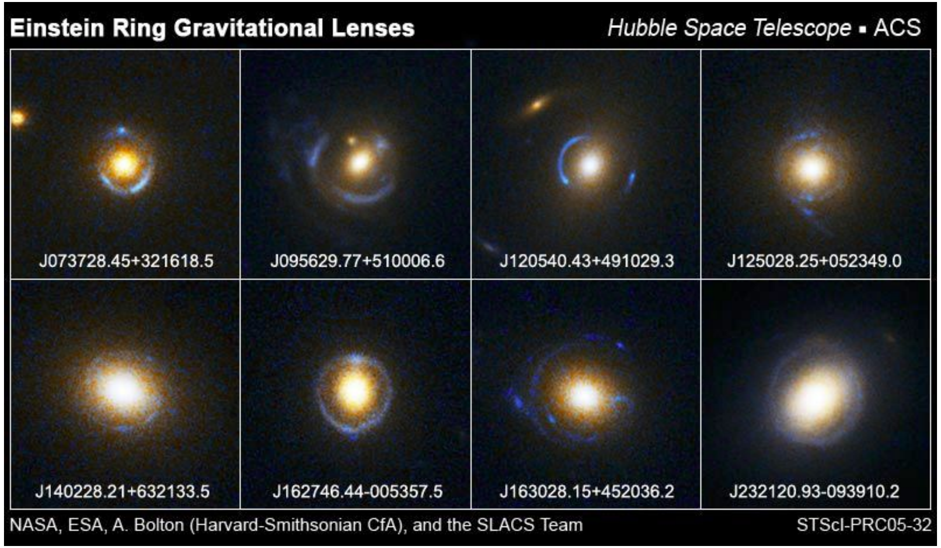
Alcock, C., Allsman, R.A., Alves, D. et al. 1997, ApJ, 479, 119
Paczynski, B. 1986b, ApJ, 304, 1

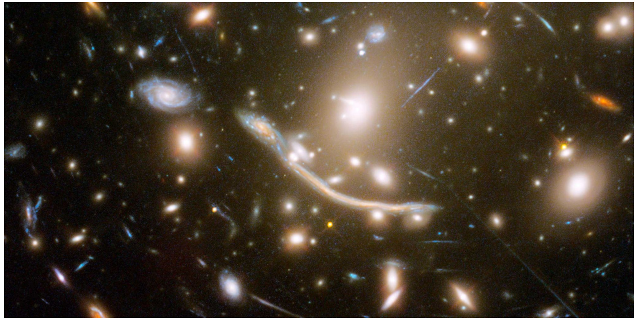
Some events from nature's telescope….



Einstein Cross

- Lenses as gravitational telescopes
- Cosmography
- Dark Matter detection





Abell 370

Image Credit: NASA/ESA

Thin Screen approximation

They again made an approximation

LENS !!!

Lens equation

$$\boxed{\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})}$$

$$\vec{\alpha}(\theta) = \frac{D_{LS}}{D_S}\hat{\alpha}(\vec{\theta})$$

Effective lensing potential

Newtonian Potential

$$\hat{\Psi}(\vec{\theta}) = \frac{D_{LS}}{D_L D_S}\frac{2}{c^2}\int \Phi(D_L\vec{\theta}, z)dz$$

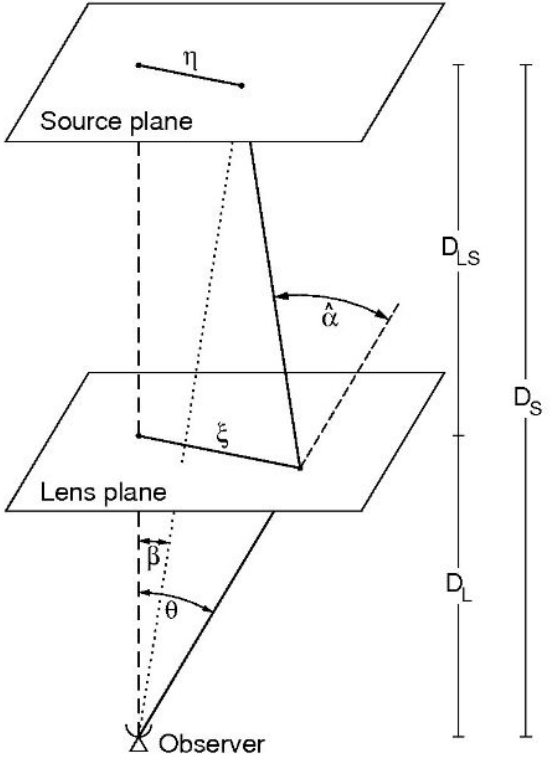
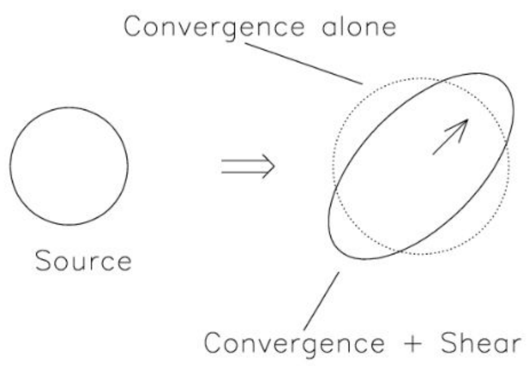


Figure :Introduction to Gravitational Lensing by Massimo Meneghetti

Effective lensing potential

$$\vec{\nabla}_x \Psi(\vec{x}) = \vec{\alpha}(\vec{x})$$

Deflection angle



Magnification and distortion of an image

$$A = \delta_{ij} - \Psi_{ij}$$ **(Hessian Matrix)**

$$\Psi_{ij} \equiv \frac{\partial^2 \Psi(\vec{x})}{\partial x_i \partial x_j}$$

Shear terms

$$A = \begin{pmatrix} 1-\kappa & 0 \\ 0 & 1-\kappa \end{pmatrix} - \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix}$$

Convergence

As a consequence of Liouville Theorem and conservation of photons the source surface brightness is conserved

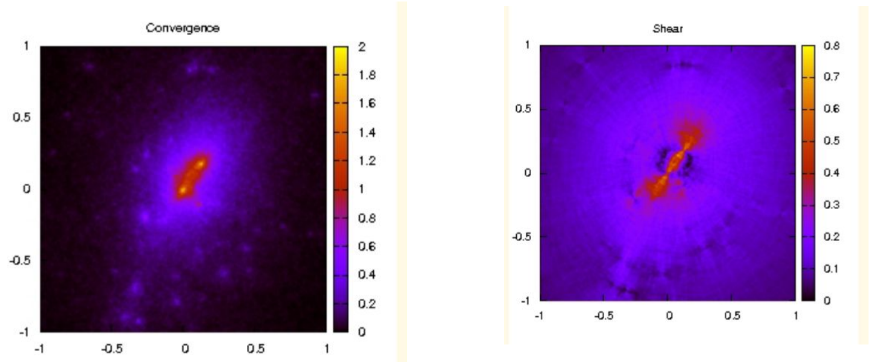
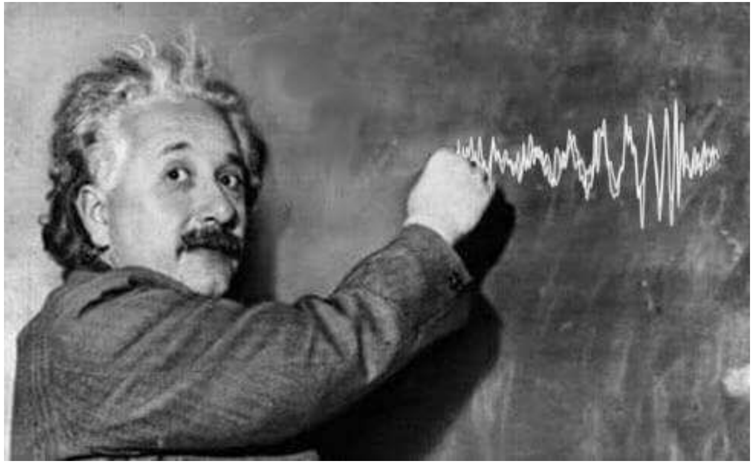
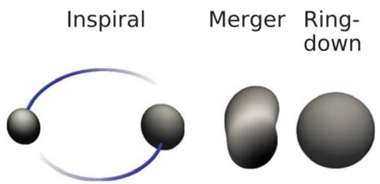


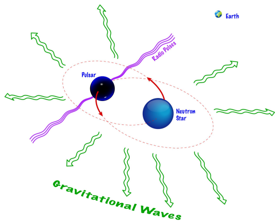
Figure :Introduction to Gravitational Lensing by Massimo Meneghetti, Narayan & Bartelmann, 1995

GRAVITATIONAL WAVES



Gravitational waves are ripples in space





Hulse & Taylor pulsar

The Nobel Prize in Physics 2017

© Nobel Media. Ill. N. Elmehed
Rainer Weiss
Prize share: 1/2

© Nobel Media. Ill. N. Elmehed
Barry C. Barish
Prize share: 1/4

© Nobel Media. Ill. N. Elmehed
Kip S. Thorne
Prize share: 1/4

On September 14, 2015 , first GW was observed by LIGO Science Collaboration **GW150914**

Predicted by Einstein in 1916.................but after many many decades of doubts ......arguments.......................... Chapel-Hill-conference...Feynman-Bondi argument.................Hulse Taylor.......................... Pulsar.....Interferometry.............it was detected in *2015*.

