Exclusive processes, factorization and parton distributions

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Work in collaboration with:
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- **Jakub Wagner** (NCBJ)

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Outline

- Preliminaries: SM, PDFs, experiments in DIS
- DVCS, TCS & DDVCS: GPD function as an extension of a PDF
- Worldwide experiments on hadron structure
- Double deeply virtual Compton scattering **(DDVCS)**
  - Formulation *à la* Kleiss & Stirling
  - Tests of our KS-based formulation
- Summary

Preliminaries

- **Modern Particle Physics = Standard Model**

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

- *Gauge group of QCD*
- Strongly coupled: quarks & gluons coupled to form hadrons

Preliminaries

- **Modern Particle Physics = Standard Model**

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- *Gauge group of QCD*
- Strongly coupled: quarks & gluons coupled to form hadrons

Elementary:
bounded as
hadrons

Composite:
what you find in
nature

Preliminaries

- **How to access quarks & gluons?**
  - “Hitting” hadrons with high-energy particles: (virtual) photon

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    - Exclusive: **full** control over all final state particles

Preliminaries

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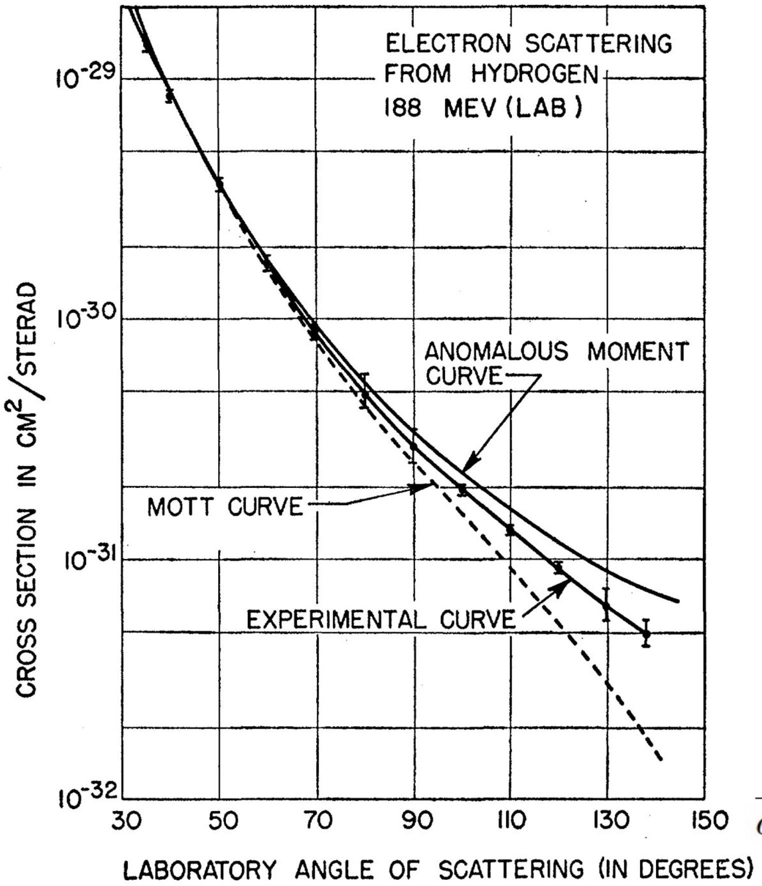
Preliminaries

- **How to access quarks & gluons?**
  - “Hitting” hadrons with high-energy particles: (virtual) photon
  - Inelastic scattering:
    - Inclusive: **no control over all** final state particles
    - Exclusive: **full** control over all final state particles
  - Quarks & gluons are collectively called *partons*
  - Late 1960s: SLAC & MIT prove the composite nature of proton

Preliminaries: elastic scattering



- **Elastic scattering of electron and proton**

Robert Hofstadter and Robert W. McAllister. Electron scattering from the proton. *Phys. Rev.*, 98:217–218, Apr 1955.

SLAC

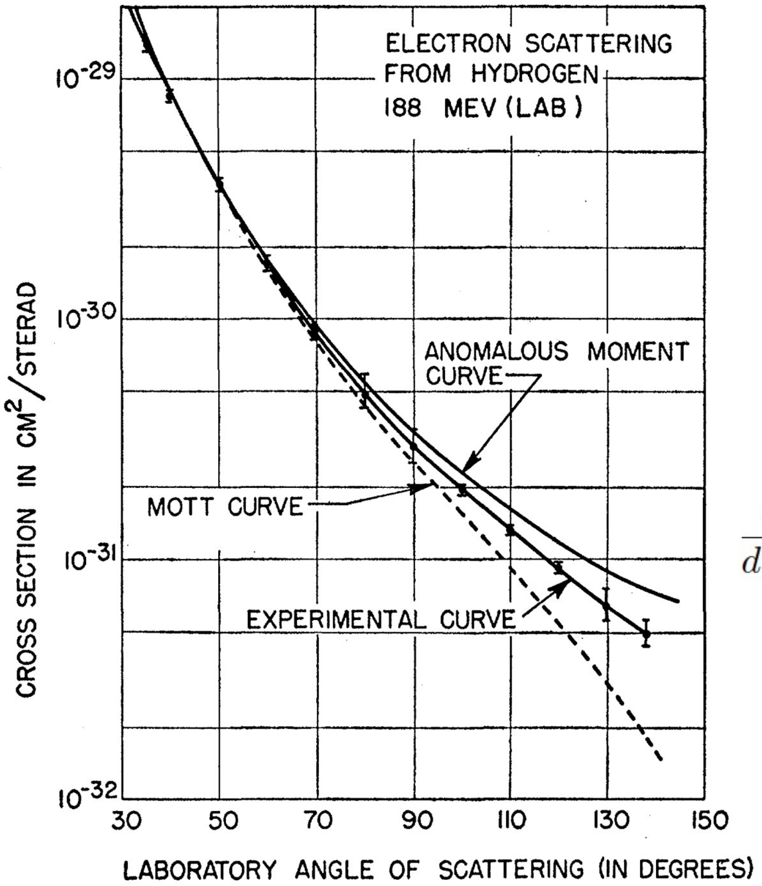
Mott xsec: spinless and point-like target

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{Mott}} = \underbrace{\frac{Z^2\alpha_{em}^2}{4E^2\sin^2(\theta/2)}}_{\text{Rutherford}}\cos^2(\theta/2)$$

Rosenbluth xsec: spin-1/2 and point-like target

$$\left.\frac{d^2\sigma}{d\Omega dE'}\right|_{\substack{\text{Ros,}\\ \text{point}}} = \left.\frac{d\sigma}{d\Omega}\right|_{\text{Mott}} \cdot \left[1 + 2\frac{Q^2}{4M^2}\tan^2(\theta/2)\right]\delta\left(\frac{Q^2}{2M} - \nu\right), \quad \nu = E - E'$$

Preliminaries: elastic scattering



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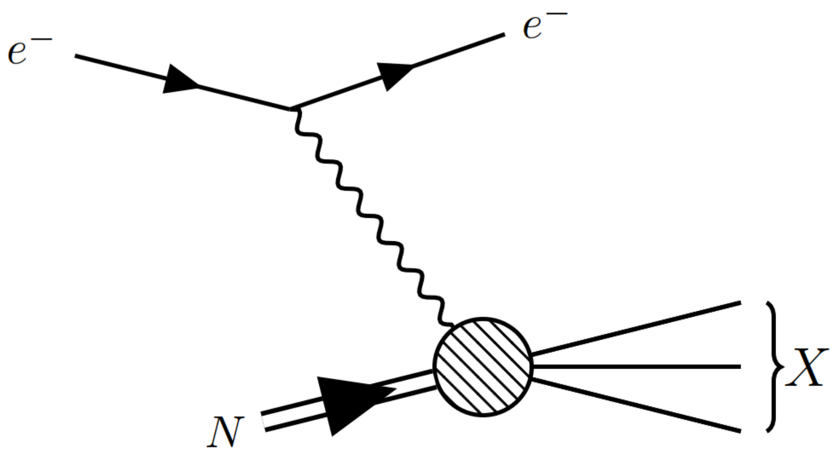
SLAC

Rosenbluth xsec: spin-1/2 and extended target

$$\frac{d^2\sigma}{d\Omega dE'}\bigg|_{\substack{\text{Ros,}\\\text{extended}}} = \frac{d\sigma}{d\Omega}\bigg|_{\text{Mott}} \cdot \left[F_E^2(|\vec{q}|^2) + 2\frac{Q^2}{4M^2}\tan^2(\theta/2)F_M^2(|\vec{q}|^2)\right]\delta\left(\frac{Q^2}{2M} - \nu\right)$$

Proton is an extended object of radius ~10^{-13} cm

- **First experiments breaking the proton**
  - Deep inelastic scattering (DIS): inclusive



$$d\sigma_{\mathrm{DIS}} \sim L^{\mu\nu} W_{\mu\nu}$$

pQED

Non-trivial QCD part: perturbative (coefficient function) convoluted with non-perturbative term (PDF)

