In Search of Precision in QCD at High Energy Physics
Beyond Eikonal Order

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Work done in collaboration with:
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Outline

- Basics and Intro to what is saturation physics?
- Tools to calculate observables
- Eikonal approximation
- Gluon propagator
- Application of gluon propagator
- Conclusion and remarks

Introduction: Quantum Chromodynamics

How this theory come into existence!?

⟹ To understand deepest secrets of matter, which give rise to two main questions:

- What is the smallest building block of matter?
- What are the forces that hold them together?

Quantum Chromodynamics

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To understand deepest secrets of matter, which give rise to two main questions:

- What is the smallest building block of matter?
- What are the forces that hold them together?

Several forces governing subatomic world: strongest of them **Strong Force**

Several forces governing subatomic world: strongest of them

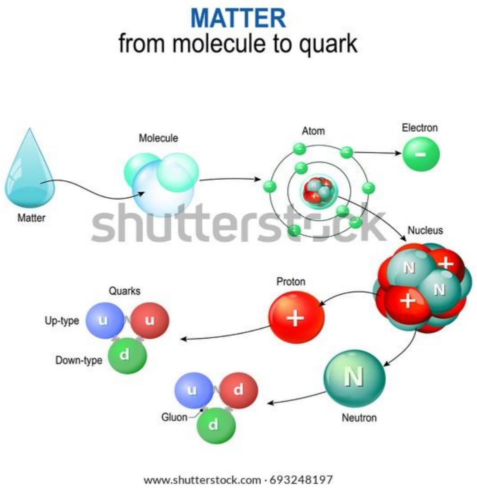
➢ **Strong Force**

If we go deep inside the atom, in nucleus: protons and neutrons contains quarks!

Basically,

Quarks are the elementary particles in all hadrons and

Gluons are exchange particles(like photons in QED) for strong interactions between quarks.



….and Quantum chromodynamics is the theory of interactions between **quarks** and **gluons.**

Basics of Quantum Chromodynamics(QCD)

- Quarks: 6 of them and each have antiparticle
  - Spin half particle
- Gluons : 8 of them
  - Spin 1 particle
  - Have color charge
  - Self interact

Cannot observe individual quarks and gluons

Always bounded!

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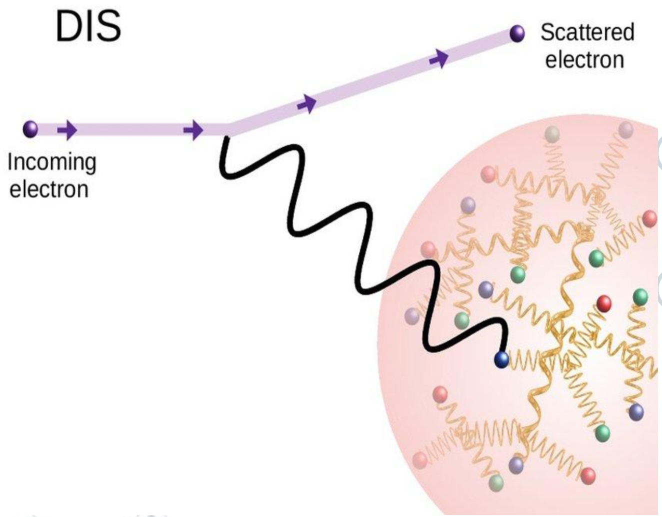
But how to apply these thing to study insides of atoms?.............

To look inside any hadrons(for example proton), we have to probe it! And inside of it depends on how we setup an experiment.

Generally, deep inelastic scattering (DIS) is used to probe the hadrons.

Credits: quanta magazine

Deep Inelastic Scattering: General View



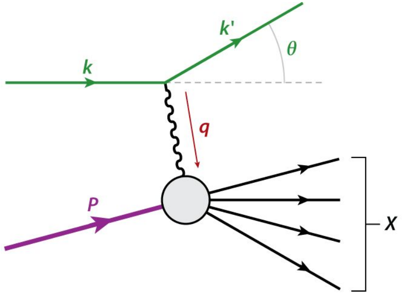
Credits: 10.1007/JHEP12(2021)102

- The process in which leptons(electron) hit the hadrons(proton) hard enough to shatter it.
- Provided first evidence for reality of quarks.
- **Deep** refers to very high energy of leptons, which gives it very short wavelength and hence the ability to probe the distances that are small compared to size of target hadron, so it can probe “deep inside” the hadron.

Deep Inelastic Scattering : Kinematics

Inclusive cross-section in DIS can be expressed in terms of Lorentz invariants as:

- $x \equiv$ longitudinal momentum fraction carried by parton in the hadron; $x = p_i/P$
  p_i = initial momentum of struck parton
  P = momentum transfer of target hadron