What are gravitational waves ?

Ricci Scalar

Stress energy tensor

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Ricci tensor

A small perturbation

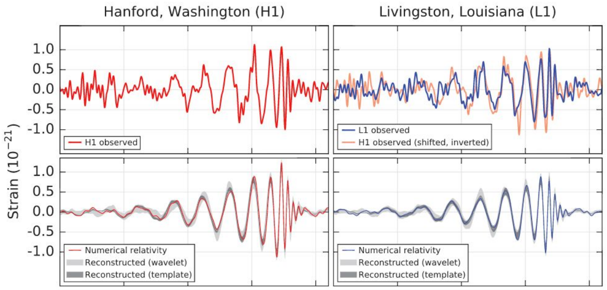
Now you slightly perturb it……

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad |h_{\mu\nu}| << 1$$

(Ripples over a flat background…..)

GRAVITATIONAL WAVE

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

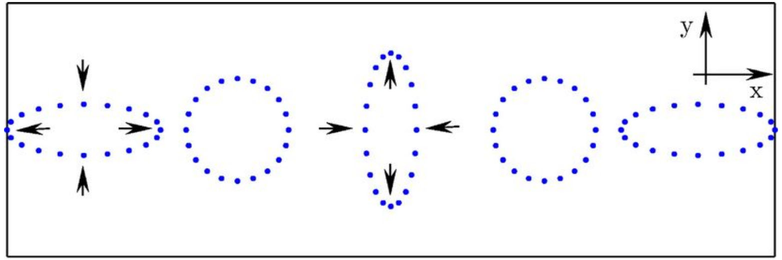


Binaries

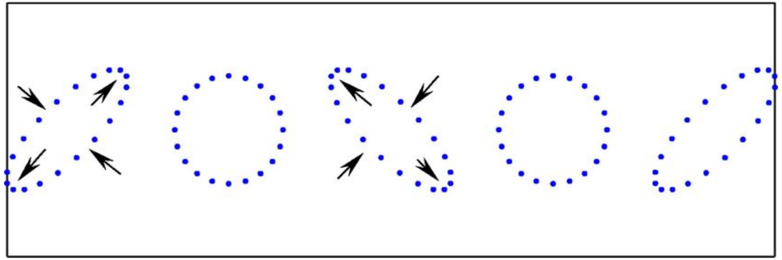
- BH-BH
- BH-NS
- NS-NS

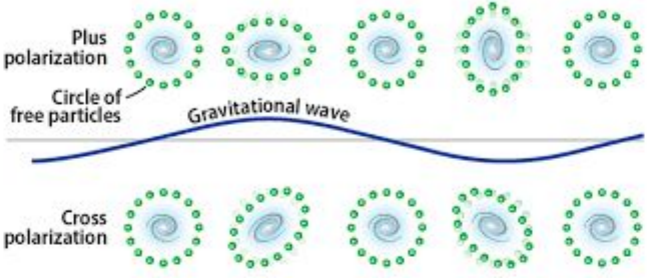
Wave tensor in Transverse Traceless (TT) gauge

$$h_{ij}^{TT}(t,z) = \begin{bmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{bmatrix} Cos[\omega(t - z/c)]$$



Gravitational Waves in General Relativity has ***2*** *polarizations*





From geodesic deviation equation

$$\frac{d^2\eta_i}{dt^2} = -R_{i0j0}\eta^j$$

Components of curvature tensor

Figure : Giles Hammond, Stefan Hild & Matthew Pitkin (2014)
10.1080/09500340.2014.920934

GW Detectors:

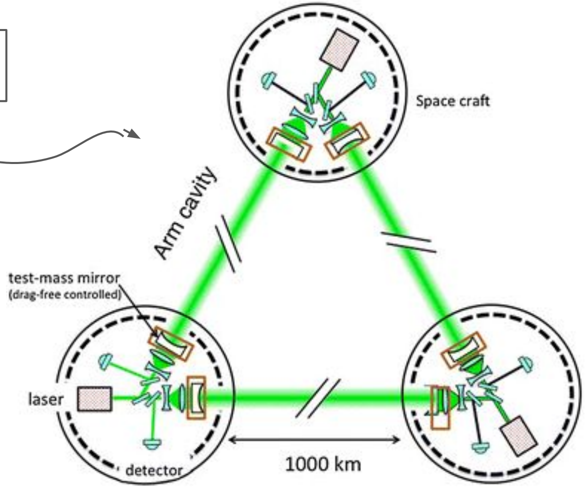
LIGO- Hanford, Livingston
Virgo- Italy

More than
90
detections
until now

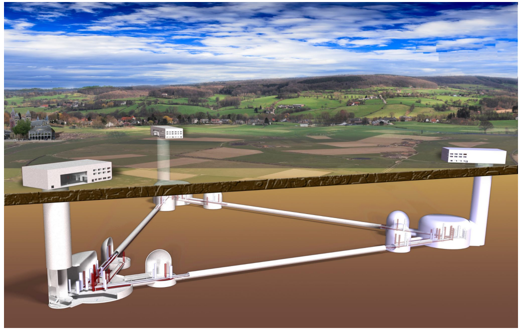
Planned:
Laser Interferometer Space Antenna (LISA)
Deci-hertz Gravitational Wave Observatory (DECIGO)

Einstein Telescope (ET)
Cosmic Explorer (CE)

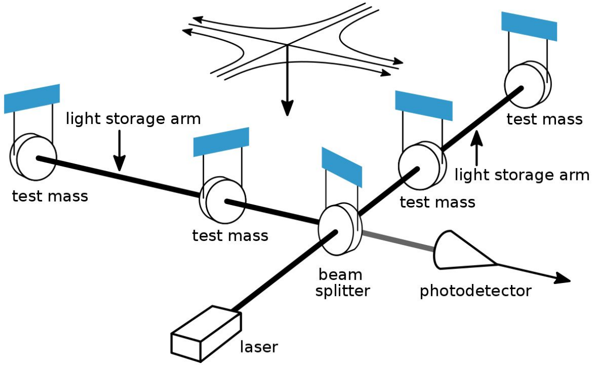
DECIGO



S Kawamura et al 2008 J. Phys.: Conf. Ser. 122 012006



Einstein Telescope (ET)