Preliminaries: inelastic scattering

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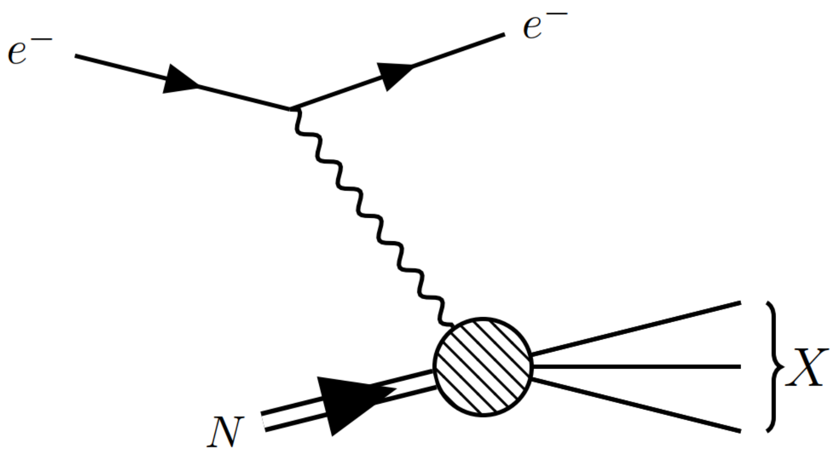
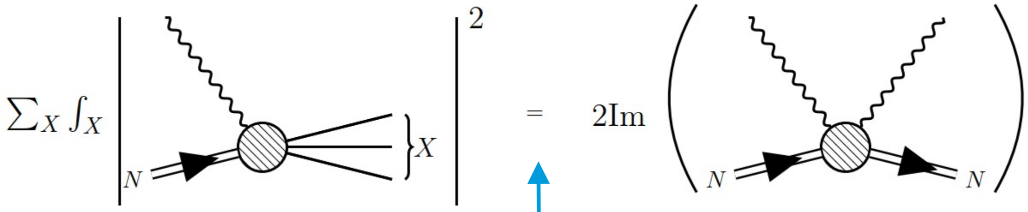




$$\sum_X \int_X |X\rangle\langle X| = 1$$

Inclusiveness in practice

Preliminaries: form factors & geometric shape

- DIS xsec in terms of form factors (F_1, F_2):

$$\left.\frac{d^2\sigma}{d\Omega dE'}\right|_{\mathrm{DIS}} = \left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{Mott}} \cdot \left[\frac{1}{\nu}F_2(x,Q^2) + \frac{2}{M}\tan^2(\theta/2)F_1(x,Q^2)\right], \quad x = \frac{Q^2}{2pq} \underbrace{=}_{\substack{\text{proton}\\\text{rest}\\\text{frame}}} \frac{Q^2}{2M\nu}$$

- If F_1, F_2 are independent of Q^2, then DIS = scattering with point-like particles = proton is not elementary

$$\rho(\vec{r}) \sim \int d^3\vec{q}\; e^{i\vec{q}\vec{r}} F(|\vec{q}|^2)$$

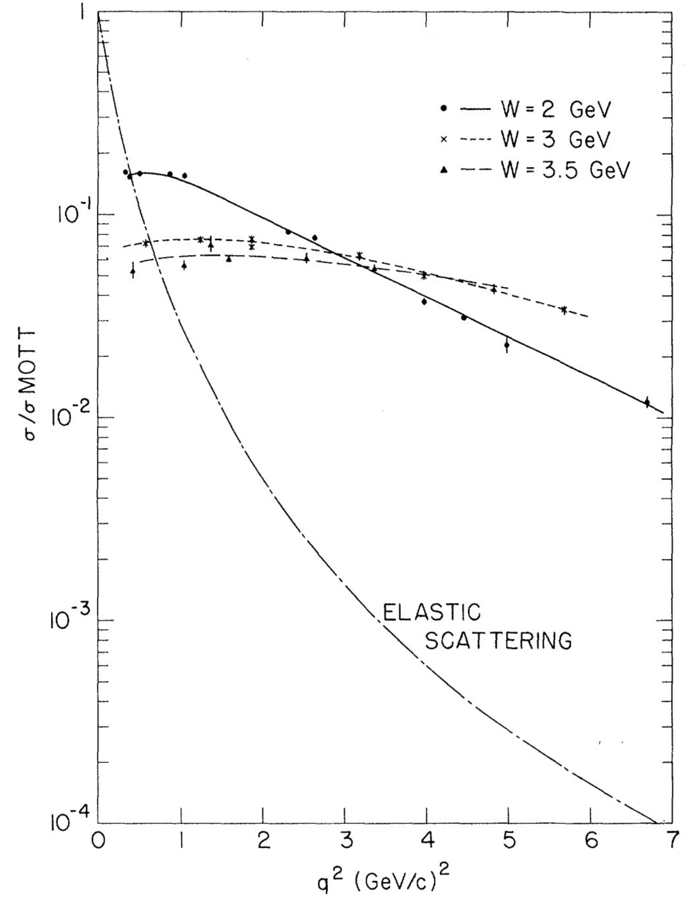
If $F(|\vec{q}|^2) \sim$ constant, then $\rho(\vec{r}) \sim \delta(\vec{r}) \Rightarrow$ point-like target

Preliminaries: inelastic scattering

- **Deep inelastic scattering of electrons off the proton**

M. Breidenbach et al., Phys. Rev. Lett. 23, 935 (1969).
DOI 10.1103/PhysRevLett.23.935

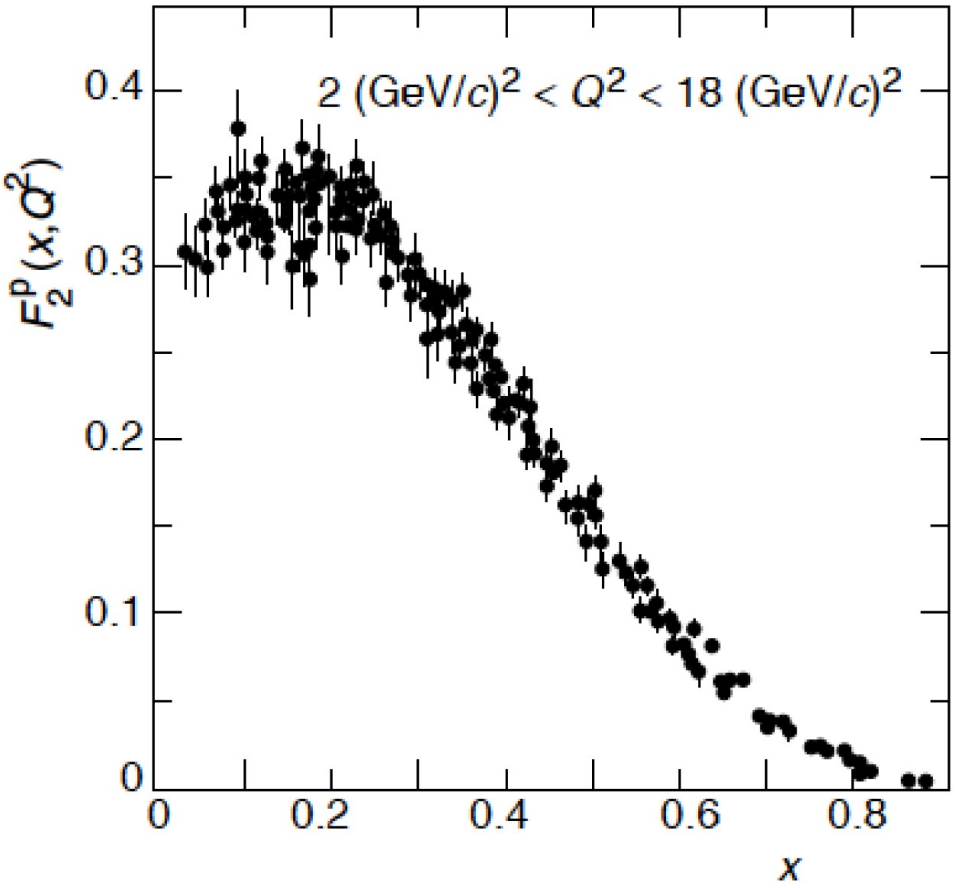
SLAC + MIT



- W = 3 and 3.5 GeV:

Almost independence with q^2 suggests scattering off elementary (point-like or very small) particles

Preliminaries: inelastic scattering



SLAC

Almost independent of Q^2: very small particles inside the proton that are interacting with each other, i.e.

$F_2(x, Q^2) \sim F_2(x)$

Plot taken from Carolina Riedl's talk at the 61 Cracow summer school of theoretical physics: Electron-Ion Collider physics (2021)

- **What does the xsec look like?**
  - Factorization in inclusive processes = **PDFs** in xsec

$$\sigma_{\gamma N \to X} = \sum_f \int_0^1 dx\ \sigma_{\gamma \mathfrak{q}_f \to X}(x) \mathrm{PDF}_f(x), \quad f = \text{flavor}$$

x = longitudinal momentum fraction

$$\mathrm{PDF}_f(x) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ix\bar{p}^+ z^-} \langle N | \bar{\mathfrak{q}}_f(-z/2) \gamma^+ \mathcal{W}[-z/2, z/2] \mathfrak{q}_f(z/2) | N \rangle \Big|_{z_\perp = z^+ = 0}$$

1D distribution

Preliminaries: understanding the PDF

- **Wilson line: definition**

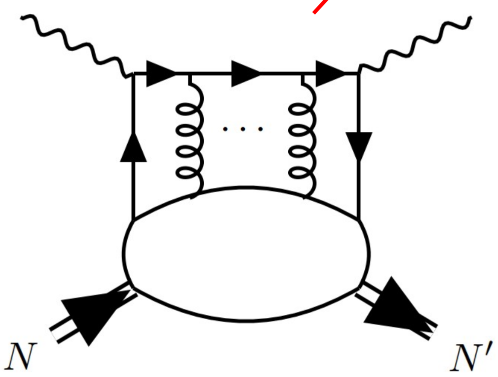
$$\mathrm{PDF}_f(x) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix\bar{p}^+ z^-} \langle N|\bar{\mathfrak{q}}_f(-z/2)\gamma^+ \mathcal{W}[-z/2, z/2]\mathfrak{q}_f(z/2)|N\rangle\Big|_{z_\perp = z^+ = 0}$$

$$\mathcal{W}[z_1^-, z_2^-] = \mathbb{P}\exp\left[ig\int_{z_2^-}^{z_1^-} da^- A^+(a^-)\right]$$

Preliminaries: understanding the PDF

- **Wilson line: origin**

$$\mathrm{PDF}_f(x) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix\bar{p}^+z^-} \langle N|\bar{\mathfrak{q}}_f(-z/2)\gamma^+\mathcal{W}[-z/2, z/2]\mathfrak{q}_f(z/2)|N\rangle\Big|_{z_\perp=z^+=0}$$