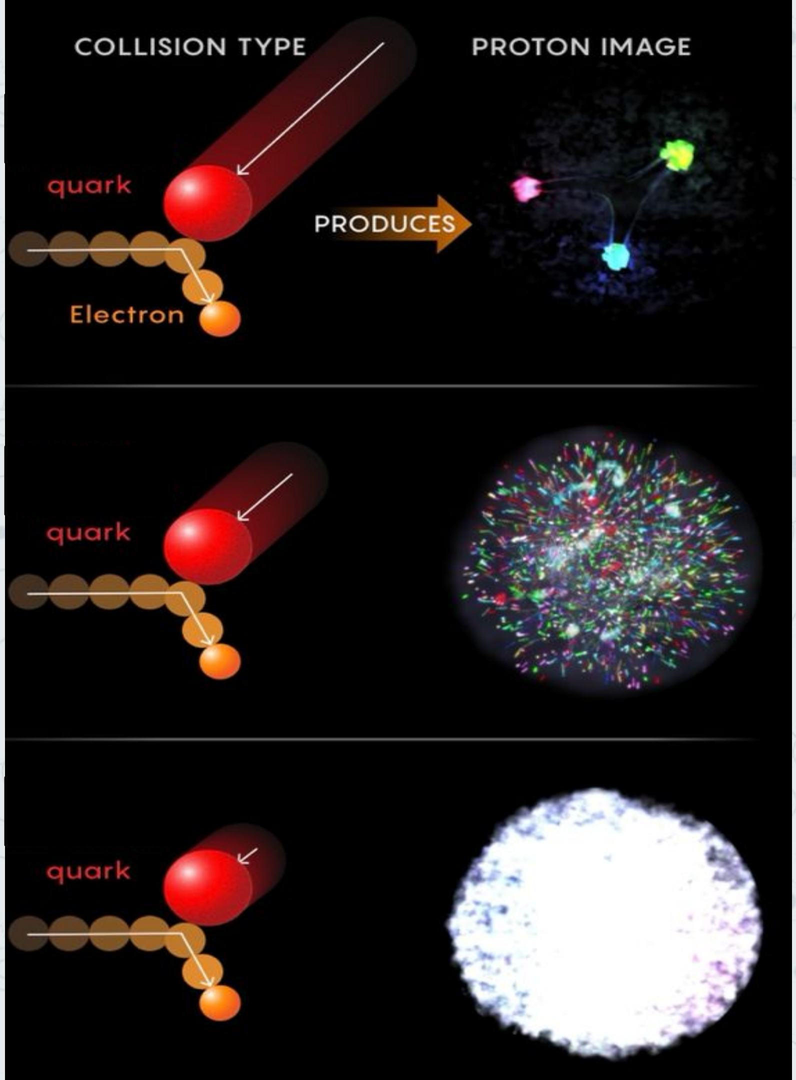
Gelis F, et al. 2010.
Annu. Rev. Nucl. Part. Sci. 60:463–89

- The virtual photon four momentum squared ($q^2 = -Q^2 < 0$), exchanged between electron and hadron, where $q = k - k'$,
  k= initial momentum of lepton, k' = final momentum of lepton
- y= elasticity= ratio of photon energy to the electron energy in the hadron rest frame
- s= the square of center of mass energy; $s = (p+k)^2$

“

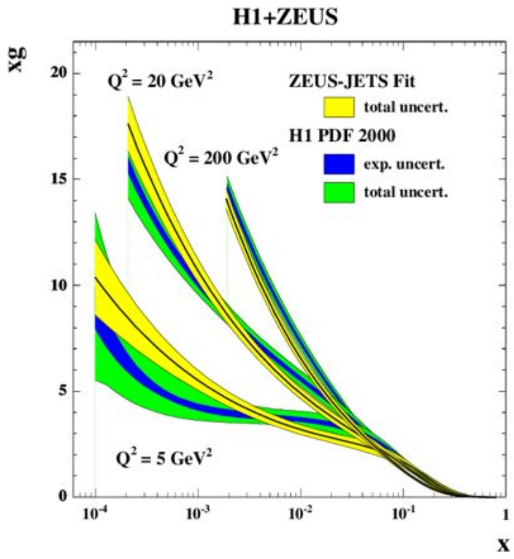
Depending on the energy scale of the lepton in deep inelastic scattering, interior of the hadron is observed!

For example :
In case of proton



Credit: quanta magazine

At very high energy, hadron is probed as sea of gluons, where self interacting gluons produce further gluons!
……and this is **saturation region**



Credit: arXiv:0901.0986

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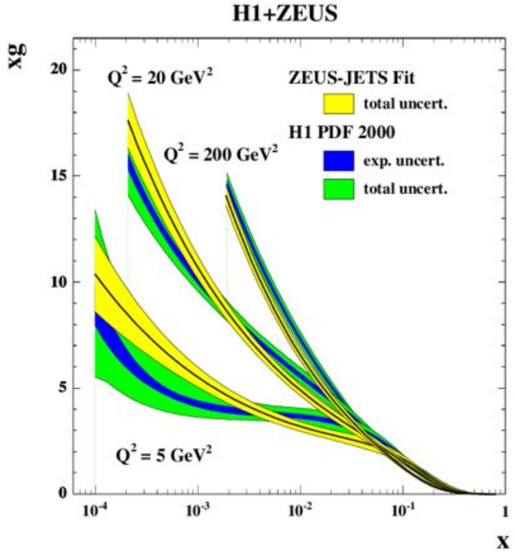
How to deal with scattering processes involving large number of partons?