LIGO & VIRGO

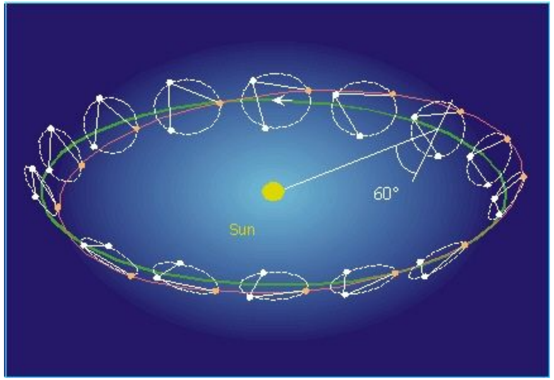
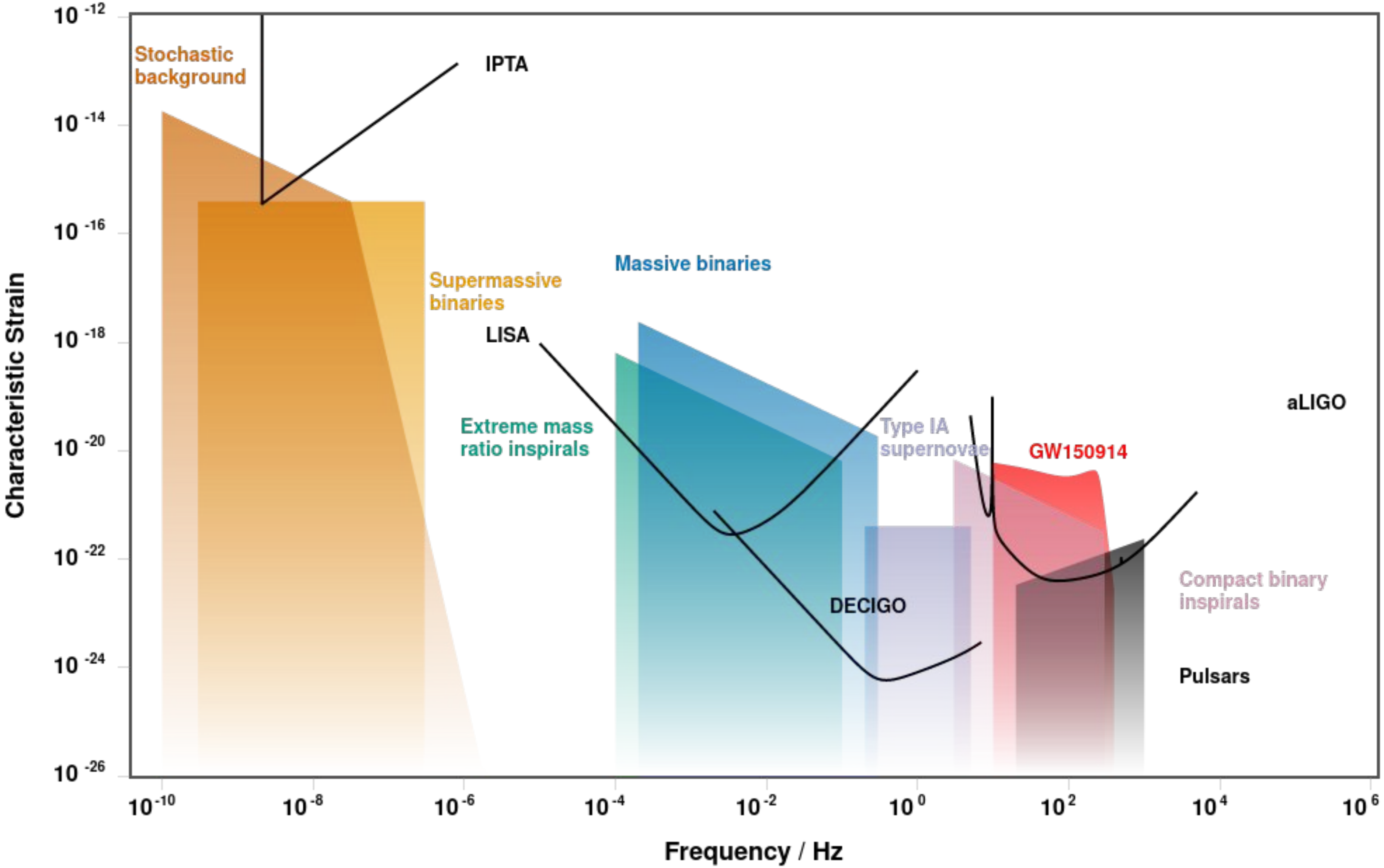


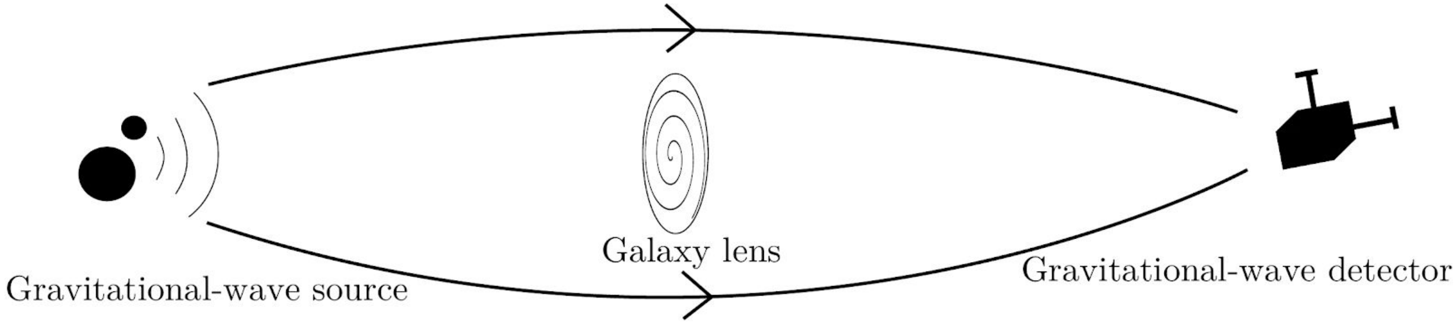
Figure: http://www.et-gw.eu/index.php/etimages

https://en.wikipedia.org/wiki/Gravitational-wave_observatory



http://gwplotter.com/

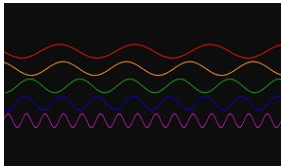
Gravitational Wave Lensing



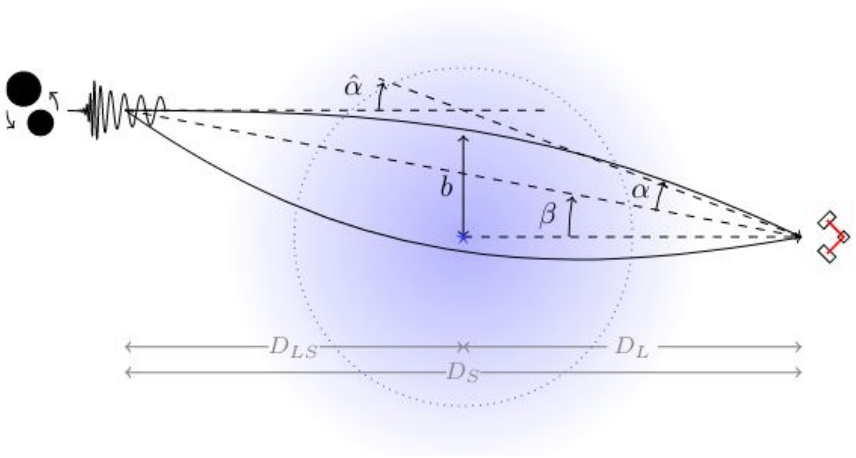
Like EM waves gravitational wave can also be **lensed** !!!!

Marek Biesiada & **Sreekanth Harikumar**, *Universe* **2021**, *7*(12), 502 https://doi.org/10.3390/universe7120502

Frequency dependent amplification



Multiple signals



1. Beat Patterns
2. Faraday rotation
3. Poisson Spot

Comparable to astrophysical objects

$$\lambda_{em} \sim 10^{-8} - 10^{-2} m$$

$$\boxed{\lambda_{gw} \sim 10^{10} - 10\text{km}}$$

This is too big !!!

Fig. Jose María Ezquiaga and Miguel Zumalacárregui PRD 102, 124048 (2020)

Comparable to astrophysical objects

$$\boxed{\lambda_{gw} \sim 10^{10} - 10\text{km}}$$

This is too big !!!

Two different regimes

Geometric Optics limit

$$\lambda_{gw} << r_s = \frac{2GM_L}{c^2}$$

G - Gravitational constant
M_L - Mass of the lens
r_S - Schwarszchild radius

Wave optics limit

$$\lambda_{gw} \geq r_S$$

WAVE OPTICS

$$\lambda_{gw} \geq r_S$$

Polarization information is neglected in the lensing study!!!

Scalar wave

Scalar wave

$$h_{\mu\nu} = \psi e_{\mu\nu}$$

Polarization tensor

Polarization tensor

$$g_{\mu\nu} = g_{\mu\nu}^{(L)} + h_{\mu\nu}$$

Lens background

Lens background

You study a scalar wave in the curved background of lens

$$\partial_\mu \left(\sqrt{-g^{(L)}} g^{(L)\mu\nu} \partial_\nu \psi \right) = 0$$

Solving this......