Preliminaries: understanding the PDF

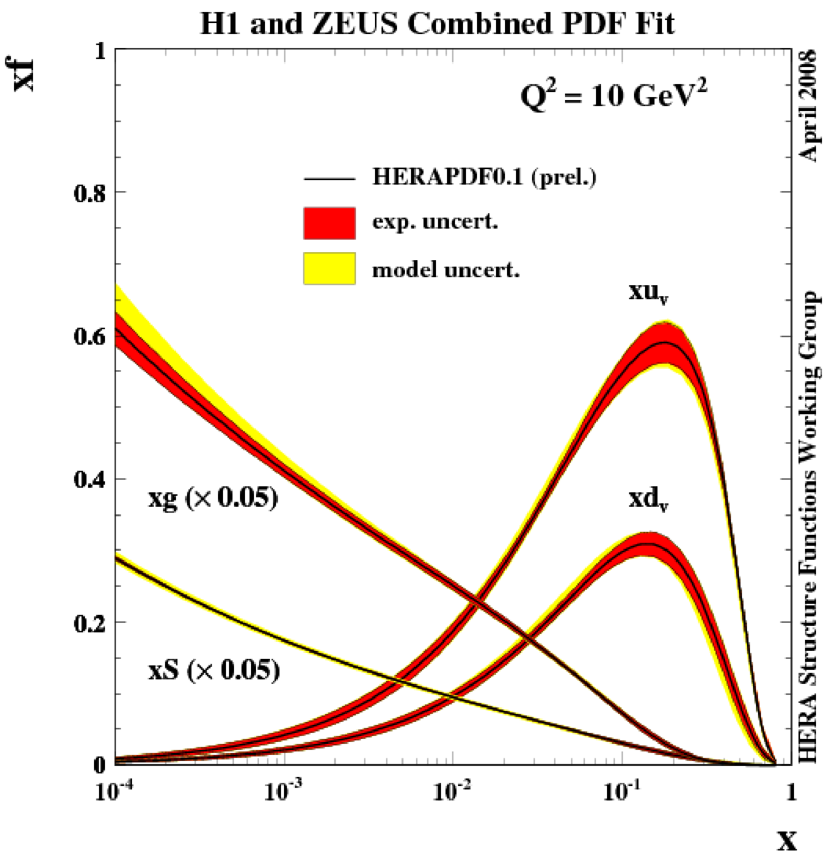
- **Gamma “plus”**

$$\mathrm{PDF}_f(x) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix\bar{p}^+ z^-} \langle N|\bar{\mathfrak{q}}_f(-z/2)\gamma^+\mathcal{W}[-z/2, z/2]\mathfrak{q}_f(z/2)|N\rangle\Big|_{z_\perp = z^+ = 0}$$

$$\gamma^+ = \frac{\gamma^0 + \gamma^3}{\sqrt{2}}$$

for a particular choice of the “plus” direction

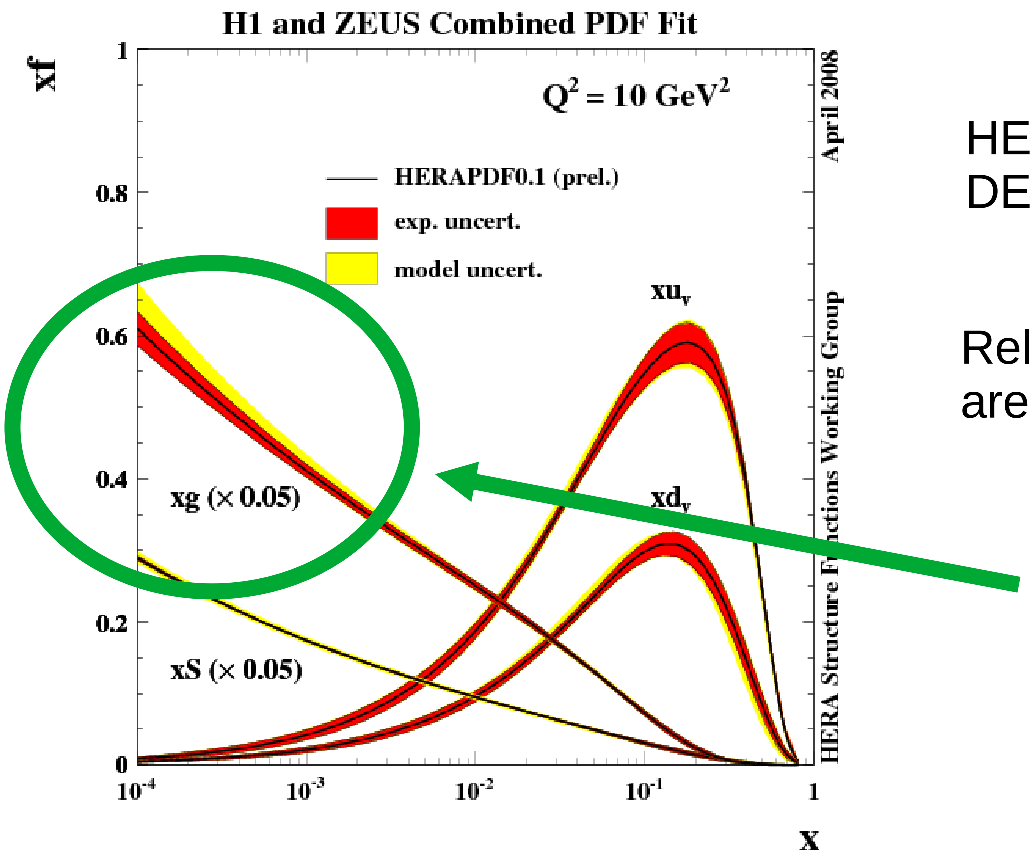
Preliminaries: PDFs as basis for experiments



HERA = Hadron-Electron Ring Accelerator
DESY, Germany

Relevance of PDFs: pp collisions at LHC (CERN) are parton collisions

Preliminaries: PDFs as basis for experiments



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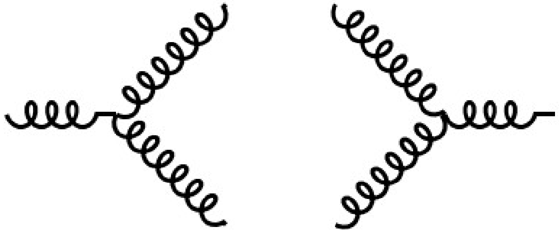
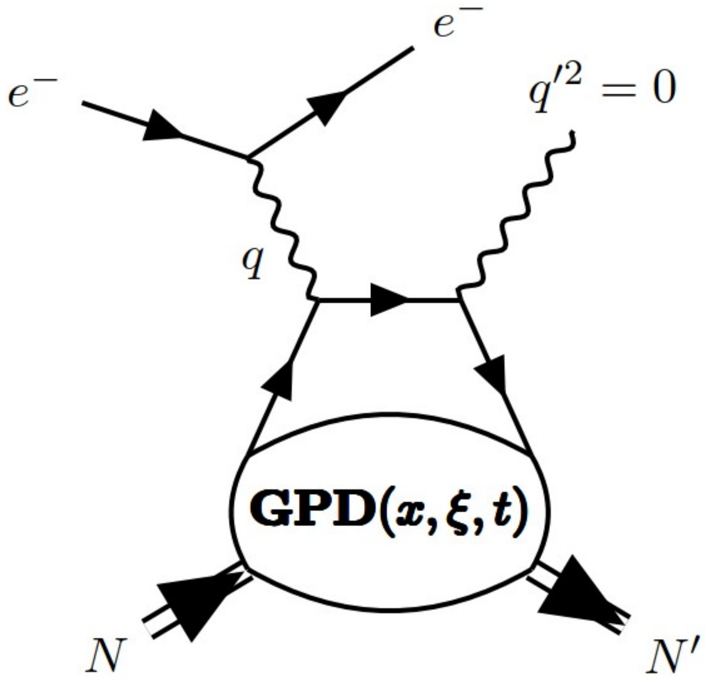


Improving PDF's picture: DVCS

- In the late '90s, Ji, Müller and Radyushkin introduced the Generalized Parton Distributions (GPDs) through Deeply Virtual Compton Scattering (DVCS) process



Electroproduction of a real photon

$$\xi = -\frac{\Delta \bar{q}}{2\bar{p}\bar{q}}$$

$$-t = -(p' - p)^2$$

$$\Delta = p' - p = q - q', \quad \bar{q} = \frac{q + q'}{2}, \quad \bar{p} = \frac{p + p'}{2}$$

GPD: 3D distribution

- **GPD** = Generalized Parton Distribution ≈ "3D version of a PDF (Parton Distribution Function)." With x the fraction of the hadron's longitudinal momentum carried by a quark:

$$\mathrm{GPD}_f(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix\bar{p}^+z^-} \langle N'|\bar{\mathfrak{q}}_f(-z/2)\gamma^+\mathcal{W}[-z/2,z/2]\mathfrak{q}_f(z/2)|N\rangle\Big|_{z_\perp=z^+=0}$$

$$t=\Delta^2=(p'-p)^2,\quad \xi=-\frac{\bar{q}\Delta}{2\bar{p}\bar{q}},\quad \rho=\frac{-\bar{q}^2}{2\bar{p}\bar{q}},\quad \bar{q}=\frac{q+q'}{2},\quad \bar{p}=\frac{p+p'}{2}$$

- **Importance:**
  - Connected to QCD energy-momentum tensor (**Ji's sum rules**): hadron's **spin puzzle**
  - **Tomography:** distribution of longitudinal momentum on the transverse (to hadron's motion) plane

$$q(x,\mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}}{4\pi^2} e^{-i\mathbf{b}_\perp\cdot\boldsymbol{\Delta}} \underbrace{H^q(x,0,t=-\boldsymbol{\Delta}^2)}_{\text{A particular GPD}}$$

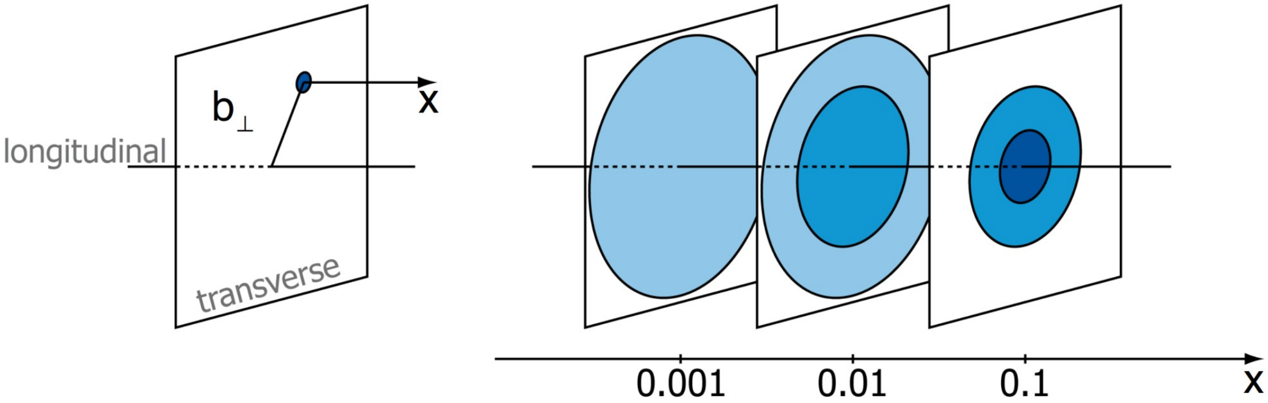
GPDs and spin crisis

- GPDs can be related to the energy-momentum tensor and therefore to total angular momentum
- GPDs allows to compute this amount through Ji's sum rule:

$$A^q(0) + B^q(0) = \int_{-1}^{1} x\left[H^q(x,\xi,0) + E^q(x,\xi,0)\right] = 2J^q$$