quarks gluons

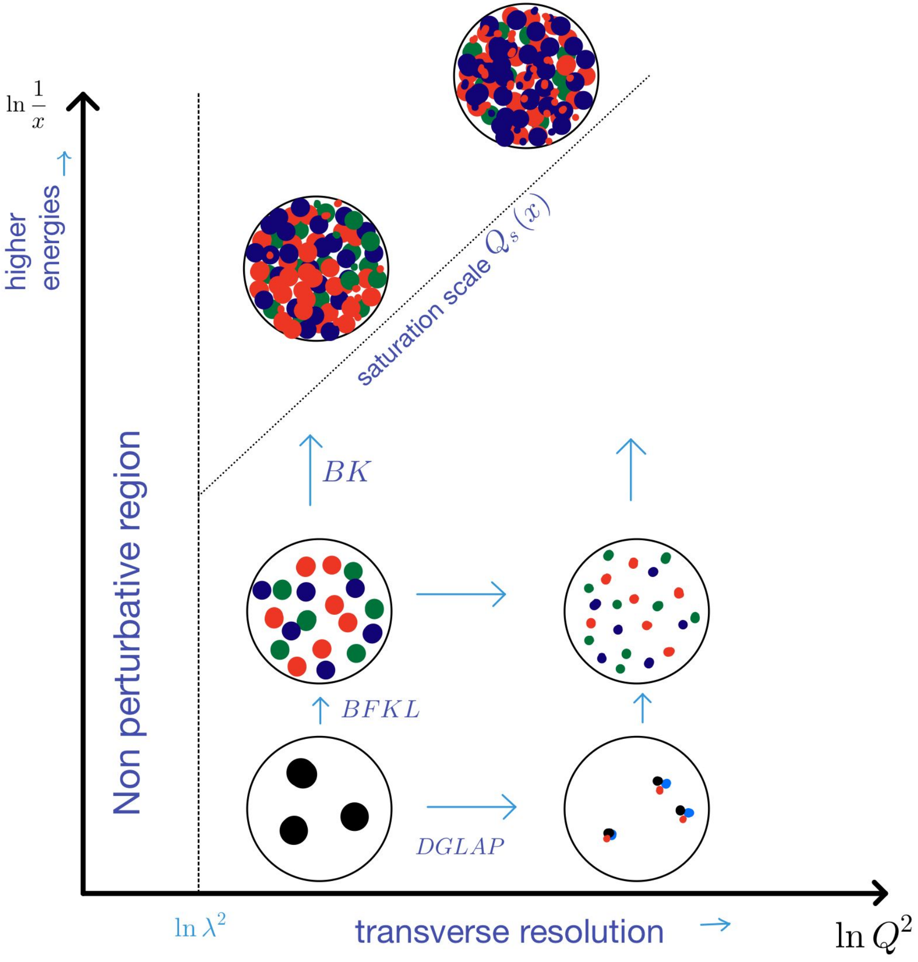


Credit: arXiv:0901.0986

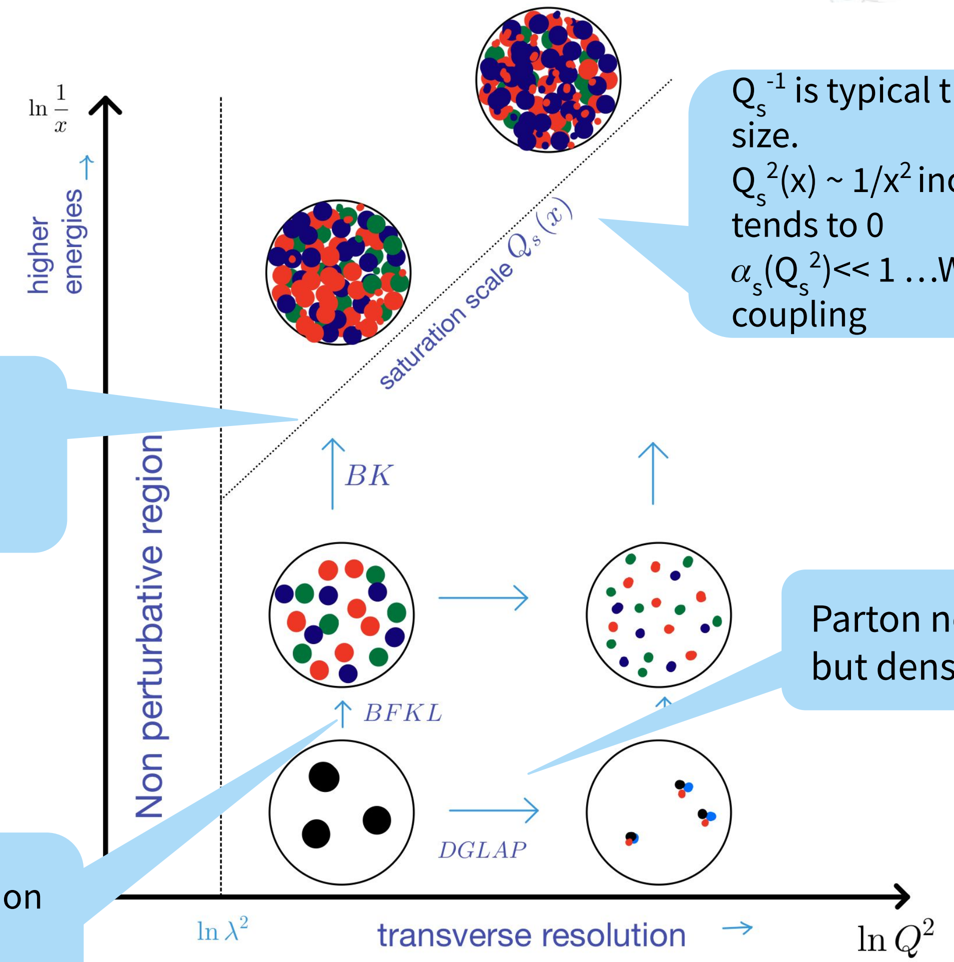
1.

Saturation physics

How much high is high energy physics!



Phase Diagram of Gluon Saturation



Q_s^{-1} is typical transverse size.
$Q_s^2(x) \sim 1/x^2$ increases x tends to 0
$\alpha_s(Q_s^2) \ll 1$ …Weak coupling