Diffraction Integral :

$$F(f) = \frac{D_S R_E^2}{D_L D_{LS}} \frac{f(1+z_l)}{i} \int d^2x \exp[2\pi i f t_d(\mathbf{x}, \mathbf{y})]$$

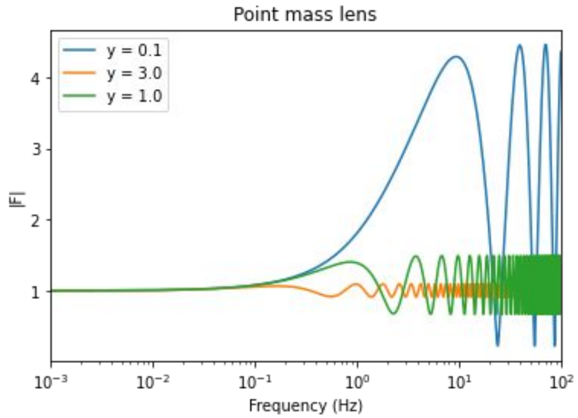
AMPLIFICATION FACTOR

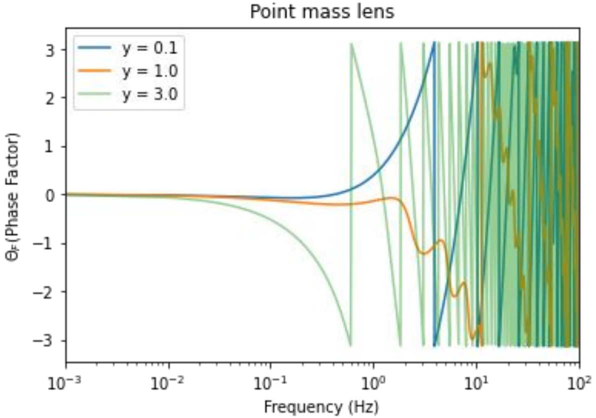
$$F(f) = \frac{\tilde{\psi}^L}{\tilde{\psi}}$$

Diffraction Integral :

$$F(f) = \frac{D_S R_E^2}{D_L D_{LS}} \frac{f(1+z_l)}{i} \int d^2x \exp[2\pi i f t_d(\mathbf{x}, \mathbf{y})]$$

This is how it looks !!!!





y - impact parameter

Time delay

$$t_d(\mathbf{x},\mathbf{y}) = \frac{D_s R_E^2 (1+z_l)}{D_l D_{ls}} \left[\frac{1}{2} |\mathbf{x}\text{-}\mathbf{y}|^2 - \psi(\mathbf{x}) + \phi_m(\mathbf{y}) \right]$$

Information about lens

Credits: Sreekanth H

LENS MODELS

Point mass lens

(Simplest of all lenses.....)

But the solution is complicated.....

$$F(f) = \exp\left[\frac{\pi w}{4} + i\frac{w}{2}\left(ln\left(\frac{w}{2}\right) - 2\phi_m(y)\right)\right]\Gamma\left(1 - \frac{i}{2}w\right)\ {}_1F_1\left(\frac{i}{2}w, 1; wy^2\right)$$

Isolated point sources

$$w = \frac{8\pi G}{c^3}M_l(\theta_E)(1+z_l)f$$

Singular Isothermal Sphere (SIS)

$$F(f) = -iw\ e^{iwy^2/2}\int_0^\infty dx\ xJ_0(wxy)\exp\left[iw\left(\frac{1}{2}x^2 - x + \phi_m(y)\right)\right]$$

Axially-symmetric sources

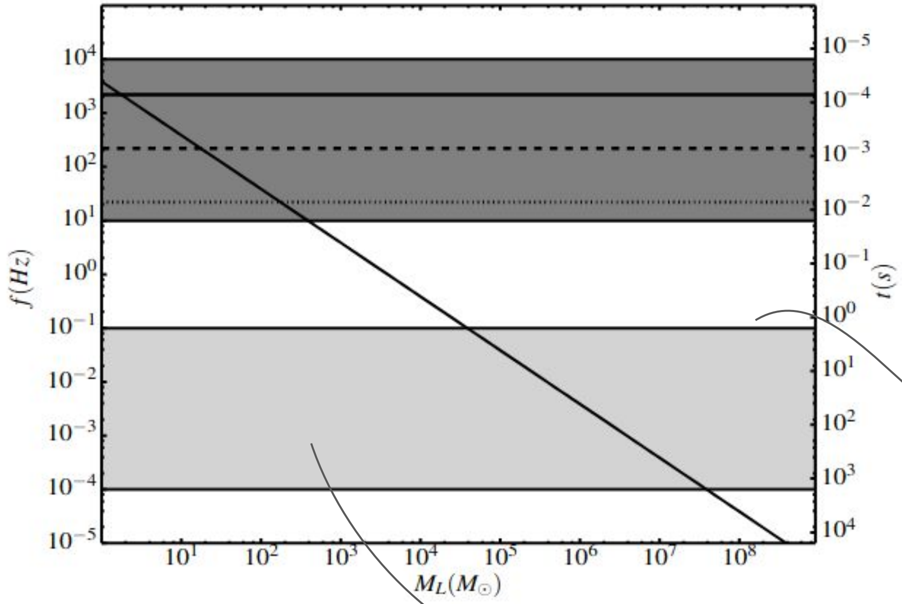
Geometric optics

$$f >> t_d^{-1}$$

For point mass lens....

$$|F(f)|^2 = |\mu_+|^2 + |\mu_-|^2 + 2|\mu_+\mu_-|\sin\left(2\pi f \Delta t_d\right)$$

LIGO band: 10 Hz to 10 kHz



This corresponds to lenses of mass 10 - 1000 Solar masses

Wave optics

Wave optics

Credit: A.K. Meena & J.S.Bagla et. al MNRAS 10.1093/mnras/stz3509

Geometric optics regime

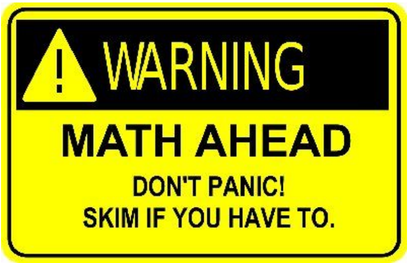
Geometric optics regime

Landscape of modified theories of gravity and lensing

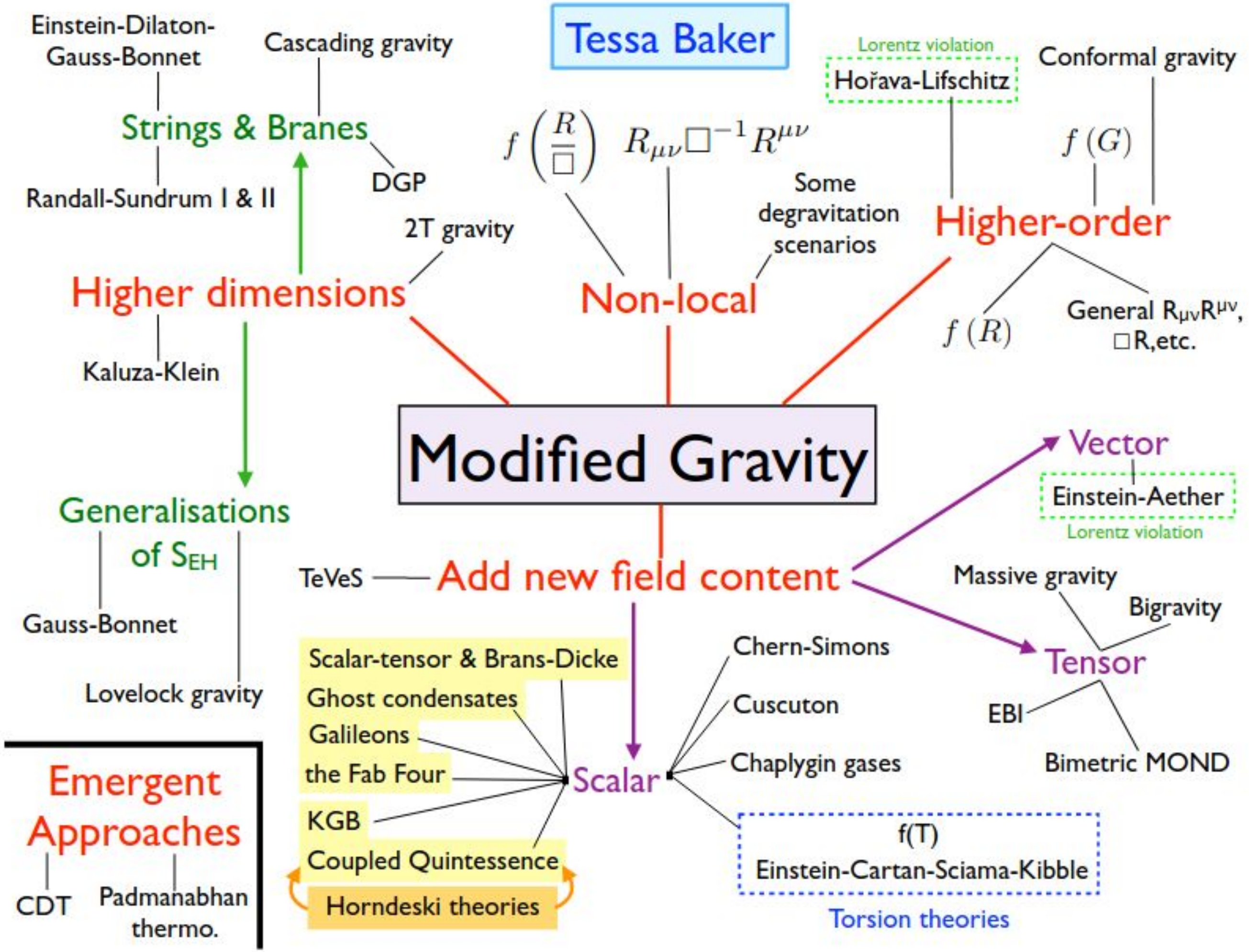
Our work:

Lensing of gravitational waves in Palatini f(R) gravity (in preparation)
Sreekanth Harikumar,Laur Jarv, Margus Saal, Aneta Wojnar,and Marek Biesiada

Dark matter
Dark energy
Singularities
Incompatibility with Quantum Mechanics



Tree diagram of modified theories of gravity



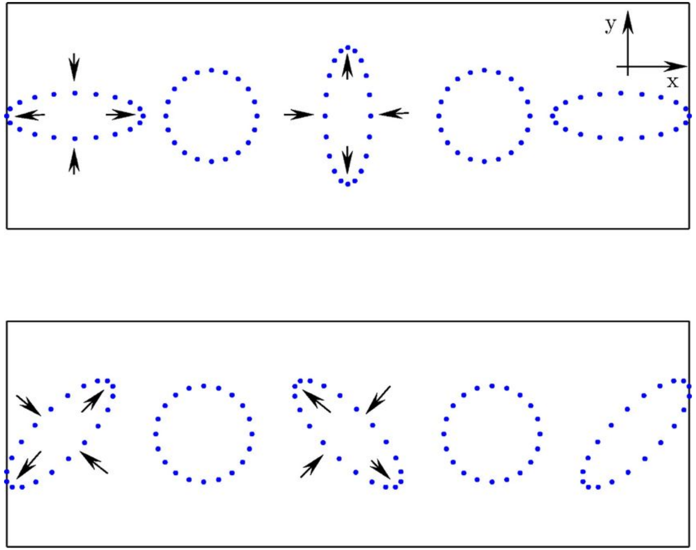
Credit : 1512.05356

Can we distinguish theories based on gravitational waves ?

EXTRA - POLARIZATION

General relativity has 2 polarizations

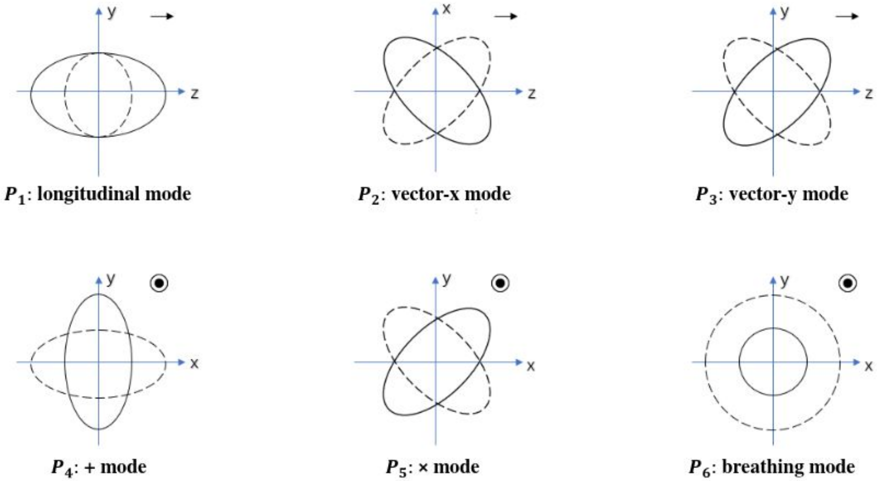
GW's in modified theories of gravity can wave non-tensor polarizations.....



From Geodesic deviation equation

$$\frac{d^2\eta_i}{dt^2} = -R_{i0j0}\eta^j$$

Most general metric theory of gravity can have at-most 6 different polarization.



Components of curvature tensor:

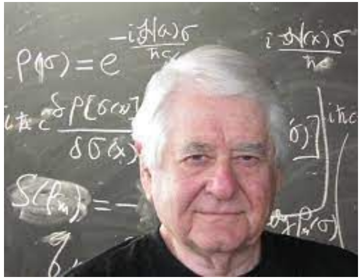
$$R_{i0j0} = \begin{pmatrix} p_4 + p_6 & p_5 & p_2 \\ p_5 & -p_4 + p_6 & p_3 \\ P_2 & p_3 & p_1 \end{pmatrix}$$

Assume the wave to be passing in **z**-direction

Example

Scalar Tensor Vector Gravity (STVG)

Also known as MOG



John.W.Moffat

GW's in this theory has all six polarizations......

Formulated in 2006 as a alternative to Dark Matter

$$\Phi_{eff}\left(\vec{x}\right) = -G_N \int \frac{\rho\left(\mathbf{x}'\right)}{\left|\mathbf{x}-\mathbf{x}'\right|}\left(1+\alpha-\alpha e^{-\mu\left|\mathbf{x}-\mathbf{x}'\right|}\right) d^3\mathbf{x}'.$$

In the weak-field limit the massive vector can mimic dark-matter behaviour.

Moffat's modified gravity tested on X-COP galaxy clusters

Sreekanth Harikumar & Marek Biesiada *The European Physical Journal C* **volume 82**, Article number: 241 (2022)

Moffat MOdified Gravity (MOG)

***Sreekanth Harikumar,** *Universe* **2022**, *8*(5), 259; https://doi.org/10.3390/universe8050259

Palatini $f(\hat{R})$ gravity

Palatini curvature scalar

$$\hat{R} = g^{\mu\nu}\hat{R}_{\mu\nu}(\Gamma)$$

Metric and the connection are independent

Action:

$$S[g,\Gamma,\psi_m] = \frac{1}{2\kappa}\int \sqrt{-g} f(\hat{R}) d^4x + S_{\mathrm{matter}}[g,\psi_m]$$

Variation w.r.t metric

$$f'(\hat{R})\hat{R}_{\mu\nu} - \frac{1}{2}f(\hat{R})g_{\mu\nu} = \kappa T_{\mu\nu}$$

Trace equation:

$$f'(\hat{R})\hat{R} - 2f(\hat{R}) = \kappa T$$

Variation w.r.t independent connection

$$\hat{\nabla}_\beta(\sqrt{-g} f'(\hat{R}(T)) g^{\mu\nu}) = 0$$

Linearised gravitational waves in Palatini f(R) gravity

Perturbation

$$\hat{g}_{\mu\nu} = \hat{g}^{(B)}_{\mu\nu} + \hat{h}_{\mu\nu} \Longrightarrow \boxed{\text{Field Equations}} \Longrightarrow \hat{\nabla}_\alpha \hat{\nabla}^\alpha h_{\mu\nu} - 2\hat{R}^{\tau}_{\rho\mu\nu} h^{\rho}_{\tau} = 0$$

Lens background

Linearised wave equation in curved spacetime

$$\boxed{\hat{g}_{\mu\nu} = f' g_{\mu\nu}}$$

Conformally related to the physical metric

$$\hat{\nabla}_\alpha \hat{\nabla}^\alpha h_{\mu\nu} - 2\hat{R}^{\tau}_{\rho\mu\nu} h^{\rho}_{\tau} = 0$$

Information about lens goes here…..

WKB

approximation…..

WKB approximation.....

$$h_{\mu\nu} = \mathrm{R}\left\{\left[\xi^{(0)}_{\mu\nu} + \epsilon\xi^{(1)}_{\mu\nu} + \epsilon^2\xi^{(2)}_{\mu\nu} + ...\right]e^{i\Phi/\epsilon}\right\}$$

($\epsilon \longrightarrow 0$ defines the geometrics optics limit.)

Linearised wave equation

$$\hat{\nabla}_\alpha\hat{\nabla}^\alpha h_{\mu\nu} - 2\hat{R}^\tau_{\rho\mu\nu}h^\rho_\tau = 0$$

Phase function

wave vector

$$\hat{k}^\mu = \hat{g}^{\mu\nu}\partial_\nu\Phi(x)$$

$$\begin{aligned}\hat{\nabla}^\alpha\hat{\nabla}_\alpha h_{\mu\nu} - 2h_{\alpha\beta}\hat{R}^\alpha{}_{\mu\nu}{}^\beta = e^{i\Phi/\epsilon}\Big\{&\frac{1}{\epsilon^2}[-\hat{k}^\beta\hat{k}_\beta\xi^{(0)}_{\mu\nu}] \\ &+ \frac{1}{\epsilon}[i(\hat{\nabla}_\beta\hat{k}^\beta\xi^{(0)}_{\mu\nu} + \hat{k}^\beta\hat{\nabla}_\beta\xi^{(0)}_{\mu\nu}) - \hat{k}^\beta\hat{k}_\beta\xi^{(1)}_{\mu\nu}] \\ &+ \epsilon^0[\hat{\nabla}_\beta\hat{\nabla}^\beta\xi^{(0)}_{\mu\nu} + i[\hat{\nabla}_\beta\hat{k}^\beta\xi^{(1)}_{\mu\nu} + \hat{k}^\beta\hat{\nabla}_\beta\xi^{(1)}_{\mu\nu} \\ &+ \hat{k}^\beta\hat{\nabla}_\beta\xi^{(1)}_{\mu\nu}]\Big\} - 2h_{\alpha\beta}\hat{R}^\alpha{}_{\mu\nu}{}^\beta = 0.\end{aligned}$$

$$\frac{1}{\epsilon^2} \longrightarrow \text{leading order}$$

$$\frac{1}{\epsilon}, \longrightarrow \text{next to leading order}$$

$$\epsilon^0, \epsilon^1 \longrightarrow \text{leading order}$$

Geometric optics limit....