- **"An investigation of the spin structure of the proton in deep inelastic scattering of polarized muons on polarized protons," J. Ashman et al., Nucl. Phys. B328, 1 (1989) – *The European Muon Collaboration***
  - ***Conclusion of the paper: only a small fraction of the hadron spin comes from the spin of the quarks***

Nucleon tomography



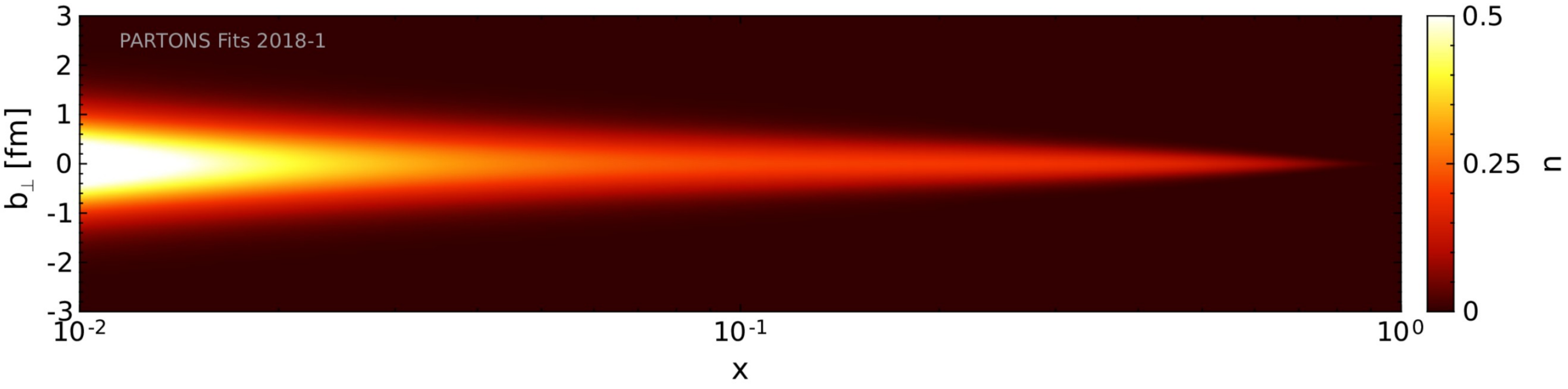
$$\sum x\, \mathbf{b}_\perp = 0$$

$$q(x, \mathbf{b}_\perp) = \int \frac{\mathrm{d}^2 \mathbf{\Delta}}{4\pi^2} e^{-i \mathbf{b}_\perp \cdot \mathbf{\Delta}} H^q(x, 0, t = -\mathbf{\Delta}^2)$$

Images and formulas from seminar at IJCLab (Dec 18th, 2020) by Paweł Sznajder

Nucleon tomography

Position vs momentum fraction of up quarks in an unpolarized proton



"Border and skewness functions from a leading order fit to DVCS data," H. Moutarde, P. Sznajder, J. Wagner, *Eur.Phys.J.C* 78 (2018) 11, 890,
DOI: 10.1140/epjc/s10052-018-6359-y

DVCS amplitude

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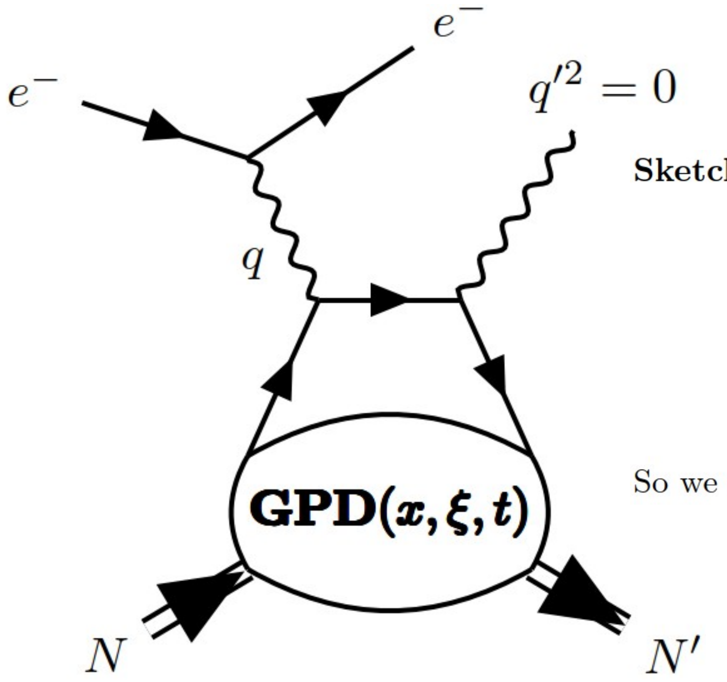

Electroproduction of a real photon

Sketch of DVCS amplitude (LO)

$$\mathcal{A}_{\mathrm{DVCS}} \sim \int_{-1}^{1} dx \, \frac{1}{x - \xi + i0} \mathrm{GPD}(x, \xi, t) + \cdots$$
$$= \mathrm{PV}\left(\int_{-1}^{1} dx \frac{1}{x - \xi} \mathrm{GPD}(x, \xi, t) \right) - \int_{-1}^{1} dx \; i\pi\delta(x - \xi) \mathrm{GPD}(x, \xi, t) + \cdots$$

So we can measure GPDs at $x = \xi$ only, i.e., we can access $\mathrm{GPD}(\xi, \xi, t)$

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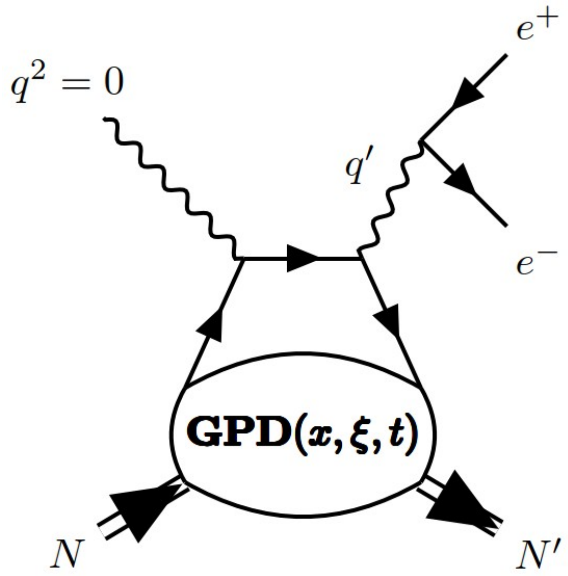
$$\begin{aligned}\mathcal{A}_{\mathrm{DVCS}} &\sim \int_{-1}^{1} dx\,\frac{1}{x-\xi+i0}\mathrm{GPD}(x,\xi,t)+\cdots \\ &= \mathrm{PV}\left(\int_{-1}^{1} dx\frac{1}{x-\xi}\mathrm{GPD}(x,\xi,t)\right)-\int_{-1}^{1} dx\; i\pi\delta(x-\xi)\mathrm{GPD}(x,\xi,t)+\cdots\end{aligned}$$

Dispersion relation: real part can be computed in terms of imaginary

$$\Re e\mathcal{A}(\xi,t)=\mathrm{PV}\left(\int_{-1}^{1} d\xi'\,\frac{\Im m\mathcal{A}(\xi',t)}{\xi'-\xi}\right)+D(t)$$

Other channels: TCS

- TCS is experimentally more challenging: xsec smaller than DVCS' (but already measured: "First Measurement of Timelike Compton Scattering," by P. Chatagnon et al., Phys. Rev. Lett. 127, 262501 (2021) – CLAS collab.)
- TCS = timelike Compton scattering = *photoproduction of a lepton pair*



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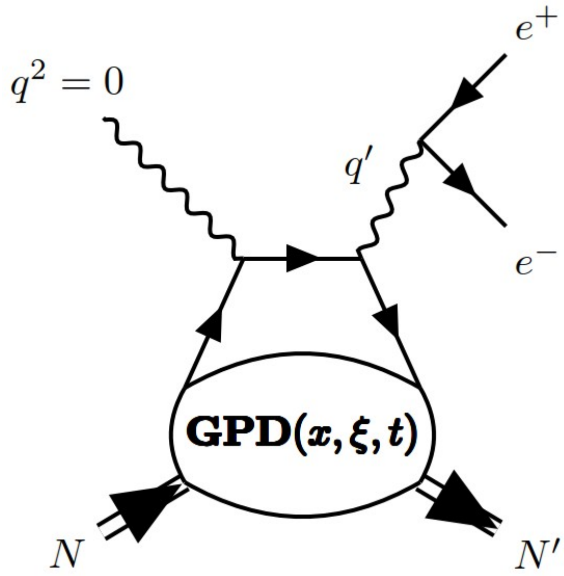