Recombination of gluons
$N_g \leq 1$
unitarity

Parton no. increases, but density decreases

Multiple Gluon emissions
$N_g \sim e^{w\ln(1/x)}$

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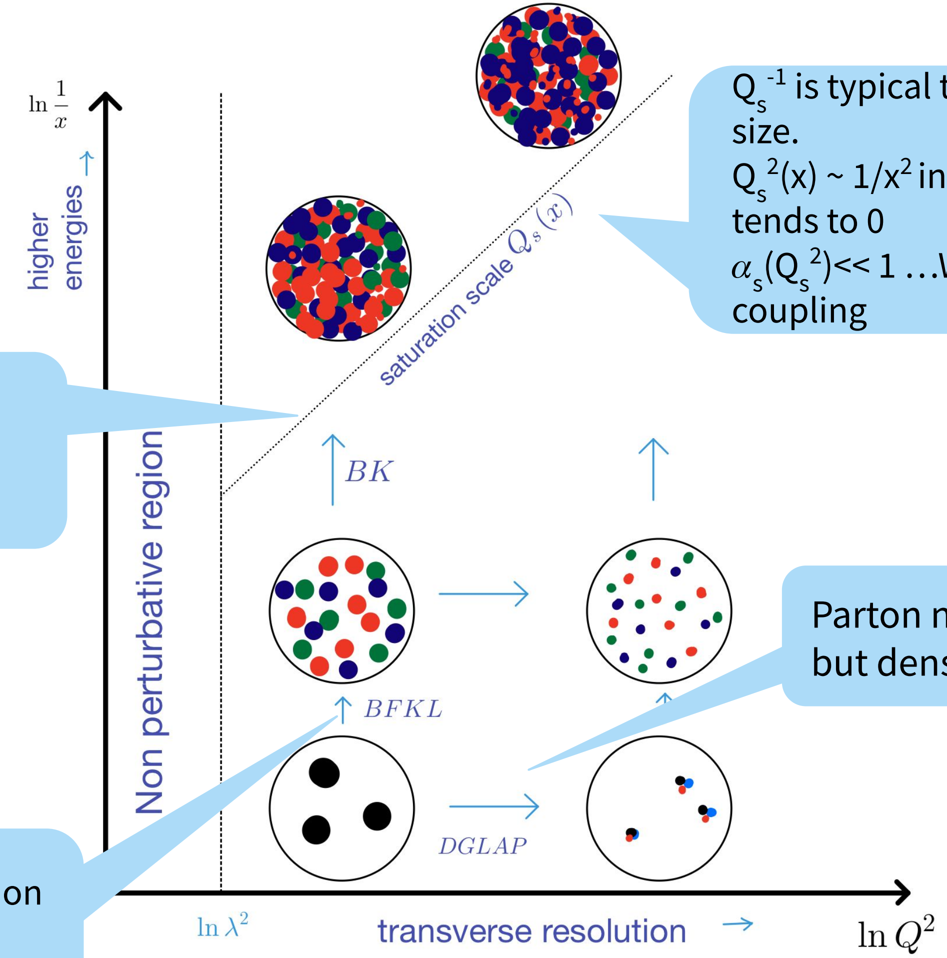

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Color Glass Condensate

Gluon Density

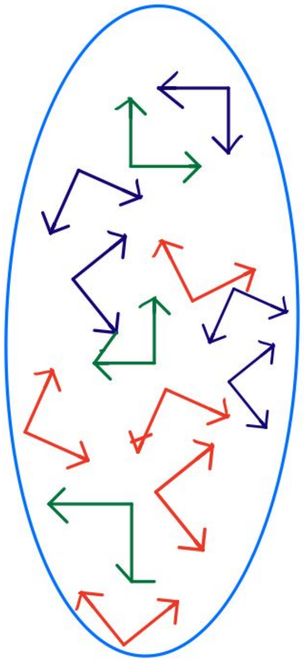
Parton gas

Color Glass Condensate(CGC)

color : Gluons have “Color”

Glass: the small-x gluons are created by slowly moving partons(with large x) which are randomly distributed over transverse plane → it looks like almost frozen over natural time scale of scattering(This is very similar to **spin glass**, where spins are distributed randomly and move very slowly)

Condensate: It is dense matter of gluons. Can be better described as fields rather than point particles!



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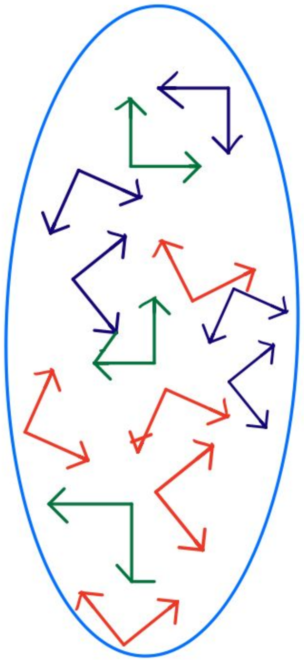

Low x gluons = Static classical field(A_μ)

But what are the tools to study this!

Color Glass Condensate(CGC)

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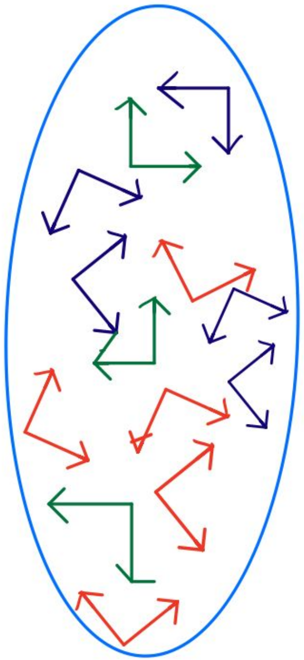
Condensate: It is dense matter of gluons and saturated. Can be better described as fields rather than point particles!



Low x gluons = Static classical field(A_μ)

2.

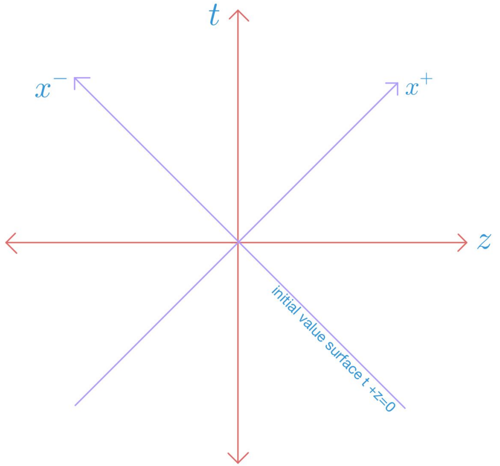
Tools used

Machineries of theoretical physicist!

