### Studying CPT with Neutral Mesons -Standard Model Extension Approach (SME)

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### Outline

- 1. Basic concepts (mesons?, discrete sym. ?).
- 2. Neutral meson oscillations phenomenological description.
- 3. CPT, Lorentz invariance and SME.
- 4. SME description of CPTV in meson mixing.
- 5. Analysis steps of CPTV measurement in charm.

### **Discrete Symmetries**

- C:  $\hat{C} | c, \vec{p}, t \rangle = e^{i\alpha_c} | \bar{c}, \vec{p}, t \rangle$ , charge conjugation transformation
- P:  $\hat{P} | c, \vec{p}, t \rangle = e^{i\alpha_P} | c, -\vec{p}, t \rangle$ , parity transformation
- $T: \hat{T} | c, \vec{p}, t \rangle = e^{i\alpha_T} \langle c, \vec{p}, -t |$ , time reversal transformation where  $e^{i\alpha_C}$ ,  $e^{i\alpha_P}$ ,  $e^{i\alpha_T}$  are phase factors
- when [H, X] = 0 then X is conserved
- CPT symmetry represented by  $\hat{C}\hat{P}\hat{T}$
- QFT+(Locality, Unitarity, Lorentz Invariance)  $\Rightarrow$  CPT invariant

#### **CPT** theorem

• CPT theorem states that local quantum field theories with Lorentz symmetry and hermitian Hamiltonian also have CPT symmetry [arXiv:1204.4674, R. Jost, Hel. Phys. Acta, 30:409–416, 1957.].

• Greenberg: An interacting theory that violates CPT invariance necessarily violates Lorentz invariance." [PRL 89 (2002) 231602]

Hence: We can study CPT with Lorentz Invariance

#### **Neutral Mesons**

1. meson = quark + antiquark:



2. neutral meson => charge = 0;

3. Flavour is not conserved in weak interactions.

4. This enables oscillations.

Flavour roughly tells us how massive our particles are. Heavy quark content.

Neutral meson $Q = 0$	Flavor F
K <sup>0</sup>	Strangeness
ds	S = 1
D <sup>0</sup>	Charm
cū	C = 1
<b>B</b> <sup>0</sup>	Beauty
db	B = 1
sb	B = 1, S = -1

Credits: Agnes Roberts, Presentation IUCSS CPT 21

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### **Neutral Meson Oscillations**

- In the SM full Hamiltonian can be written as:  $H = H_{st} + H_{em} + H_{wk}$
- We can treat weak interactions as a perturbation to strong Hamiltonian.
- Flavour states e.g.  $|P^0>, |\bar{P}^0>$  are not eigenvectors of the full Hamiltonian.
- Flavour states are eigenstates of the strong Hamiltonian.
- Due to neutral el. charge  $H_{em} = 0$ .
- $H = H_{st} + H_{wk}$

https://arxiv.org/pdf/hep-ph/0104120.pdf

#### Weisskopf-Wiger Approx. (WWA)

What happens to our Hamiltonian H?

WWA=>We get an effective 2x2 Hamiltonian H<sup>eff</sup>.

- We assume times longer than strong interaction scale.
- Space resticted to 2-dim  $P^0 \overline{P}^0$ .
- Eff. Hamiltonian is non-hermitian (possiblity of decays).
- Eff. Hamiltonian is time-independent.

V. Weisskopf and E. P. Wigner. Z. Phys. 63, 54 (1930); Z. Phys. 65, 18 (1930).

We can express the eff. Hamiltonian as a sum of to 2x2 hermitian matrices **M** and **C** (mass and decay matrix respectively).

### Neutral Meson Math. Description

Neutral meson system in the ( $P^0$ ,  $\overline{P}^0$ ) basis can be described by the 2x2 effective hamiltonian  $H^{eff}$ :

$$H^{eff} = \begin{bmatrix} \begin{pmatrix} M_{11} & M_{12} \\ M_{21}^* & M_{22} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21}^* & \Gamma_{22} \end{pmatrix} \end{bmatrix}$$
**CPT**

$$(M_{11} - M_{22}) - \frac{i}{2} (\Gamma_{11} - \Gamma_{22}) = 0 \iff M_{11} = M_{22}; \Gamma_{11} = \Gamma_{22}$$
**T**

$$\frac{|H_{12}|}{|H_{21}|} = 1 \iff H_{12} = e^{2i\eta_{CP}} H_{21}$$
**CP**
Conserved when both of the above are fulfilled.