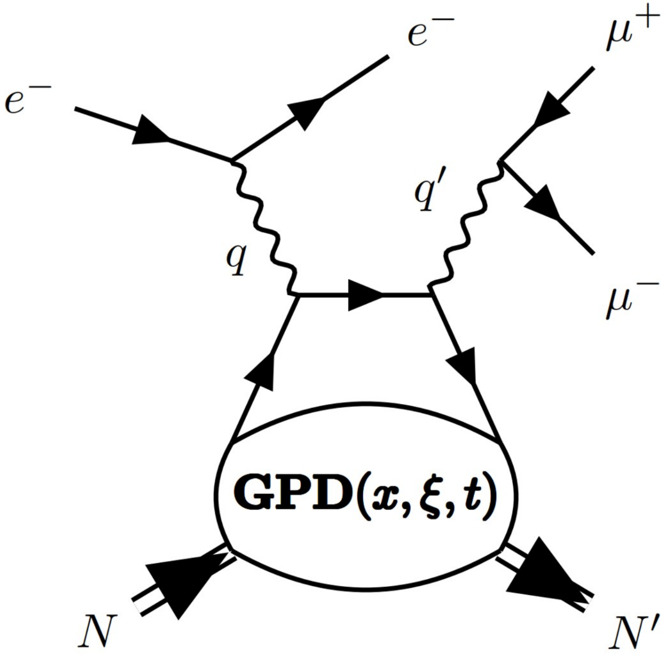
Sketch of TCS amplitude (LO)

$$
\begin{aligned}
\mathcal{A}_{\mathrm{TCS}} &\sim \int_{-1}^{1} dx\, \frac{1}{x+\xi+i0} \mathrm{GPD}(x,\xi,t) + \cdots \\
&= \mathrm{PV}\left(\int_{-1}^{1} dx \frac{1}{x+\xi} \mathrm{GPD}(x,\xi,t)\right) - \int_{-1}^{1} dx\; i\pi\delta(x+\xi)\mathrm{GPD}(x,\xi,t) + \cdots
\end{aligned}
$$

So we can measure GPDs at $x = -\xi$ only, i.e., we can access $\mathrm{GPD}(-\xi, \xi, t)$

Other channels: DDVCS

- DDVCS = double DVCS = *electroproduction of a lepton pair*
- Even more experimentally challenging than TCS



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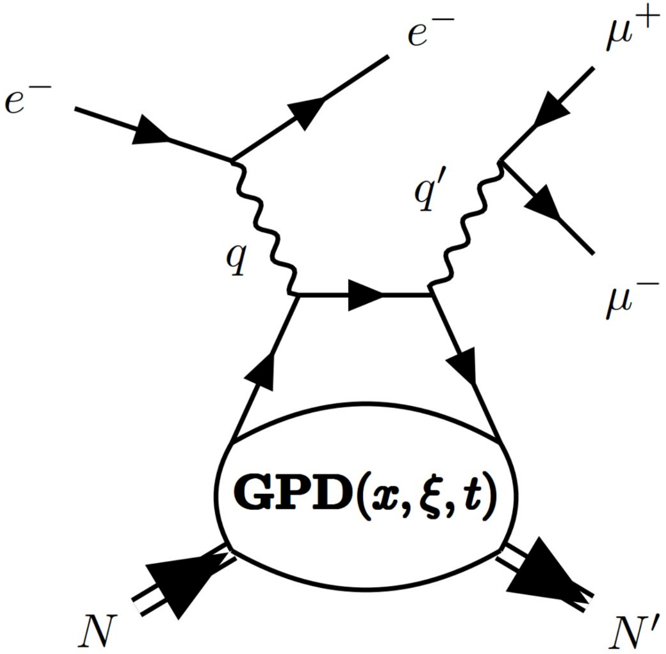

Sketch of DDVCS amplitude (LO)

$$
\begin{aligned}
\mathcal{A}_{\mathrm{DDVCS}} &\sim \int_{-1}^{1} dx\, \frac{1}{x-\rho+i0} \mathrm{GPD}(x,\xi,t) + \cdots \\
&= \mathrm{PV}\left(\int_{-1}^{1} dx \frac{1}{x-\rho} \mathrm{GPD}(x,\xi,t)\right) - \int_{-1}^{1} dx\; i\pi\delta(x-\rho)\mathrm{GPD}(x,\xi,t) + \cdots
\end{aligned}
$$

$$
\xi = -\frac{\bar{q}\Delta}{2\bar{p}\bar{q}}, \qquad \rho = \frac{-\bar{q}^2}{2\bar{p}\bar{q}}
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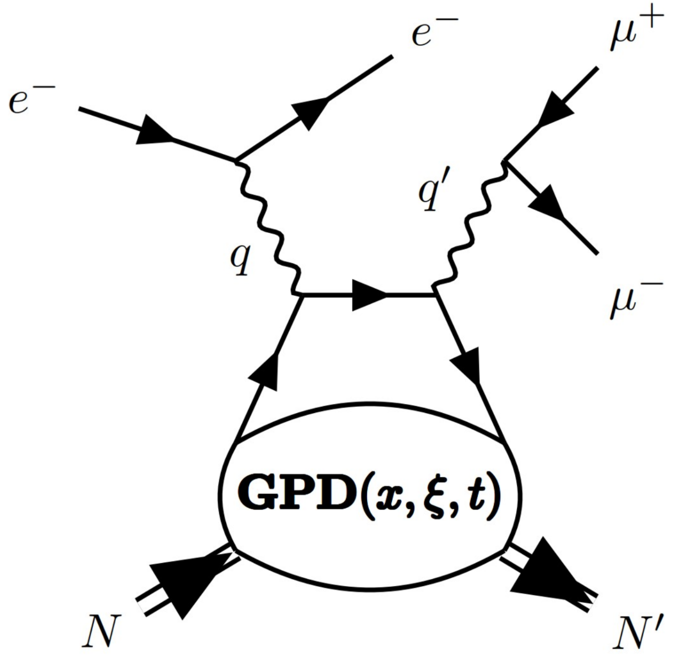

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we can access $\mathrm{GPD}(\rho,\xi,t)$

Experiments devoted to hadron structure

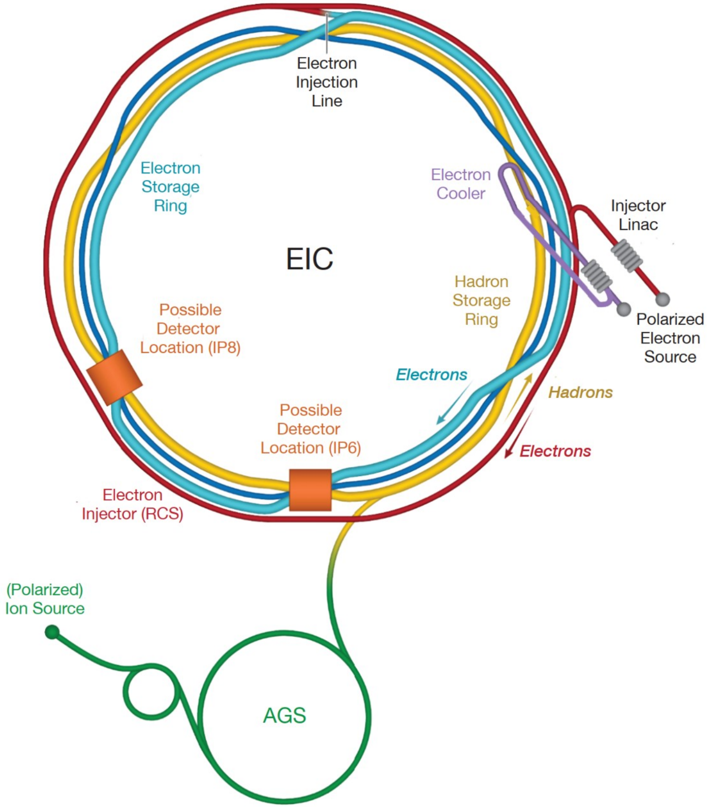


Figure 1.1: Schematic layout of the planned EIC accelerator based on the existing RHIC complex at Brookhaven National Laboratory.

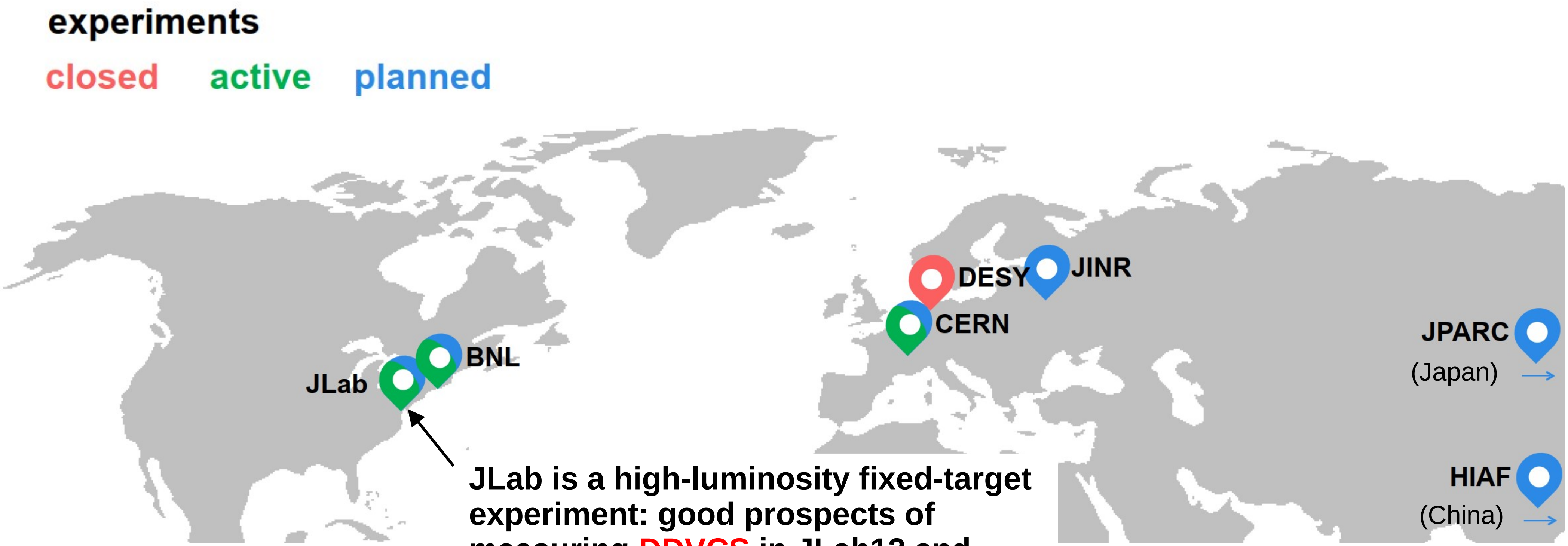
The **Electron-Ion Collider (EIC)**
to be built within the next decade
at Brookhaven National Laboratory (BNL), USA

"Science Requirements and Detector Concepts for the Electron-Ion Collider: EIC Yellow Report," R. A. Khalek et al.