Light Cone Coordinates

- We have to construct Lorentz invariant wavefunction to study high energy physics
- ➔ Quantize theory on light like surface $x^+ = 0$
- $$x^\mu = (x^0, x^1, x^2, x^3) = (t, \vec{x})$$
$$\Rightarrow x^\pm = \frac{t \pm z}{\sqrt{2}}$$

Light cone coordinates:
$[x^+, x^-, x_1, x_2]$



- Also, Light-cone gauge($A^+ = 0$) is used to simplify calculations.

2.

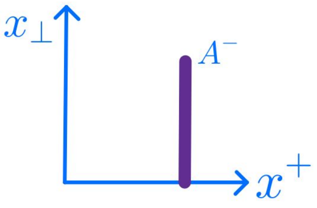
Eikonal Approximation

And beyond …..
(some technical stuff 📰)

What is Eikonal Approximation?

- In practice, it means describing scattering processes by considering leading order energy contributions only and neglecting power suppressed contributions.
- In this approximation, it is assumed that:
  a. Highly boosted background field(target) is localised in the longitudinal direction($x^+ = 0$).
  b. Only leading component of target (- component) is considered and subleading components are neglected (suppressed by Lorentz boost factor).
  c. Dynamics of the target are neglected(x^- dependence of target neglected).

Background field of target is : $A^{\mu}\left(x^{-}, x^{+}, \boldsymbol{x}\right) \approx \delta^{\mu-} \delta\left(x^{+}\right) A^{-}(\boldsymbol{x})$



Relaxing Eikonal Approximation

Going beyond Eikonal order means considering power suppressed component

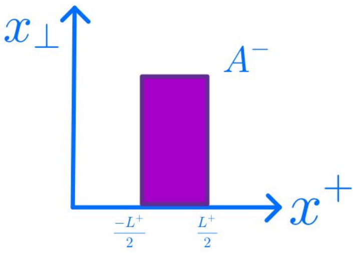
- The term is suppressed as compared to leading term by order of $1/\gamma$ (γ is Lorentz boost factor); corresponds to next-to-eikonal approximation term.
- In case of background field of target, each component is separated by hierarchy:

$$A^- = O(\gamma) \gg A_j = O(1) \gg A^+ = O\left(\frac{1}{\gamma}\right)$$

Relaxing Eikonal Approximation

a. Finite Width

Instead of infinite thin shockwave as a target, we consider finite width of a target



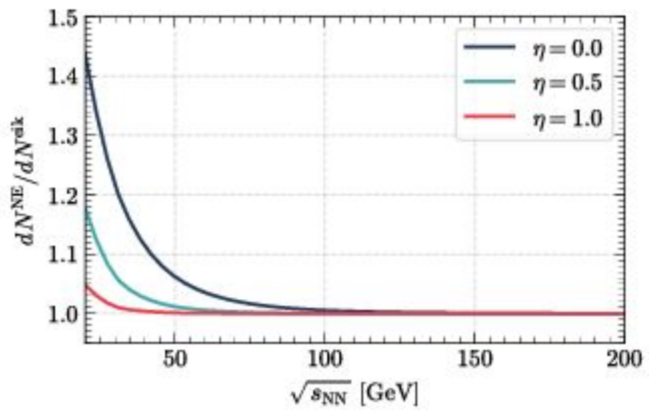
b. Transverse Component

Instead of neglecting sub-leading components, we include transverse component of background field

c. x^- dependence

We take into account corrections coming due to the x^- dependence of a target(consider background field is x^- dependent)

Why beyond Eikonal!



Credit: P. Agostini et.al.(arXiv:2207.10472v2)

- In very high energy accelerators like **LHC,** NEik order terms **are negligible** while calculating observables
- But to analyze the data from **RHIC** and future electron ion collider (**EIC**), NEik order terms will be **sizable**!

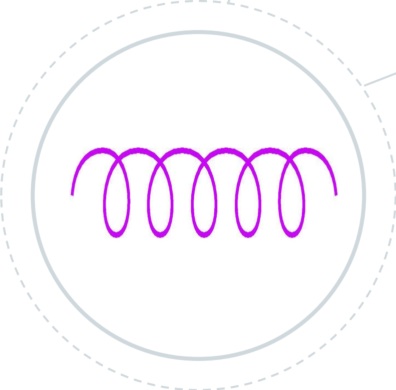
Currently, we are calculating Gluon propagator at NEik approximation…..

2.

Gluon Propagator

In presence of medium

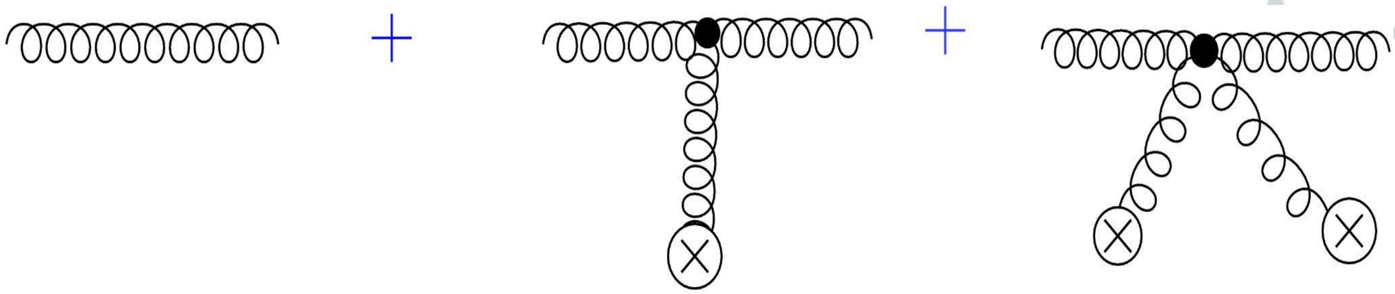
Gluon Propagator



In vacuum Gluon propagator in momentum space is give as:

$$G^{\mu\nu}_{0,F}(p) = \frac{i}{p^2 + i\varepsilon}\left[-g^{\mu\nu} + \frac{p^\mu\eta^\nu + \eta^\mu p^\nu}{p\cdot\eta}\right]$$