https://arxiv.org/pdf/hep-ph/0104120.pdf 15.12.2022

С

#### Let's find H<sup>eff</sup> eigenvalues

Eigen vectors of the eff. Hamil.

$$|P_H>, |P_L>$$

mass eigenstates

Interpretation of the coefficients will come later!

$$|P_L \rangle = c_1 |P^0 \rangle + c_2 |\bar{P}^0 \rangle |P_H \rangle = d_1 |P^0 \rangle - d_2 |\bar{P}^0 \rangle$$

Mass eigenstates are linear combinations of flavour eigenstates!

$$H^{eff}|P_{H/L} >= \left(M - \frac{i}{2}\Gamma\right)|P_{H/L} >= \left(m_{H/L} - \frac{i}{2}\Gamma_{H/L}\right)|P_{H/L} >$$



# Let's try to visualise how these oscillations occur.

#### **Neutral Meson Oscillations**

#### $H^{eff}$

described by 2x2 effective hamiltonian – accuracy up to the second order in weak interactions

D0

PO



mass eigenstates  $\rightarrow$  propagation

 $P^0 - \bar{P}^0$  flavour eigenstates

 $\rightarrow$  production, decay



Credits: Fernando Martinez Vidal arXiv:0910.5061

Fig. 1. SM processes contributing to  $D^0-\overline{D}^0$  mixing: (Left) Short-range box diagram and (Right) long-range interactions with intermediate states.



#### Oscillations – diff. species

 $Pr[P^{0} \rightarrow P^{0}] \sim e^{-\Gamma t} (cosh(y\Gamma t) + cos(x\Gamma t))$ 



PDF distributions for diff. meson species. No non-direct transitions and CPV assumed.

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### CPT parametrisation in mixing

$$|P_L >= p\sqrt{1-z}|P^0 > +q\sqrt{1+z}|\bar{P}^0 > |P_H >= p\sqrt{1+z}|P^0 > -q\sqrt{1-z}|\bar{P}^0 > |P_H > = p\sqrt{1+z}|P^0 > -q\sqrt{1-z}|\bar{P}^0 > |P_H > -q\sqrt{1-z}|\bar{P}^0 > -q\sqrt{1-z}|\bar{P}^0 > |P_H > -q\sqrt{1-z}|\bar{P}^0 > -q\sqrt{1-z}|\bar{P}^0$$

$$z = \frac{H_{11}^{eff} - H_{22}^{eff}}{\Delta m - i\Delta\Gamma/2} = \frac{\delta m - i\delta\Gamma/2}{\Delta m - i\Delta\Gamma/2}$$

CPT conserved => z=0 !!!

 $\delta m = M_{11} - M_{22}; \delta \Gamma = \Gamma_{11} - \Gamma_{22}$  $\Delta m = m_H - m_L; \Delta \Gamma = \Gamma_H - \Gamma_L$ 

T is conserved then 
$$\left|\frac{p}{q}\right| = 1$$
.  
CPT is conserved then  $z = 0$ .  
CP is conserved then  $z = 0$  and  $|p| = |q|$ .  
 $0 \le |p| \le 1$ ;  $0 \le |q| \le 1$ ;  $0 \le |z| \le 1$ .  
When  $z = 0$  then  $|p|^2 + |q|^2 = 1$ .

## Lorentz Symmetry

(in this presentation: arbitrary rotation or boost invariance violation)

- Foundational assumption of both the SM and GR.
- Continually tested since Einstein (many potential LV sources untested)
- Theory predictions => possibility of LV at Planck scale.
- Resurgence of interest due to searches of physics at the Planck scale.
- In interacting theories CPTV necessitates LV.

[Rep. Prog. Phys. 77, 062901 (2014)]

# Standard Model Extension (SME)

- SME effective field theory, which includes all possible Lorentz violating terms that can be constructed from the associated fields [V.A.Kostelecky, PRD55 (1997), 6760].
- During the last 50 years the SM was established.
- We are interested in the small deviations from the SM using tools of the effective field theory.
- Planck-scale physics using Planck sensitivity rather than Planck energy.