Worldwide picture

Image courtesy of Paweł Sznajder

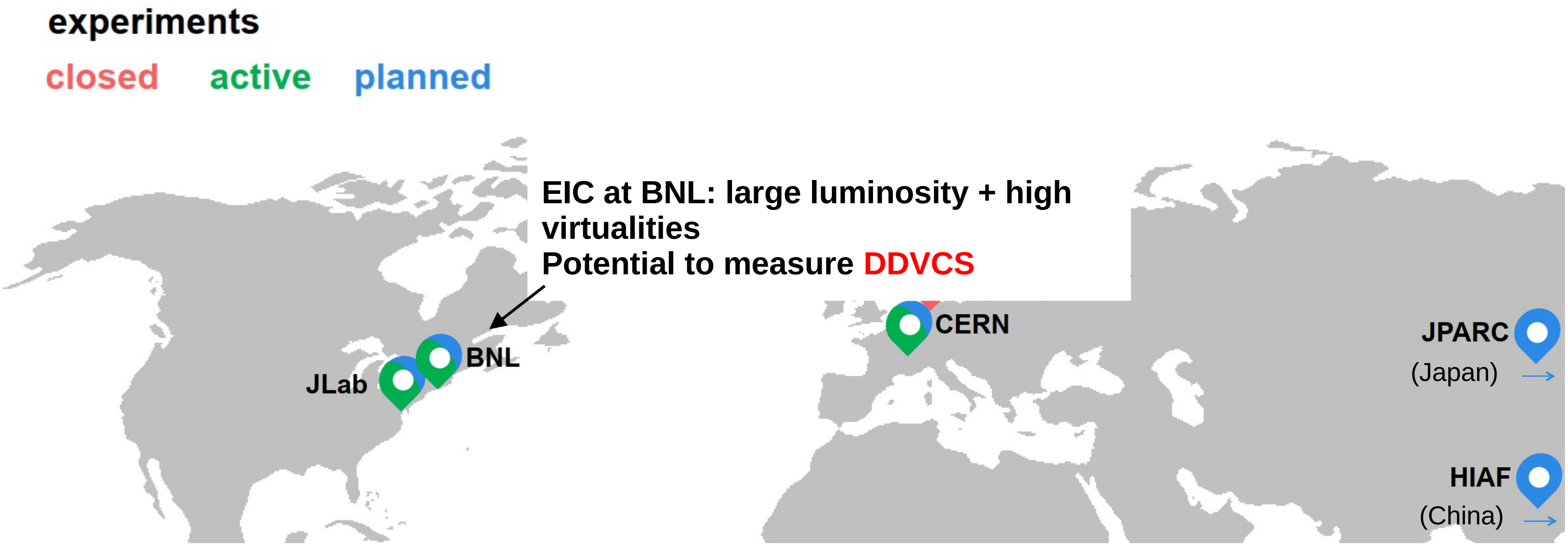
Worldwide picture



JLab is a high-luminosity fixed-target experiment: good prospects of measuring DDVCS in JLab12 and JLab20+ (if it gets approved)

Image courtesy of Paweł Sznajder

Worldwide picture



EIC at BNL: large luminosity + high virtualities
Potential to measure DDVCS

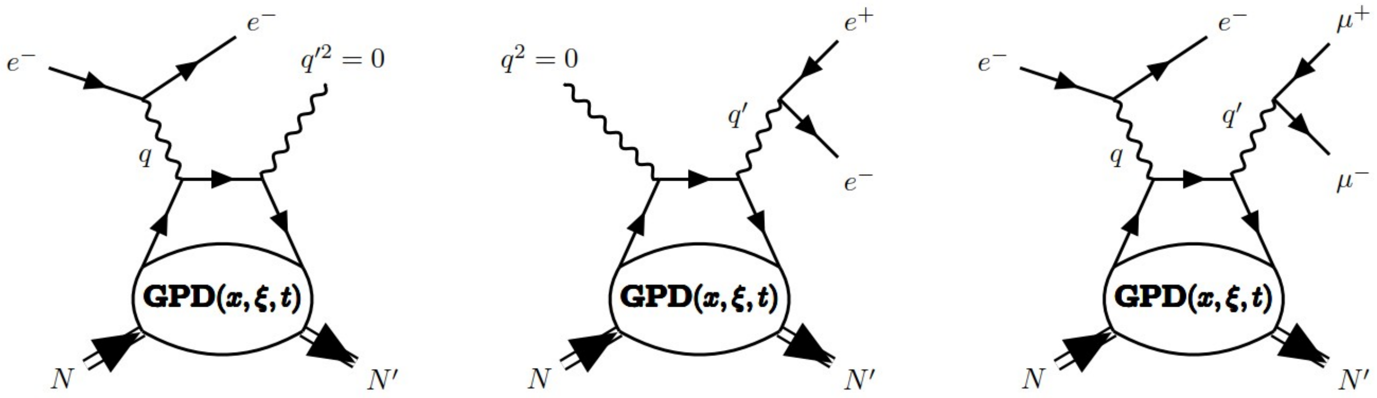
Image courtesy of Paweł Sznajder

Working out DDVCS

- DDVCS for firstly proposed by Belitsky, Mueller, Guidal and Vanderhaeghen in:
  - ***Exclusive Electroproduction of Lepton Pairs as a Probe of Nucleon Structure***, PRL 90, 022001 (2003)
  - ***Double Deeply Virtual Compton Scattering off the Nucleon***, PRL 90, 012001 (2003)
- Xsec by Belitsky and Mueller in ***Probing generalized parton distributions with electroproduction of lepton pairs off the nucleon***, Phys. Rev. D 68, 116005 (2003)
- That work seems to present some typos or mismatches because we cannot reproduce appropriate limits with it: taking a virtuality of DDVCS to be a reality you must recover either DVCS or TCS
- Consequently, we have performed a **rederivation of DDVCS' formulae** via Kleiss & Stirling's methods

Working out DDVCS

Recap: left to right: DVCS, TCS, DDVCS



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- Consequently, we have performed a **rederivation of DDVCS' formulae** via Kleiss & Stirling's methods

Kleiss & Stirling's techniques (KS): the basics

- **The idea of KS**: compute amplitudes, not the modulus squared of them

- Transform spinor products into new scalars *s* and *t* **(prevents the use of traces of Dirac gamma matrices)**:

$$s(p_1, p_2) := \bar{u}_+(p_1)u_-(p_2) = -s(p_2, p_1)$$

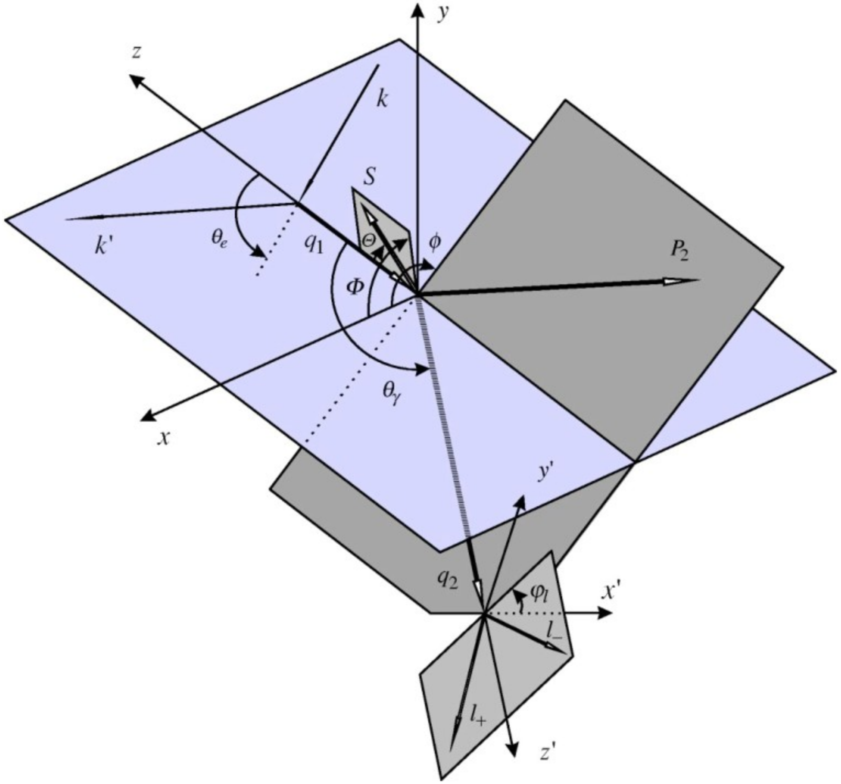
$$t(p_1, p_2) := \bar{u}_-(p_1)u_+(p_2) = [s(p_2, p_1)]^*$$

$$s(p_1, p_2) = (p_1^y + ip_1^z)\sqrt{\frac{p_2^0 - p_2^x}{p_1^0 - p_1^x}} - (p_2 \leftrightarrow p_1)$$

KS' paper: ***Spinor Techniques for Calculating p anti p → W+-/Z0 +Jets***. Nuclear Physics B262 (1985) 235-262

DDVCS' set-up

- KS methods are convenient for processes with many particles as DDVCS: 2 to 4 scattering

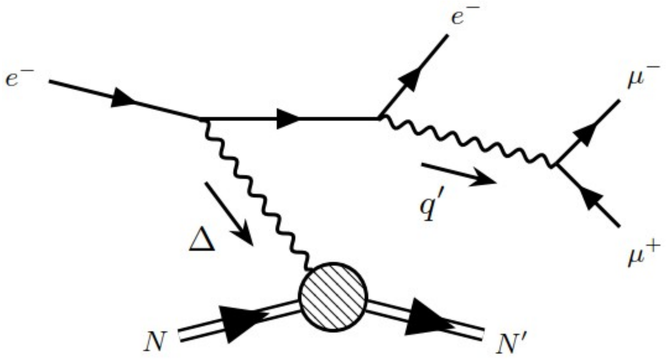


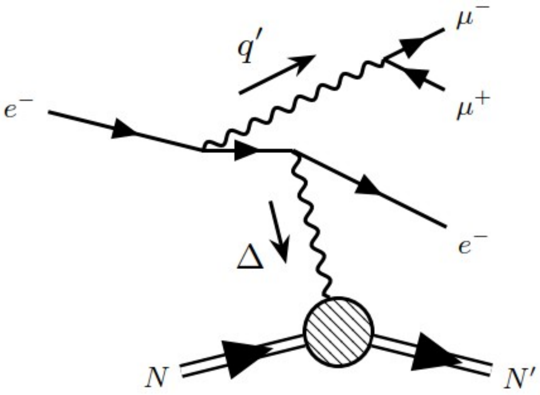
7-fold xsec for unpolarized target; 8-fold if polarized