This is in Light-cone gauge, $A^+ = 0$ and $\eta^2=0$
Where, $\eta^\mu = g^{\mu +}$

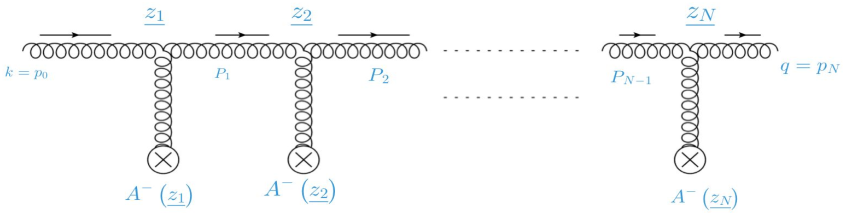


We are computing Gluon propagator in a classical background gluon field $A_\mu(x)$ at next-to-eikonal(NEik) accuracy.

- To do that we will be relaxing Eikonal approximation as mentioned before.
- We can write gluon propagator as $G_F^{\mu\nu}(x,y) = G_{0,F}^{\mu\nu}(x,y) + \delta G_F^{\mu\nu}(x,y)$
  Where, $G_{0,F}$ is vacuum contribution and δG_F is medium correction.
- We compute gluon propagator in Eikonal approximation then include NEik corrections.
- We will get NEik correction due to three different effects
    - Due to insertion of transverse components of background field
    - Due to x^- component of background field
    - Due to finite width of target

Method to calculate Eikonal order Gluon Propagator

- To do that, we re-sum multiple interaction diagrams of Gluon background field as shown in figure below
- And consider only “-” component(dynamic component) of classical gluon background field
- Then we take eikonal approximation to obtain following expression.



Gluon propagator at Eikonal approximation

$$G_F^{\mu\nu}(x,y)|_{Eik} = G_{0,F}^{\mu\nu}(x,y) + \delta G_F^{\mu\nu}(x,y)|_{Eik}$$

$$G_F^{\mu\nu}(x,y)|_{Eik} = i\delta^2(x_\perp - y_\perp)\ \delta(x^+ - y^+)\eta^\mu\eta^\nu\left[\int\frac{dk^+}{2\pi}\frac{e^{-i(x^- - y^-)k^+}}{k^+k^+}\right]$$
$$+\left\{\int\frac{d^3q}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\ \frac{e^{-ix\cdot\check{q}}\ e^{iy\cdot\check{k}}}{2k^+}\left[2\pi\ \delta(k^+ - q^+)\right]\right.$$
$$\left.\left[-g^{\mu\nu} + \frac{\check{k}^\mu\eta^\nu}{k^+} + \frac{\eta^\mu\check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k})\right]\left[\int d^2z_\perp\ e^{-i(q_\perp - k_\perp)z_\perp}\right]\right\}$$
$$\left[\theta(x^+ - y^+)\ \theta(k^+)\ U_A(x^+, y^+; z_\perp) - \theta(y^+ - x^+)\ \theta(-k^+)\ U_A^\dagger(x^+, y^+; z_\perp)\right]$$

Gluon propagator at Eikonal approximation

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$$+\left\{\int \frac{d^3q}{(2\pi)^3}\int \frac{d^3k}{(2\pi)^3}\ \frac{e^{-ix\cdot\check{q}}\ e^{iy\cdot\check{k}}}{2k^+}\Bigg[2\pi\ \delta($$

$$\left[-g^{\mu\nu} + \frac{\check{k}^\mu\eta^\nu}{k^+} + \frac{\eta^\mu\check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k})\right]\left[\int d^2z_\perp\ e^{-i(q_\perp - k_\perp)z_\perp}\right]\right\}$$

$$\left[\theta(x^+ - y^+)\ \theta(k^+)\ U_A(x^+,y^+;z_\perp) - \theta(y^+ - x^+)\ \theta(-k^+)\ U_A^\dagger(x^+,y^+;z_\perp)\right]$$

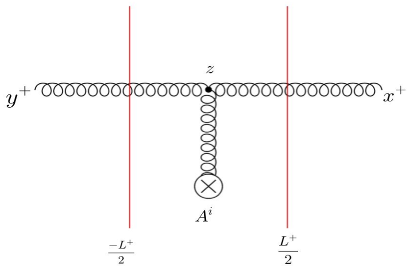
This is **gauge link**, which contains summation of all the A⁻ insertions!