[Rep. Prog. Phys. 77, 062901 (2014)]

### Structure of the SME

1. LV terms of the SME are constructed by coupling observer vector or tensor coefficients for Lorentz violation to SM operators.

2. Generality of the SME => LV could exist in nature associated with one particle but not another.

3. Lagrange density includes: LV terms that are CPT invariant + terms that violate both CPT and Lorentz symmetries.

4. SME is not a model => test framework designed for a broad search [Data tables: arXiv:0801.0287]

[Rep. Prog. Phys. 77, 062901 (2014)]

### Classical vs SME for mesons

- Classical approach: purely phenomenological description of neutral meson mixing (p,q,z).
- Alternative (EFT) approach: SME framework where Lorentz Violation and CPT violation are treated as small deviations to the SM.
- "z" can be associated with CPTV parameters in the SME.
- In the SME "z" equivalent depends on meson species boost and 3momenta.

### **Sidereal Modulations**

- Search for LV => study a system that is symmetric in conventional physics and changes in SME when rotated.
- LV effects are correlated to the background vacuum.
- Earth based LV effects are correlated with sidereal days.

### What are sidereal days?

1. Karl Jansky 1930 built a resonant electric circuit sensitive to 20.5 MHz.

2. Observed signal around noon each day. Sounds like the Sun is triggering it...?

3. Minor detail: it occurred 4 minutes earlier each day.

4. Interpretation => signal coming from outside of the solar system.

5. Strong radio source in the center of the Milky way.



https://iucss.sitehost.iu.edu/sme2021/lectures/Lane.pdf

Credits: Charles Lane (Berry)

#### **Sidereal Modulations**

CPTV z undergoes sidereal modulations, which depend on the geographical location of the lab.



# mSME for mesons (coupling to LV)

- CPTV and LV terms operator mass dim. < 4 (mSME): are introduced for fermions with coupling coefficients  $a_{\mu}$ .
- All components of a "good" QFT remain (renormalisation, locality, spinstatistics etc.).
- Observable effect determined by the contribution from the coupling of the two valence quarks  $q_1, \overline{q}_2$  with the LV field:

$$\Delta a_{\mu} \simeq a_{\mu}^{q_1} - a_{\mu}^{q_2}$$

[Phys.Rev.D61:016002,2000; Phys. Lett. B 730 (2014) 89-94]

### CPT violating parameter z

Phys.Rev.D61:016002,2000



 $\frac{\beta^{\mu}\Delta a_{\mu}}{\Delta m - i\Delta\Gamma/2}$  $z \simeq$ 

$$\beta^{\mu} = \gamma\left(1, \overrightarrow{\beta}\right)$$



blue arrows-LV/CPTV field

four-velocity (lab frame)

sidereal modulations

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FIG. 1. Bases in the laboratory and nonrotating frames.

fixed stars frame

$$\beta^{\mu} \Delta a_{\mu} = \gamma [\Delta a_0 + \beta \Delta a_Z \cos \chi + \beta \sin \chi (\Delta a_Y \sin \Omega \hat{t} + \Delta a_X \cos \Omega \hat{t})]$$

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lab frame

#### Sidereal Modulations at LHCb

 $Re(z) = \frac{\gamma}{\Delta m} [\Delta a_0 + \beta \Delta a_Z \cos \chi + \beta \sin \chi (\Delta a_Y \sin \Omega \hat{t} + \Delta a_X \cos \Omega \hat{t})].$ 

Ω and t̂ - sidereal frequency and time respectively,
 Due to the LHCb geographical location: cos χ = -0.38 (const term), sin χ = 0.92 (sidereal modulated term)

At CERN site these modulations are close to being maximal!

 $\operatorname{Re}(z)$  as a function of sidereal phase



### Current Limits on CPTV

- KLOE:  $K^0$ :  $\Delta a_0$ ,  $\Delta a_X$ ,  $_Y$ ,  $_Z \approx 10^{-18}$  GeV
- FOCUS:  $D^0$ :  $\Delta a_0$ ,  $\Delta a_X$ ,  $\gamma$ ,  $Z \approx 10^{-13}$  GeV
- LHCb:  $B^0$ :  $\Delta a_0, \Delta a_Z \approx 10^{-15}$  GeV,  $\Delta a_X, \Delta a_Y \approx 10^{-15}$  GeV
- LHCb:  $B_s^0$ :  $\Delta a_0, \Delta a_Z \approx 10^{-12}$  GeV,  $\Delta a_X, \Delta a_Y \approx 10^{-14}$  GeV

Measurements in different sectors are complementary [Data tables: arXiv:0801.0287].