Belitsky and Mueller in “Probing generalized parton distributions with electroproduction of lepton pairs off the nucleon,” Phys. Rev. D 68, 116005 (2003)

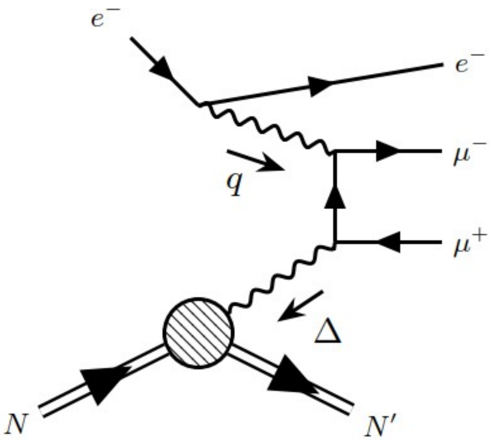
Electroproduction of lepton pair = DDVCS + **BH (pure QED)**

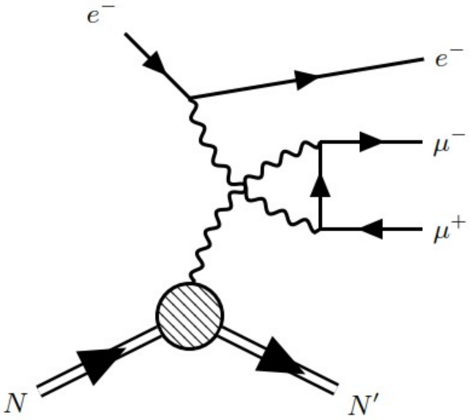
BH1 + BH1-crossed =





BH2 + BH2-crossed =





Example: BH1 *à la* KS

- KS application to BH1 diagram of DDVCS:

amplitude

$$i\widetilde{\mathcal{M}}_{\text{BH1}} = \left(\frac{ie^4}{(q'^2+i0)(\Delta^2+i0)((k-\Delta)^2+i0)}\right)^{-1} i\mathcal{M}_{\text{BH1}}$$

$$\begin{aligned} i\widetilde{\mathcal{M}}_{\text{BH1}} =& (F_1+F_2)\sum_L \Big(Y_{s_2 s_1} f(s_\ell,\ell_-,\ell_+;s,k',L) f(s,L,k;+,r'_{s_2},r_{s_1}) + \\ & Z_{s_2 s_1} f(s_\ell,\ell_-,\ell_+;s,k',L) f(s,L,k;-,r'_{-s_2},r_{-s_1})\Big) - \frac{F_2}{2M} J^{(2)}_{s_2 s_1} \sum_{L,R} f(s_\ell,\ell_-,\ell_+;s,k',L) g(s,L,R,k) \end{aligned}$$

- For example, *f* function is defined as

$$f(s,k_0,k_1;s',k_2,k_3) = \bar{u}_s(k_0)\gamma^\mu u_s(k_1)\bar{u}_{s'}(k_2)\gamma_\mu u_{s'}(k_3)$$

that can be expressed by means of *s* and *t* KS scalars

Dedicated softwares

➔ PARTONS platform: open-source C++ program

- Contains several GPD models
- Leading twist… but higher twist corrections will be included in near future
- Useful for theorists and experimentalists
- Provides xsecs, Compton Form Factors, etc
- DVCS, TCS and DVMP are already included

PARtonic Tomography Of Nucleon Software

To download and for tutorials:
http://partons.cea.fr

Description of architecture:
Eur. Phys. J. C78 (2018), 478

Dedicated softwares

➔ EpIC Monte Carlo event generator in C++

- Uses PARTONS framework
- Includes radiative corrections
- Generates the kinematic configurations following the probability distributions given by PARTONS
- DVCS, TCS and DVMP are already included

Access EpIC via GitHub:

https://github.com/pawelsznajder/epic

Detail description and architecture:

arXiv:2205.01762 [hep-ph]

DVCS & TCS limits of DDVCS

- **DDVCS to DVCS**

$$\int d\Omega_\ell \underbrace{\frac{d^7\sigma}{dx_B dQ^2 dQ'^2 d|t| d\phi d\Omega_\ell}}_{\text{DDVCS}} \xrightarrow{Q'^2\to 0} \underbrace{\left(\frac{d^4\sigma}{dx_B dQ^2 d|t| d\phi}\right)}_{\text{DVCS}} \frac{N}{Q'^2}, \qquad N = \frac{\alpha_{em}}{3\pi}$$

- **DDVCS to TCS**

Evaluate energy of incoming virtual photon to be used as energy of TCS photon beam

$$\nu = \frac{Q^2}{2Mx_B}$$

Divide by flux Γ and get rid of x_B and Q^2 differentiation

$$\Gamma = \frac{\alpha_{em}}{2\pi Q^2}\left(1 + \frac{(1-y)^2}{y} - \frac{2(1-y)Q^2_{\min}}{yQ^2}\right)\frac{\nu}{Ex_B}, \qquad Q^2_{\min} = \frac{(ym_e)^2}{1-y}$$

DVCS & TCS limits of DDVCS

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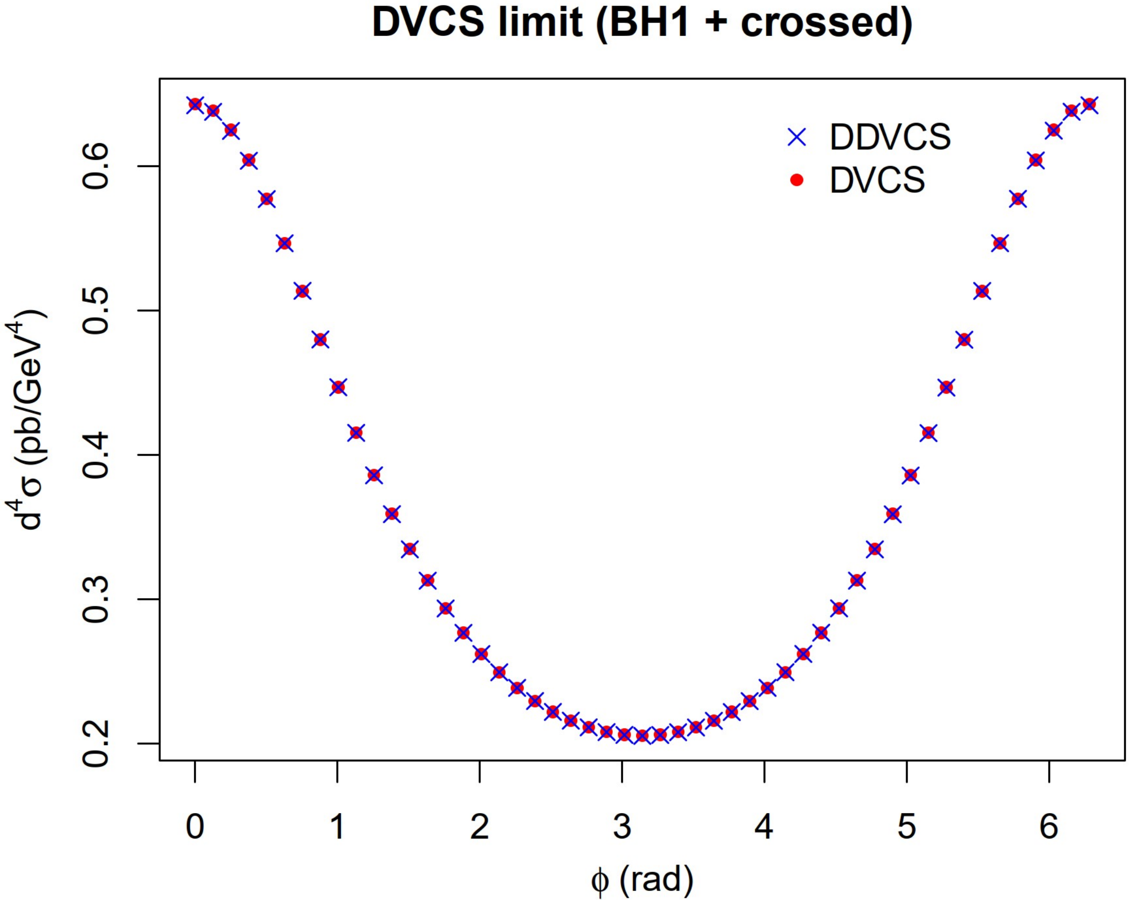
In what follows, preliminary results of these limits are shown