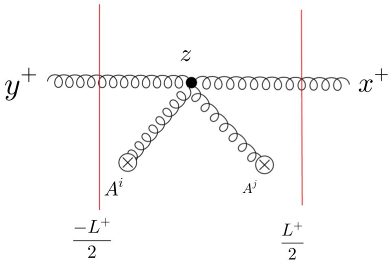
NEik Corrections: Due to Insertion of transverse component

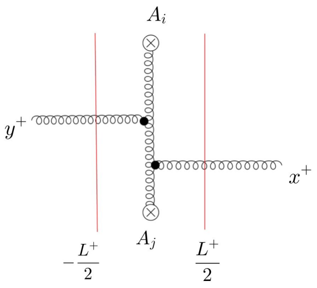
Single Insertion of transverse component

Double transverse component insertion

Instantaneous double transverse component insertion







Calculating corrections

We will calculate NEik corrections to Gluon propagator by evaluating following kind of equations:

$$\delta G_F^{\mu\nu}(x,y) = \int d^4z \; G_F^{\mu\mu'}(x,z)\Big|_{Eik} X_{\mu'\nu'}(\underline{z}) \; G_F^{\nu'\nu}(z,y)\Big|_{Eik}$$

Where, $X_{\mu'\nu'}(\underline{z})$ is insertion factor.(depends upon insertions of background field)

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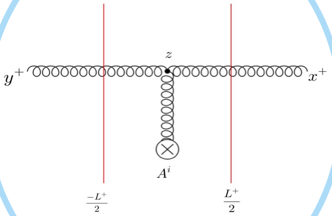
Where, $X_{\mu'\nu'}(\underline{z})$ is insertion factor.(depends upon insertions of background field)

After many of filled blackboards and simplifications of integration, we get……

Full neik result

Correction due to single insertion of A$_\perp$:

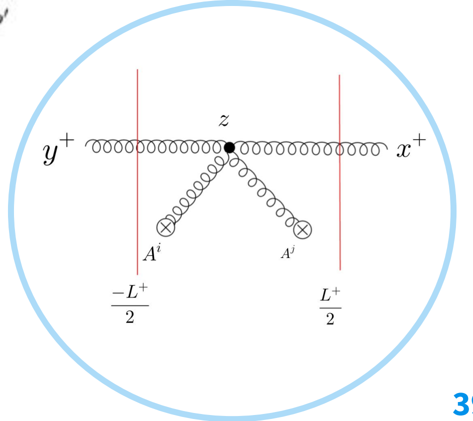


$$
\begin{aligned}
\delta G_{F\ ab}^{\mu\nu}(x,y)|_{\mathrm{single}A_\perp} = g \int d^3z \int \frac{d^3q}{(2\pi)^3}\frac{e^{-ix\cdot\check{q}}}{2q^+}\theta(x^+-z^+)\theta(q^+) \int \frac{d^3k}{(2\pi)^3}\frac{e^{iy\cdot\check{k}}}{2k^+}\theta(k^+)\left(2\pi\delta(q^+-k^+)\right) \\
\left[U_A(x^+,z^+;z_\perp)\right]_{aa'} e^{-iq_\perp z_\perp}\Bigg\{2\Bigg[\left(g^{\mu j}g^{\nu i}-\frac{\eta^\mu g^{\nu i}q^j}{q^+}-\frac{g^{\mu j}k^i\eta^\nu}{q^+}+\frac{\eta^\mu\eta^\nu k^i q^j}{q^+q^+}\right) \\
-\left(g^{\mu i}g^{j\nu}-\frac{\eta^\mu q^i g^{j\nu}}{q^+}-\frac{g^{\mu i}k^j\eta^\nu}{q^+}+\frac{\eta^\mu\eta^\nu q^i k^j}{q^+q^+}\right)\Bigg]\left[\frac{\overleftarrow{d}}{dz^i}\left(T\cdot A^j(z)\right)+\left(T\cdot A^j(z)\right)\frac{\overrightarrow{d}}{dz^i}\right] \\
+\left[g^{\mu\nu}-\frac{\eta^\mu\check{q}^\nu}{q^+}-\frac{\check{k}^\mu\eta^\nu}{q^+}+\frac{\eta^\mu\eta^\nu}{q^+q^+}(\check{q}\cdot\check{k})\right]\left[\frac{\overleftarrow{d}}{dz^j}\left(T\cdot A^j(z)\right)-\left(T\cdot A^j(z)\right)\frac{\overrightarrow{d}}{dz^j}\right] \\
\Bigg\} e^{ik_\perp z_\perp}\left[U_A(z^+,y^+;z_\perp)\right]_{b'b}\theta(z^+-y^+)
\end{aligned}
$$

Full neik result

Correction due to double insertion of $A_\perp$:

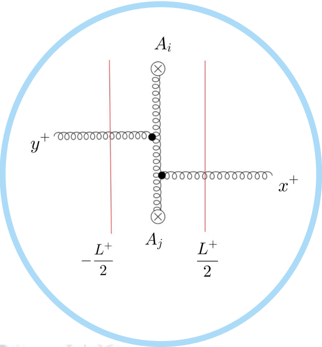
$$\begin{aligned}
\delta G_F^{\mu\nu}{}_{ab}(x,y)|_{\text{double}A_\perp} = \frac{1}{2}\int d^3z \int \frac{d^3q}{(2\pi)^3}\,\frac{e^{-ix\cdot\check{q}}}{2q^+}\,\theta(x^+-z^+)\theta(q^+)\int\frac{d^3k}{(2\pi)^3}\,\frac{e^{iy\cdot\check{k}}}{2k^+}\,\theta(k^+)\,\Big[U_A(x^+,z^+;z_\perp)\Big]_{aa'} \\
\Big[2\pi\delta(q^+-k^+)\Big]\, e^{-i(q_\perp-k_\perp)z_\perp}\,\left(2ig^2\right)\Big[T\cdot A^i(z)\Big]_{a'e}\Big[T\cdot A^j(z)\Big]_{eb'} \\
\Bigg[\Bigg(-g^{\mu\nu}g_{ij}+\frac{\check{k}^\mu\eta^\nu g_{ij}}{q^+}+\frac{\eta^\mu\check{q}^\nu g_{ij}}{q^+}-\frac{\eta^\mu\eta^\nu g_{ij}}{q^+q^+}(\check{k}\cdot\check{q})\Bigg) \\
+\Bigg(g_i^\nu g_j^\mu - \frac{k_i g_j^\mu\eta^\nu}{q^+}-\frac{\eta^\mu g_i^\nu q_j}{q^+}+\frac{\eta^\mu\eta^\nu k_i q_j}{q^+q^+}\Bigg)\Bigg] \\
\Big[U_A(z^+,y^+;z_\perp)\Big]_{b'b}\,\theta(z^+-y^+)
\end{aligned}$$