Let us consider the leading order terms

$$\frac{1}{\epsilon^2}[-\hat{k}^\beta \hat{k}_\beta \xi^{(0)}_{\mu\nu}] + \frac{1}{\epsilon}[i(\hat{\nabla}_\beta \hat{k}^\beta \xi^{(0)}_{\mu\nu} + \hat{k}^\beta \hat{\nabla}_\beta \xi^{(0)}_{\mu\nu}) - \hat{k}^\beta \hat{k}_\beta \xi^{(1)}_{\mu\nu}]$$

Leading term

Next to Leading term

Leading order

$$k_\beta k^\beta = 0$$

Taking the covariant derivative w.r.t the independent connection gives us geodesic equation

$$\hat{k}^\mu \hat{\nabla}_\mu \hat{k}_\nu = 0$$

Expanding...

$$\frac{dx^\beta}{d\hat{\lambda}^2} + \hat{\Gamma}^\beta_{\alpha\mu} \frac{dx^\alpha}{d\hat{\lambda}} \frac{dx^\mu}{d\hat{\lambda}} = 0.$$

$$k^\mu \nabla_\mu k_\nu = 0$$

These are not geodesics but autoparallel curves...

Phase function

wave vector

$$\hat{k}^\mu = \hat{g}^{\mu\nu} \partial_\nu \Phi(x)$$

But coincides with geodesics

Conformal Invariance

Next to leading order….

$$\boxed{2k^{\alpha}\hat{\nabla}_{\alpha}\xi^{(0)}_{\mu\nu} + \hat{\nabla}_{\alpha}\hat{k}^{\alpha}\xi^{(0)}_{\mu\nu} = 0}$$

$$\underbrace{\frac{1}{\epsilon^2}[-\hat{k}^{\beta}\hat{k}_{\beta}\xi^{(0)}_{\mu\nu}]}_{\text{Leading term}} + \underbrace{\frac{1}{\epsilon}[i(\hat{\nabla}_{\beta}\hat{k}^{\beta}\xi^{(0)}_{\mu\nu} + \hat{k}^{\beta}\hat{\nabla}_{\beta}\xi^{(0)}_{\mu\nu}) - \hat{k}^{\beta}\hat{k}_{\beta}\xi^{(1)}_{\mu\nu}]}_{\text{Next to Leading term}}$$

Decompose the wave tensor

$$\xi^{(0)}_{\mu\nu} = \mathcal{A}\mathcal{A}_{\mu\nu} \longrightarrow \text{Polarization tensor}$$

Polarization tensor

Amplitude

Amplitude

$$\mathcal{A} = \sqrt{\xi^{*}_{\mu\nu}\xi^{*\mu\nu}}$$

Two important information

Conservation of graviton number density along the ray bundle

$$\hat{\nabla}_{\mu}(\hat{k}^{\mu}\mathcal{A}^2) = 0$$

$$\hat{P} = \hbar\hat{k}^{\mu}$$

$$\hat{\nabla}_{\mu}\hat{N} = 0$$

Polarization tensor is parallel propagated along the direction of wave vector

$$\hat{k}^{\alpha}\hat{\nabla}_{\alpha}\mathcal{A}_{\mu\nu} = 0$$

$$\xi_{\mu\nu} = \mathcal{C}_{kk}\Theta^{kk}_{\mu\nu} + \mathcal{C}_{ll}\Theta^{ll}_{\mu\nu} + \mathcal{C}_{mm}\Theta^{mm}_{\mu\nu} + \mathcal{C}_{nn}\Theta^{nn}_{\mu\nu} + \mathcal{C}_{kl}\Theta^{kl}_{\mu\nu} + \mathcal{C}_{km}\Theta^{km}_{\mu\nu} + \mathcal{C}_{kn}\Theta^{kn}_{\mu\nu} + \mathcal{C}_{ml}\Theta^{ml}_{\mu\nu} + \mathcal{C}_{nl}\Theta^{ml}_{\mu\nu} + \mathcal{C}_{mn}\Theta^{mn}_{\mu\nu}$$

$$\Theta^{AB}_{\mu\nu} = \frac{1}{2}(A_\mu B_\nu + A_\nu B_\mu) \qquad A, B = \{k^\mu, m^\mu, l^\mu, n^\mu\}$$

Evolution of GW amplitude:

$$2\hat{k}^\mu \hat{\nabla}_\mu \mathcal{C}_{AB} + \hat{\nabla}_\mu \hat{k}^\mu \mathcal{C}_{AB} = 0$$

$$\Downarrow$$

$$\mathcal{C}^{\text{Palatini}}_{AB} = \mathcal{C}^{\text{GR}}_{AB}(\lambda) \exp\Big[-\int_{\lambda_s}^{\lambda_o} \Big(2\frac{d\ln f'}{d\lambda} + \frac{1}{2} f' \frac{d\ln f'}{d\lambda} \Big) d\lambda \Big]$$

$f'(T) \Longrightarrow$ *Depends on the lens*

General Relativity

$$2k^\mu \nabla_\mu \mathcal{C}_{AB} + \nabla_\mu k^\mu \mathcal{C}_{AB} = 0$$

$$\mathcal{C}^{\text{GR}}_{AB}(\lambda) = \frac{\mathcal{C}_{AB}(\lambda_s) D(\lambda_s)}{D(\lambda)}$$

Conclusion:

- Evolution of GW amplitude is theory dependent.

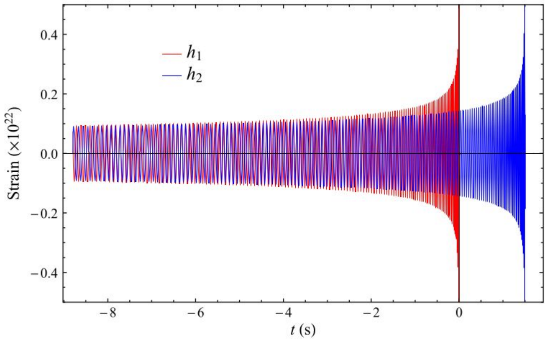
- In Palatini f (R) no extra polarizations arise in Geometric Optics limit

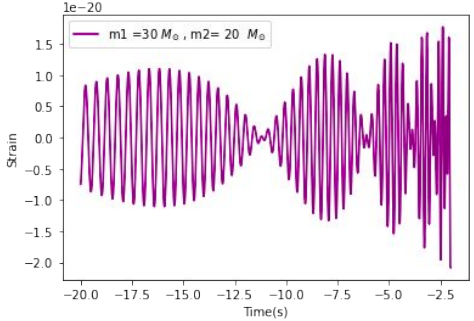
- Vector and scalar polarizations arise in beyond geometric optics along with the effects of Palatini f(R)
- The gravitational potential of structures in the line of sight depends on the theory of gravity and hence GW lensing

- Hence lensing of GW's can be used to constrain theories of gravity.

Thank you

Some interesting applications...