LHCb – good place to study CPTV in charm, 10<sup>4</sup> greater statistics compared to FOCUS and excellent decay time resolution 45fs.

Differences in accuracy stem from different masses!

### FOCUS experiment (1996-1997)

- Upper limit in the charm sector from FOCUS (PLB 556 (2003)7),
- Experiment @FERMILAB ( $\gamma$  beam with energy of  $\approx$  180 GeV on fixed BeO target),
- $D^{*+} \rightarrow D^0 \pi^+$ ,  $D^0 \rightarrow K^- \pi^+$ , both in classical and SME approach,
- $\operatorname{Re}(z) \operatorname{Im}(z) \approx O(1)$ ,
- $\Delta a_{\mu} \approx 3 \times 10^{-13}$  GeV,

#### [Phys. Lett. B 556, 7 (2003)]



**Expected from statistics:**  $N_{FOCUS}$ : ~ 10<sup>4</sup> events  $N_{\rm LHCh}$ : ~ 10<sup>8</sup> sqrt( $N_{IHCb}/N_{FOCUS}$ ) ~ 100

# Goal of my analysis

 1. Upper limits on CPTV parameters in the charm sector using LHCb data (10<sup>4</sup> greater statistics + excellent decay time resolution).

2. The search is performed in  $D^* \rightarrow \pi^+_{soft} D^0(K^- \pi^+)$  prompt channel (D\* produced in pp vertex)

#### **Data sample**

- $D^0 \rightarrow K \pi$  candidates from prompt
- $D^{*+/-} \rightarrow D^0 \pi_{soft}^{+/-}$ ,  $\pi_{soft}^{+/-}$  used to tag the flavour =>  $D^0/\overline{D}^0$  respectively
- Analysis performed with Run 2 data: {2015, 2016, 2017, 2018} x {MagUp, MagDown} (today only 2015+2016)
- L~6 fb<sup>-1</sup>@13 TeV p-p collision in Run 2



This selected mode was used as a control channel in other LHCb analyses.

p



D0 lifetime ~= 0.41 ps B hadrons ~= 1.5 ps Displacement from PV

#### **Analysis Steps**

1) Trigger and offlline selection

(done)

- 2) Combinatorial background estimation with m(D\*) distribution
- 3) Kinematic weighting to reduce the second-order effect of detector-induced time-depend asymmetry
- 4) D<sup>o</sup> secondary decays contamination estimation
- **5)** Control of Misreconstructed  $D^0/\overline{D}^0 \rightarrow K^-\pi^+$  ( $K^+\pi^-$ ) background

6) Formation of time-dependent right-sign A<sub>CPT</sub> asymmetry

(ongoing)

- 7) Extraction of bounds on CPT violation parameter z from asymmetry fit.
- 8) Analysis and systematic studies of observables and errors in sidereal time.

#### Observable

right-sign time-dependent CPT asymmetry  

$$A_{CPT}(t) = \frac{\Gamma\left(\bar{P}^0 \to \bar{f}\right)(t) - \Gamma\left(P^0 \to f\right)(t)}{\Gamma\left(\bar{P}^0 \to \bar{f}\right)(t) + \Gamma\left(P^0 \to f\right)(t)}$$

$$P^0 = D^0; f = K^+ \pi^-$$



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#### **General Experimental Idea**



- 1. determine flavour in the initial state
- 2. determine decay time
- 3. determine flavour in the final state
- 4. construct (time-dependent) asymmetry A<sub>CPT</sub>(t)
- 5. extract (mixing/CP/CPT) parameters from the fit

Naturally, the analysis can be performed in the time-bins enabling SME parameter extraction and sidereal modulations analysis. 15.12.2022



Expected	from	statistics:
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 $N_{FOCUS}$ : ~ 10<sup>4</sup> events  $N_{LHCb}$ : ~ 10<sup>8</sup> sqrt(  $N_{LHCb}/N_{FOCUS}$ ) ~ 100 Run 2: 15-16 Bounds on z: x,y ~ O(10<sup>-3</sup>) in charm  $\rightarrow$  z bound ~ O(10<sup>-2</sup>)

Compared with the best strange sector (KLOE, KteV) : z bound  $\sim O(10^{-4})$ 

#### **Misidentification BKG sources**

•  $D^0 \rightarrow K^- \pi^+$  SIGNAL!