Full neik result

Correction due to instantaneous insertion of $A_\perp$:

$$\delta G^{\mu\nu}_{ab}(x,y)|_{inst.A_iA_j} = g^2 \int d^3z \int \frac{d^3q}{(2\pi)^3} \frac{e^{-ix\cdot\check{q}}}{2q^+} \theta(x^+ - z^+)\, \theta(q^+) \int \frac{d^3k}{(2\pi)^3} \frac{e^{iy\cdot\check{k}}}{2k^+} U_A(x^+, z^+; z_\perp)\, e^{-iq_\perp z_\perp}$$
$$\left[2\pi\, \delta(q^+ - k^+)\right] (-i) \left[T \cdot A^i(z)\right] \left[T \cdot A^j(z)\right]$$
$$\left[g^{\mu i} g^{j\nu} - \frac{g^{\mu i} k^j \eta^\nu}{k^+} - \frac{\eta^\mu q^i g^{j\nu}}{q^+} + \frac{\eta^\mu \eta^\nu q^i k^j}{q^+ k^+}\right] e^{ik_\perp z_\perp}\, \theta(z^+ - y^+)\, \theta(k^+)\, U_A(z^+, y^+; z_\perp)$$



Full NEik Gluon Propagator

We add all the mentioned corrections to the gluon propagator at eikonal order and simplify the total expression to write it in compact form. (For the case $x^+ > L^+/2$ and $y^+ < -L^+/2$)

To write in compact form, we split the gluon propagator in two part, such as :

$$\delta G_F^{\mu\nu}(x,y) = \delta G_{1F}^{\mu\nu}(x,y) + \delta G_{2F}^{\mu\nu}(x,y)$$

Where, one part is

$$\delta G_{1F}^{\mu\nu}(x,y) = \int \frac{d^3\underline{q}}{(2\pi)^3} \int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+ - k^+) \frac{e^{-ix\cdot\check{q}}\; e^{iy\cdot\check{k}}}{2k^+\; 2k^+} \bigg(- g^{\mu\nu} + \frac{\check{k}^\mu \eta^\nu}{k^+} + \frac{\eta^\mu \check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k}) \bigg)$$
$$\theta(k^+) \int d^2 z_\perp\; e^{-iz_\perp(q_\perp - k_\perp)} \bigg\{ - \frac{q^j + k^j}{2} \int_{\frac{-L^+}{2}}^{\frac{L^+}{2}} dz^+ \bigg[U_A\Big(\frac{L^+}{2}, z^+; z_\perp\Big) \Big(\overrightarrow{D}_{z^j} - \overleftarrow{D}_{z^j}\Big)$$
$$U_A\Big(z^+, -\frac{L^+}{2}; z_\perp\Big)\bigg] - i \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \bigg[U_A\Big(\frac{L^+}{2}, z^+; z_\perp\Big) \Big(\overleftarrow{D}_{z^j} \overrightarrow{D}_{z^j}\Big)\, U_A\Big(z^+, -\frac{L^+}{2}; z_\perp\Big)\bigg]$$
$$\bigg\} + \text{NNEik}$$

Where, $D_i = \partial_i + igT\cdot A_i$ is covariant derivative.

Full NEik Gluon Propagator

And other one is in terms of field strength given as:

$$\delta G^{\mu\nu}_{2F\,ab}(x,y) = \int dz^+ \int \frac{d^3q}{(2\pi)^3}\,\frac{e^{-ix\cdot\check{q}}}{2q^+} \int \frac{d^3k}{(2\pi)^3}\,\frac{e^{iy\cdot\check{k}}}{2k^+}\theta(k^+)\;2\pi\delta(q^+-k^+)\bigg(g^{\mu j}g^{\nu i} - \frac{\eta^\mu g^{\nu i} q^j}{q^+} - \frac{g^{\mu j}k^i\eta^\nu}{q^+} + \frac{\eta^\mu\eta^\nu k^i q^j}{q^+q^+}\bigg)\int d^2z_\perp e^{-i(-q_\perp-k_\perp)z_\perp} U_A\Big(\frac{L^+}{2}, z^+; z_\perp\Big)\; gT\cdot F_{ij}\; U_A\Big(z^+, -\frac{L^+}{2}; z_\perp\Big)$$

Where field strength is given as $F_{ij} = [D_i, D_j]$

Applications

- In future we will use this computed Gluon propagator to calculate cross sections for different scattering process.
- This expression of NEik order gluon propagator will be used to calculate single inclusive gluon production in forward pA collisions.

Conclusions and remarks

- In saturation region, we can study hadrons with perturbation theory.
- CGC in eikonal approximation is generally used to study high energy scattering processes.
- NEik corrections will be sizable in future EIC collider.

Thank you!

I hope you are still with me😂

Backup

Coupling Constant

- In QCD, coupling constant is not fixed.
- It changes with the momentum scale involved in the interaction, $\alpha_s = \alpha_s(Q)$
- For short distances x<0.2 fm or large momenta k > 1 GeV coupling constant is small $\boldsymbol{\alpha_s \ll 1}$ and interactions are weak.

Saturation physics

- Large parton density gives large momentum scale Qs(Saturation scale) Qs~ no. of partons per unit transverse area.
- Saturation physics is based on large internal momentum scale Qs which grows with decreasing bjorken x and increasing nuclear atomic number! ($Q_s \sim A^{1/3} (x)^{-\lambda}$ for α_s(Qs)<<1)

  We can use **perturbation theory** to study saturation physics(to calculate total cross sections,correlations etc.)
- We can use classical gluon field A_μ for calculations.