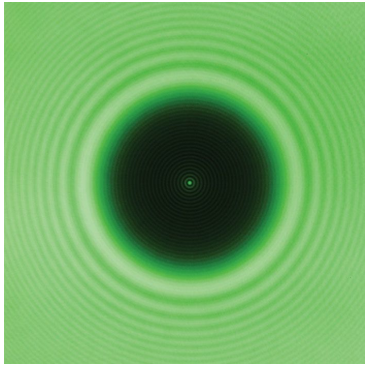
Beat frequency

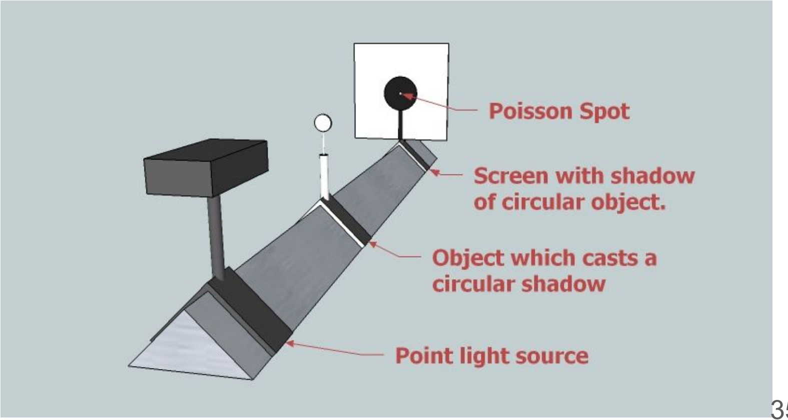
$$\omega_b = \frac{96}{5}\left(\frac{\omega_f}{2}\right)^{11/3} \mathcal{M}\Delta t$$

$$\omega_b = \frac{\omega_1 - \omega_2}{2} \qquad \omega_f = \frac{\omega_1 + \omega_2}{2}$$

Poisson- Arago spot

$$|F(w, \beta = 0)|^2 = \frac{\pi w}{1 - e^{-\pi w}}$$





A.2

Gravitational Waves in General Relativity

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad |h_{\mu\nu}| << 1$$

$$R_{\mu\nu\rho\sigma} = \frac{1}{2}(h_{\mu\sigma,\rho\nu} + h_{\nu\rho,\sigma\mu} - h_{\nu\sigma,\rho\mu} - h_{\mu\rho,\sigma\nu})$$

Trace Reversed tensor

$$\bar{h}^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} h$$

***Lorentz gauge condition** $\quad \bar{h}^{\mu\nu}_{\ ,\nu} = 0$

Linearised EFE

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

Gauge transformation:

$$x^\mu \longrightarrow x'^\mu + \xi^\mu(x)$$

$$\bar{h}_{\mu\nu} \longrightarrow \bar{h}'_{\mu\nu} = \bar{h}_{\mu\nu} - (\partial_\mu \xi_\nu + \partial_\nu \xi_\mu - \eta_{\mu\nu}\partial_\rho \xi^\rho)$$

$$\partial^\nu \bar{h}_{\mu\nu} \longrightarrow (\partial^\nu \bar{h}_{\mu\nu})' = \partial^\nu \bar{h}_{\mu\nu} - \Box \xi_\mu$$

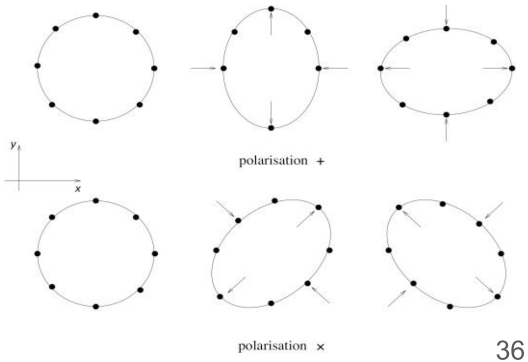
$$h^{0\mu} = 0 \ \ ; h^i_i = 0 \ \ ; \partial^j h_{ij} = 0$$

Plane monochromatic waves: $\quad \Box h_{\mu\nu} = 0$

$$\bar{h}_{\mu\nu}(x^\alpha) = A_{\mu\nu} \exp(i k_\alpha x^\alpha)$$

$$\eta_{\mu\nu} k^\mu k^\nu = 0$$

$$A^{\mu\nu} k_\nu = 0$$



Detection of Compact Dark Matter

$$\tilde{h}^L(f) = F(f)\tilde{h}(f)$$

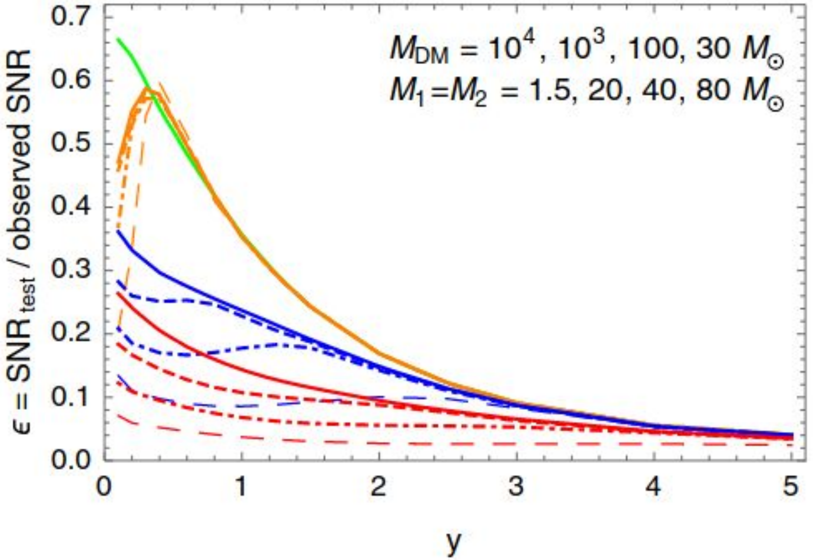
In freq.domain

$$\rho_{test}^2 \equiv 4\int_{f_0}^{f_i} \frac{|\tilde{h}^L(f) - \tilde{h}(f)|}{S_n(f)} df$$

$$\rho^2 \equiv 4\int_{f_0}^{f_i} \frac{|\tilde{h}^L(f)|^2}{S_n(f)} df$$

Lensing detection efficiency

$$\epsilon \equiv \frac{\rho_{\text{test}}}{\rho}$$



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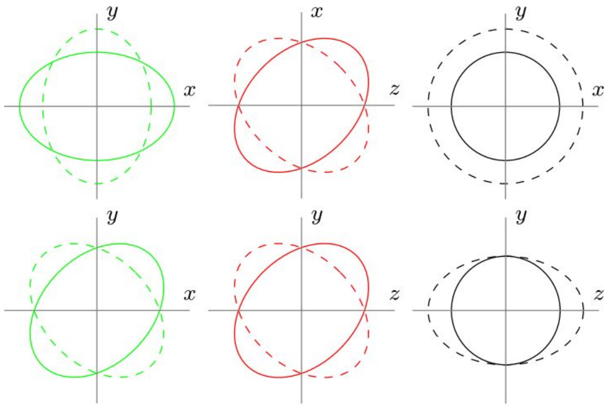
Gravitational-Wave Fringes at LIGO: Detecting Compact Dark Matter by Gravitational Lensing

Sunghoon Jung[1,*] and Chang Sub Shin[2]
[1]*Center for Theoretical Physics, Department of Physics and Astronomy, Seoul National University, Seoul 08826, Korea*
[2]*Center for Theoretical Physics of the Universe, IBS, Daejeon 34051, Korea*

A.3

Newman Penrose Formalism (NP)



$$k^\mu = \frac{1}{\sqrt{2}}(1,0,0,1); l^\mu = \frac{1}{\sqrt{2}}(1,0,0,-1)$$

$$m^\mu = \frac{1}{\sqrt{2}}(0,1,i,-0); \bar{m}^\mu = \frac{1}{\sqrt{2}}(0,1,-i,0)$$

Newman Penrose Variables

$$\Psi_2 = -\frac{1}{6}R_{klkl}$$

$$\Psi_3 = -\frac{1}{2}R_{kl\bar{m}l}$$

$$\Psi_4 = -R_{\bar{m}l\bar{m}l}$$

$$\Phi_{22} = -R_{ml\bar{m}l}$$

BD Theory

$$R^{\mathrm{BD}}_{tjtk} = \begin{pmatrix} -\frac{1}{2}(\mathcal{R}\Psi_4 + \Phi_{22}) & \frac{1}{2}\mathcal{I}\Psi_4 & 0 \\ \frac{1}{2}\mathcal{I}\Psi_4 & \frac{1}{2}(\mathcal{R}\Psi_4 - \Phi_{22}) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

D.M. Eardley, D.L. Lee, A.P. Lightman, Gravitational-wave observations as a tool for testing relativistic gravity. Phys. Rev. D **8**, 3308–3321 (1973)

D.M. Eardley, D.L. Lee, A.P. Lightman, R.V. Wagoner, C.M. Will, Gravitational-wave observations as a tool for testing relativistic gravity. Phys. Rev. Lett. **30**, 884–886 (1973)

Transformation properties under E(2) Little group

For massless particles…..

$$\Psi_2' = \Psi_2$$

$$\Psi_3' = e^{-i\theta}(\Psi_3 + 3\bar{\rho}\Psi_2)$$

$$\Psi_4' = e^{-i2\theta}(\Psi_4 + 4\bar{\rho}\Psi_3 + 6\bar{\rho}^2\Psi_2)$$

$$\Phi_{22}' = \Phi_{22} + 2\rho\Psi_3 + 2\bar{\rho}\bar{\Psi}_3 + 6\bar{\rho}\rho\Psi_2$$

Petrov Classification

Class II_6 $\Psi_2 \neq 0$. All observers measure the same nonzero amplitude of the Ψ_2 mode, but the presence or absence of all other modes is observer-dependent.