• 
$$D^0 
ightarrow K^- \pi^+ \pi^0$$
 ( $\pi^0$  not rec.)

• 
$$D^0 \rightarrow \pi^+\pi^-$$
 (K<sup>-</sup> instead of  $\pi^-$ )

• 
$$D^0 \rightarrow \pi^- \pi^+ \pi^0$$
 (K<sup>-</sup> instead of  $\pi^- + \pi^0$  not rec.)

• 
$$D^0 
ightarrow K^- e^+ 
u_e$$
 ( $\pi^+$  instead of  $e^+$ )

• 
$$D^0 
ightarrow K^- \mu^+ 
u_\mu$$
 ( $\pi^+$  instead of  $\mu^+$ )

#### **Misidentification background**



#### Produced using RapidSim simulated samples.

1. Events reweighted with: mis-PID efficiency (obtained with PIDCalib) and branching fractions (kinematic weights will be added soon).

2. Calculation based on the comparison between integrals of signal and backgrounds within D\* signal region.

	raw	weights applied	
$K^-\pi^+$	1	1	
${\cal K}^-\mu^+ u_\mu$	$4.81 \cdot 10^{-4}$	$1.07 \cdot 10^{-4}$	
$K^-e^+\nu_e$	$9.70 \cdot 10^{-4}$	$1.55 \cdot 10^{-4}$	
$\pi^{-}\pi^{+}$	$4.32 \cdot 10^{-3}$	$2.18 \cdot 10^{-6}$	
$K^-K^+$	0	0	
$\pi^{-}\pi^{+}\pi^{0}$	$2.99 \cdot 10^{-2}$	$1.81 \cdot 10^{-5}$	

#### Circles in the graphics below mark 0<|z|<1

![](_page_34_Figure_1.jpeg)

### Summary

• Due to large statistics and excelent decay time resulution at LHCb in  $D^* \rightarrow \pi^+$ soft  $D^0(K^- \pi^+)$  prompt channel =>

=> we might expect improvement on the limits on FOCUS results by the factor of 100.

- Control of systematic and statistical errors constitute a significant part of my work (not fully covered here).
- We consequently move the precision to CPTV in the direction of the Planck scale.
- We expect to achieve the highest sensitivity to SME target CPTV parameters in the charm sector.

#### BACK UP

#### **Trigger requirements**

- $D^0 \rightarrow h^+h^-$  candidates from prompt  $D^{*+} \rightarrow D^0 \pi_{soft}^{-+}$ ,  $\pi_{soft}^{-+}$  used for  $D^0$  flavour tagging
- L0 trigger: D0\_L0HadronDecision\_TOS || Dst\_L0Global\_TIS
- HLT1: D0\_HIt1TrackMVADecision\_TOS || D0\_HIt1TwoTrackMVADecision\_TOS

#### **Event Selection - Offline**

Name	Offline		20		$10^{6}$
$ \begin{array}{l} m(D^{0}) \\ \text{DLL}(K^{\pm}) \\ \text{DLL}(\pi^{\pm}) \\ \text{Rxy} \\  z(DV)  \\ \text{track-based ghost probabiliy } (\pi_{s}^{\pm}) \\ D^{*+} \text{ vertex-fit } \chi^{2}/ndf \\ p_{T}(\pi_{s}^{\pm}) \\ D^{0} \text{ proper decay time} \\ IP(D^{0}) \end{array} $	$ \begin{array}{l} \in [1840.84, 1888.84] \text{ MeV} \\ <= -5 \\ >= 5 \\ <= 4 \text{ mm} \\ \in [-200, 200] \text{ mm} \\ <= 0.15 \\ <= 25 \\ >= 200 \text{ MeV} \\ \in [0.45, 8] \tau(D^0) \\ < 9 \end{array} $	R [mm]	$ \begin{array}{c} 20 \\ 15 \\ 10 \\ 5 \\ 0 \\ -5 \\ -10 \\ -15 \\ -20 \\ -200 \\ 0 \end{array} $		10° 10 <sup>5</sup> 10 <sup>4</sup> 10 <sup>3</sup> 10 <sup>2</sup> 10
		1		Z(DV)  mm	

$$R_{xy} = \text{sgn}[x(\text{DV}) - x(\text{PV})]\sqrt{(x(\text{DV}) - x(\text{PV}))^2 + (y(\text{DV}) - y(\text{PV}))^2}$$