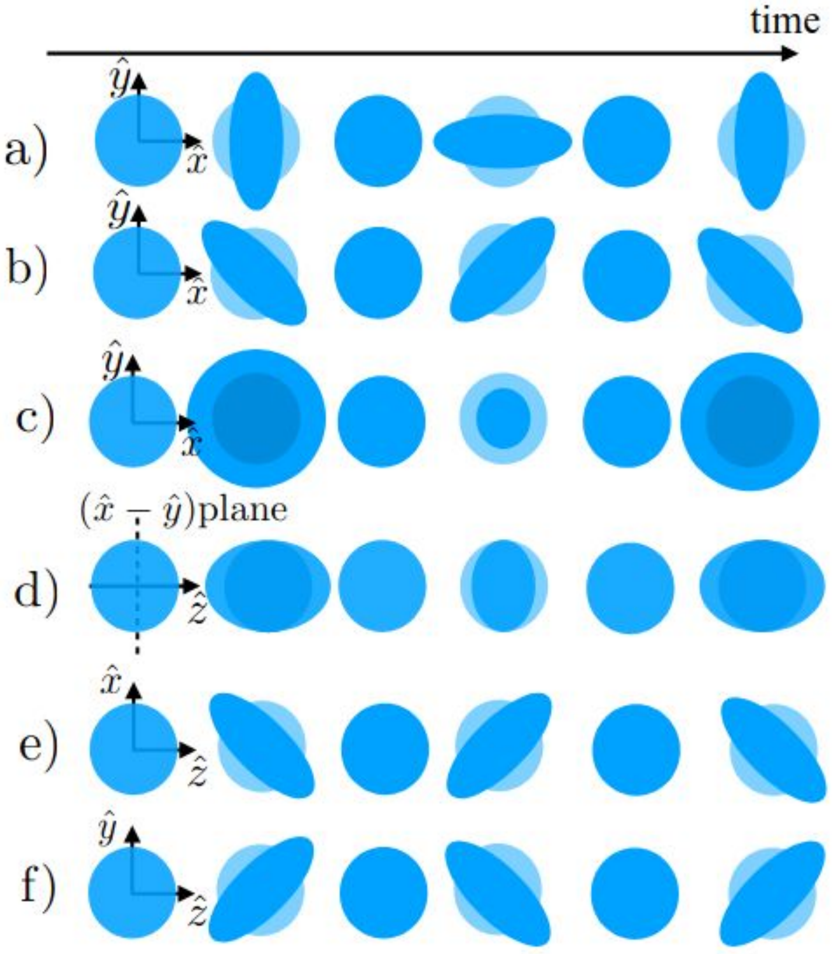
Class III_5 $\Psi_2 = 0$, $\Psi_3 \neq 0$. All observers measure the absence of the Ψ_2 mode and the presence of the Ψ_3 mode, but the presence or absence of Ψ_4 and Φ_{22} is observer-dependent.

Class N_3 $\Psi_2 = \Psi_3 = 0$, $\Psi_4 \neq 0 \neq \Phi_{22}$. The presence or absence of all modes is observer-independent.

Class N_2 $\Psi_2 = \Psi_3 = \Phi_{22} = 0$, $\Psi_4 \neq 0$. The presence or absence of all modes is observer-independent.

Class O_1 $\Psi_2 = \Psi_3 = \Psi_4 = 0$, $\Phi_{22} \neq 0$. The presence or absence of all modes is observer-independent.

Class O_0 $\Psi_2 = \Psi_3 = \Psi_4 = \Phi_{22} = 0$. No wave is observed.



A. Plus
B. Cross
C. Breathing
D. Longitudinal Scalar
E. Vector -x
F. Vector -y

The curvature of spacetime induces distortions in the polarization of the wave such that diffraction effects may be misinterpreted as effective scalar and vector polarizations.

The scalar and vector polarizations in some scenario could be unphysical.

Lorentz boost can remove vector polarization but not scalar ones

A.4

Scalar Tensor Representation

$$\hat{G}_{\mu\nu} = \frac{1}{f'}[\kappa T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} U(f')]$$

$$\hat{R}_{\mu\nu} - \frac{1}{2}\hat{g}_{\mu\nu}\hat{R} = 0$$

$$\hat{g}_{\mu\nu} = \hat{g}^{(B)}_{\mu\nu} + \hat{h}_{\mu\nu}$$

Linearzed Field Equation:

$$\boxed{\hat{\nabla}_\alpha \hat{\nabla}^\alpha h_{\mu\nu} - 2\hat{R}^\tau_{\rho\mu\nu} h^\rho_\tau = 0}$$

$$\hat{h}_{\mu\nu} = h_{\mu\nu}$$ (In the absence of an-isotropic stress !!)

Eikonal Expansion

$$\hat{\nabla}^\alpha \hat{\nabla}_\alpha h_{\mu\nu} - 2h_{\alpha\beta}\hat{R}^\alpha{}_{\mu\nu}{}^\beta = e^{i\Phi/\epsilon}\Big\{ \frac{1}{\epsilon^2}[-k^\beta k_\beta \xi^{(0)}_{\mu\nu}] + \frac{1}{\epsilon}[i(\hat{\nabla}_\beta k^\beta \xi^{(0)}_{\mu\nu} + k^\beta \hat{\nabla}_\beta \xi^{(0)}_{\mu\nu}) - k^\beta k_\beta \xi^{(1)}_{\mu\nu}] + \epsilon^0[\hat{\nabla}_\beta \hat{\nabla}^\beta \xi^{(0)}_{\mu\nu} + i[\hat{\nabla}_\beta k^\beta \xi^{(1)}_{\mu\nu} + k^\beta \hat{\nabla}_\beta \xi^{(1)}_{\mu\nu} + k^\beta \hat{\nabla}_\beta \xi^{(1)}_{\mu\nu}]\Big\} - 2h_{\alpha\beta}\hat{R}^\alpha{}_{\mu\nu}{}^\beta$$

$$\hat{\Gamma}^\alpha_{\mu\nu} = \overset{B\alpha}{\Gamma}_{\mu\nu} + \frac{1}{2} g^{(B)\alpha\rho}[\hat{\nabla}_\mu h_{\rho\nu} + \hat{\nabla}_\nu h_{\rho\mu} - \hat{\nabla}_\rho h_{\mu\nu}] + \frac{1}{2f'}\left[\delta^\alpha_\nu \partial_\mu f' + \delta^\alpha_\mu \partial_\nu f' - g^{(B)}_{\mu\nu}\partial^\alpha f' - h_{\mu\nu}\partial^\alpha f' + g^{(B)}_{\mu\nu} h^{\alpha\beta}\partial_\beta f'\right]$$

$$\boxed{\hat{\nabla}_\alpha \hat{\nabla}^\alpha h_{\mu\nu} - 2\hat{R}^\tau_{\rho\mu\nu} h^\rho_\tau = 0}$$

Leading order:

Eikonal equation

$$\overset{\mathrm{B}}{\hat{g}}_{\mu\nu} \hat{k}^\mu \hat{k}^\nu = 0$$

Geodesic equation

$$\boxed{\hat{k}^\mu \hat{\nabla}_\mu \hat{k}_\nu = 0}$$

Null geodesics enjoy conformal invariance

where $\hat{k}^\mu = \frac{dx^\mu}{d\hat{\lambda}}$ and $d\hat{\lambda} = f' d\lambda$

$$\begin{aligned}
\hat{k}^\nu \hat{\nabla}_\nu \hat{k}^\mu &= \hat{k}^\nu \partial_\nu \hat{k}^\mu + \hat{\Gamma}^\mu_{\nu\alpha} \hat{k}^\nu \hat{k}^\alpha \\
&= \frac{1}{f'} k^\nu \partial_\nu \left(\frac{k^\mu}{f'}\right) + [\Gamma^\mu_{\nu\alpha} + \frac{1}{2f'^2}(\delta^\mu_\nu \partial_\alpha \ln f' + \frac{1}{2}\delta^\mu_\alpha \partial_\nu \ln f' - g_{\nu\alpha}\partial^\mu \ln f')] k^\nu k^\alpha \\
&= \frac{1}{f'^2}\{k^\nu \partial_\nu k^\mu + \Gamma^\mu_{\nu\alpha} k^\nu k^\alpha\} + \frac{1}{f'^2} k^\mu k^\alpha \partial_\alpha \ln f' - \frac{1}{f'^2} k^\mu k^\alpha \partial_\alpha \ln f' \\
&= \frac{1}{f'^2}\{k^\nu \nabla_\nu k^\mu\} = 0
\end{aligned}$$