Selection criteria analogous to LHCb-ANA-2020

![](_page_39_Figure_0.jpeg)

![](_page_39_Figure_1.jpeg)

400

200

![](_page_39_Figure_2.jpeg)

 $10^{2}$ 

2.002

2.004

2.006

2 00

2.008

2.008

2.01

2.012

2.012

2.014

2.014

2.016

2.018

2.02 mass D\*

#### **Kinematic Weighting**

![](_page_40_Figure_1.jpeg)

- Detection should be symmetric with respect to the transformation:  $x \rightarrow -x \&\& q_s \rightarrow -q_s$
- Currently one-step procedure used reweigh the events in 3-D bins of D<sup>o</sup> momenta
- Geometric mean with additional fiducial cuts
- Weighing performed separately for Run periods and Magnet positions

Method taken from [Agamma - LHCb-ANA-2020-040]

Representation of  $D^0$ :

• 
$$\theta_x = atan2(Px/P_z)*sgn(\pi_{soft})$$

• 
$$\theta_y = atan2(P_y/P_z)$$

$$k = 1/sqrt(P_x^{2} + P_z^{2})$$

#### **Kinematic weighting**

 $p1 = Re(z) \cdot y - Im(z) \cdot x$ 

![](_page_41_Figure_2.jpeg)

![](_page_42_Figure_0.jpeg)

#### For the analysis we need to estimate the values of N<sub>prim</sub> and N<sub>sec</sub>: $\rightarrow$ next presentation

#### **Fitting Strategy**

$$A_{CPT}(t) = \frac{\frac{d\Gamma_{\bar{M}^0 \to \bar{f}}(t)}{dt} - \frac{d\Gamma_{M^0 \to f}(t)}{dt}}{\frac{d\Gamma_{\bar{M}^0 \to \bar{f}}(t)}{dt} + \frac{d\Gamma_{M^0 \to f}(t)}{dt}} \qquad A^{\text{dir}} \equiv (|\bar{A}_{\bar{f}}|^2 - |A_f|^2)/(|\bar{A}_{\bar{f}}|^2 + |A_f|^2)$$

$$A_{CPT}(t) = A^{\text{dir}} + \frac{2\text{Re}(z)\sinh\Delta\Gamma t/2 - 2\text{Im}(z)\sin\Delta mt}{(1+|z|^2)\cosh\Delta\Gamma t/2 + (1-|z|^2)\cos\Delta mt}$$

 $A_{CPT}(t) = A^{dir} + [Re(z)y - Im(z)x]\Gamma t$  (Linear term Taylor expansion in t.)

 $A_{CPT}(t) = \frac{Re(z)\left(x^2 + y^2\right)}{2x}\left(\Gamma t\right)^2 \left[\frac{xy}{3}\left(\Gamma t + \sqrt{R_{DCS}}\left(x\cos\delta + y\sin\delta\right)\right)\right]$ (Second non zero term in Taylor)

Term responsible for doubly cabibbo supressed.

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#### Sensitivity studies based toy MC generator

A<sub>CPT</sub>(t)

0.5

-0.5

0

1. MC generator of neutral meson decays. We can set CP/CPT violation scale before simulation through the choice of CPV/CPTV parameter values (p,q,z).

2. It can be used to generate times of decay of any meson.

3. Generated times of decay can be used to build right-sign asymmetry histogram.

4. Finally, CPTV parameters can be extracted from fits of the phenomenolgical/SME model to the aforementioned right-sign asymmetry.

![](_page_44_Figure_5.jpeg)

![](_page_44_Figure_6.jpeg)

2

entries = 1000000 Im $(z_m) = 0.0999 \pm 0.0005$ Re $(z_m) = -0.0007 \pm 0.001$ 

0.0000 + 0.0008

1.0932

45