DVCS & TCS limits of DDVCS



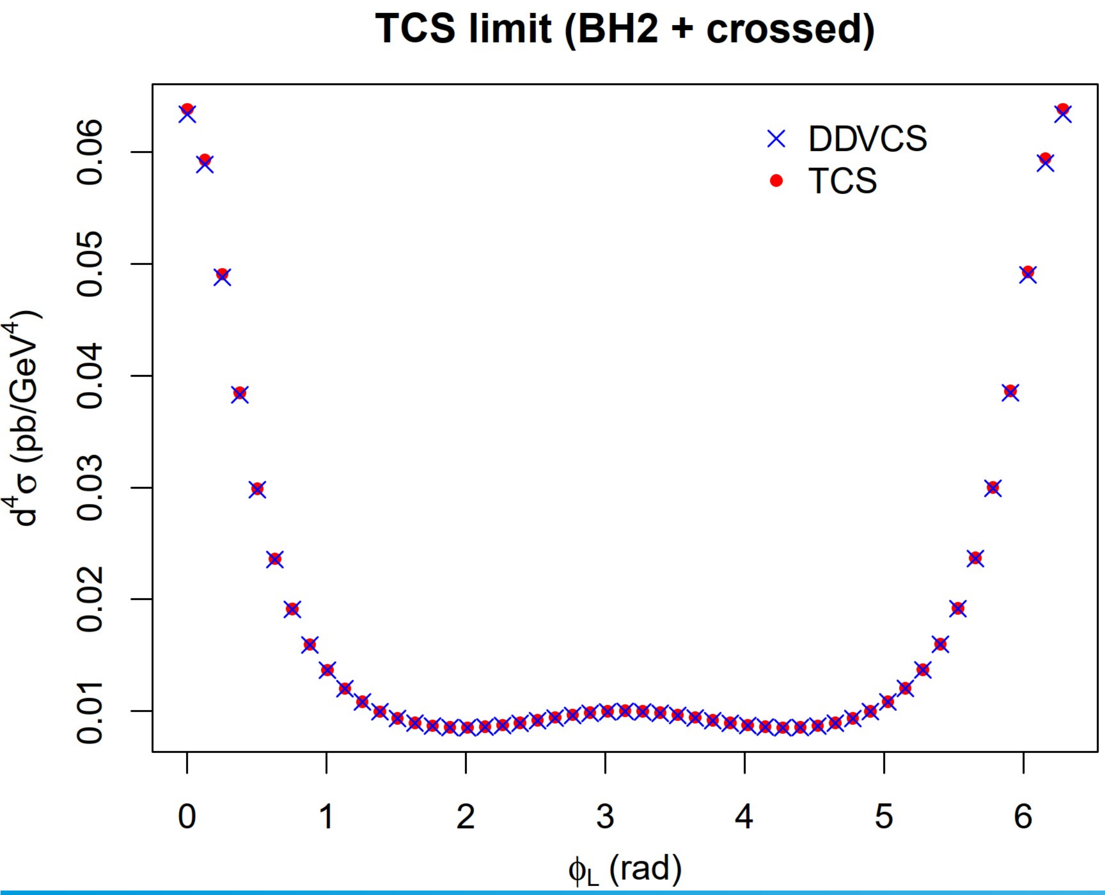
$x_B = 0.04$, $t = -0.1$ GeV^2, $-q^2 = 10$ GeV^2, $q'^2 \approx 0.001$ GeV^2, $E_{beam} = 160$ GeV

DVCS formulae:

Belitsky et al., *Theory of deeply virtual Compton scattering on the nucleon*, Nuclear Physics B629 (2002)

Belitsky et al., *Compton scattering: from deeply virtual to quasi-real*, Nuclear Physics B878 (2014)

DVCS & TCS limits of DDVCS

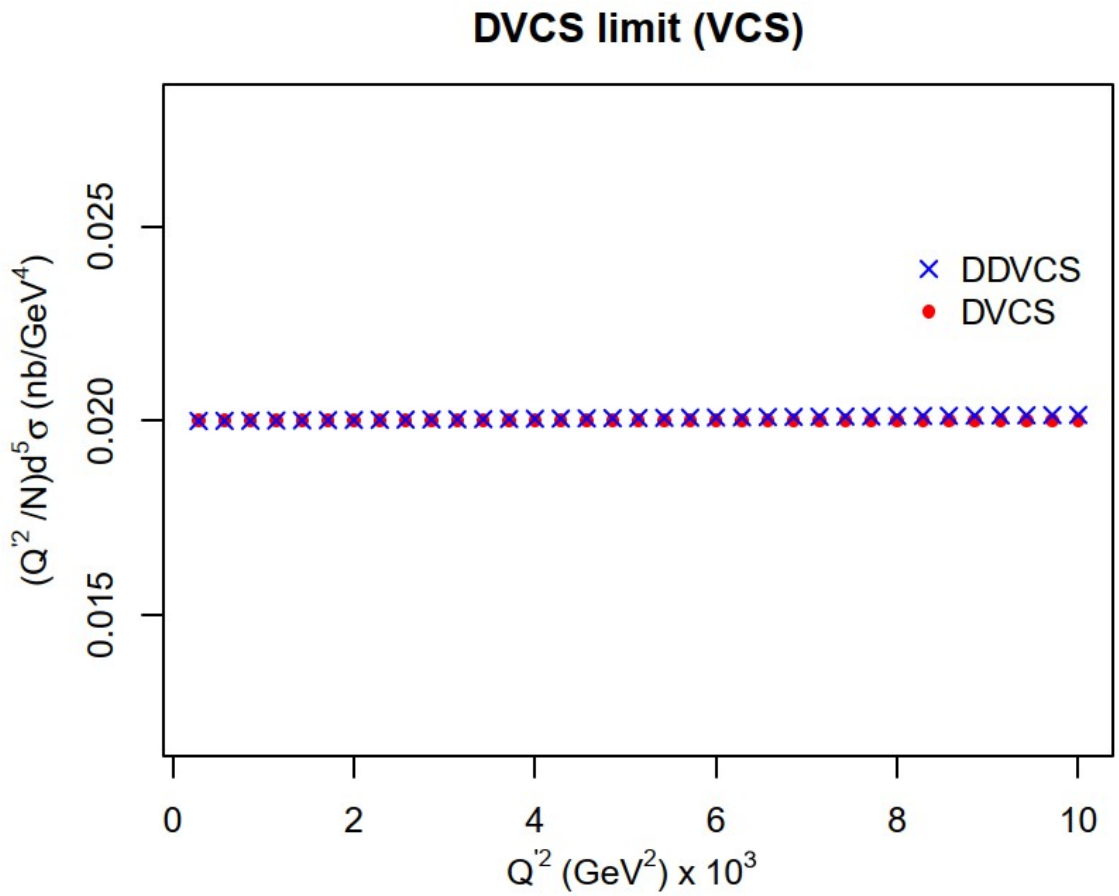


$x_B = 2 \cdot 10^{-4}$, $t = -0.5$ GeV^2, $-q^2 = 2 \cdot 10^{-3}$ GeV^2, $q'^2 = 1$ GeV^2, $E_{beam} = 12$ GeV

TCS formulae:

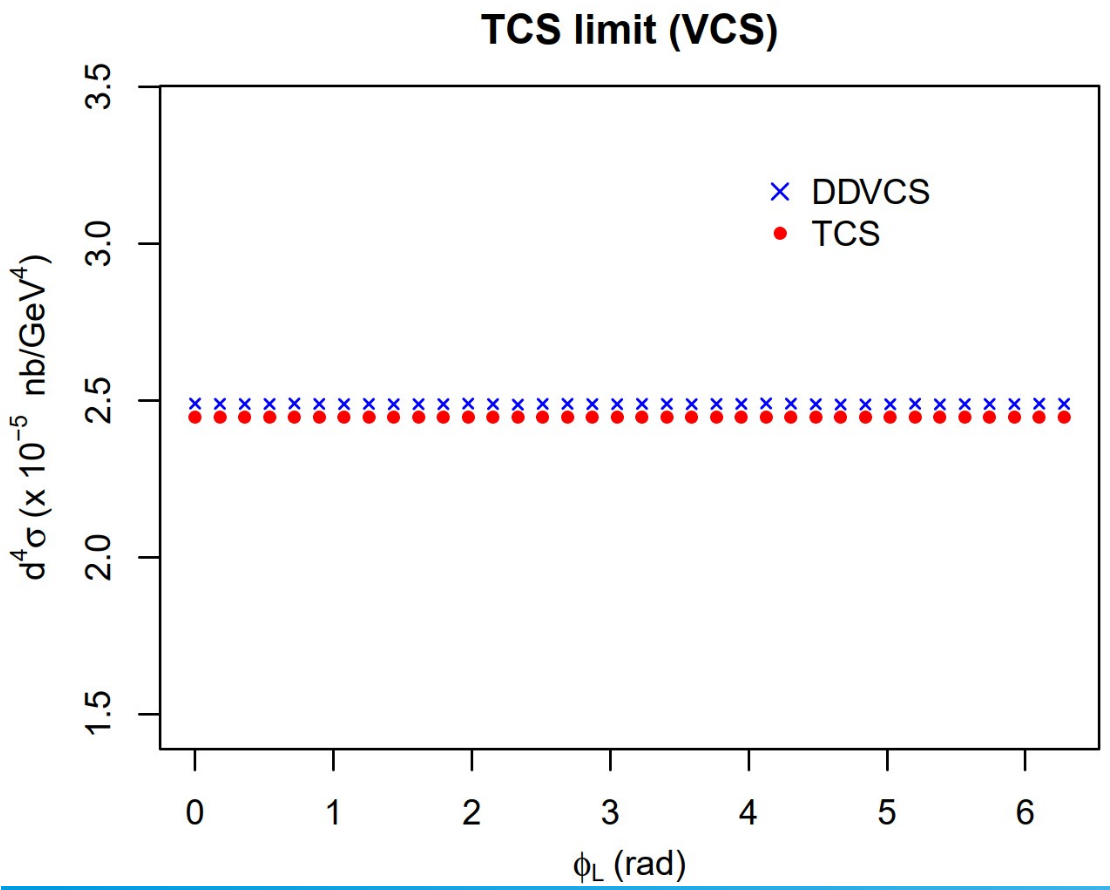
Berger et al., *Timelike Compton scattering: exclusive photoproduction of lepton pairs*, The European Physics Journal C 23 (2002)

DVCS & TCS limits of DDVCS



x_B = 0.04, t = -0.01 GeV^2, -q^2 = 10 GeV^2, E_{beam} = 160 GeV, ϕ = 0

DVCS & TCS limits of DDVCS



$x_B = 0.5 \cdot 10^{-5}$, $t = -0.01$ GeV^2, $-q^2 = 0.5 \cdot 10^{-3}$ GeV^2, $q'^2 = 10$ GeV^2, $E_{beam} = 60$ GeV, $\theta_L = \pi/2$

Relative errors in y-axis:

(DDVCS-TCS)/TCS ~ 2%

DVCS & TCS limits of DDVCS

- Xsec goes as $1/Q^2$ meaning that there is a delicate cancellation of such a dependence on the amplitude

- As shown, these limits are not trivial and require a careful analysis

Summary

- Differences between inclusive (unknown pieces in the final state) & exclusive QCD processes (full control of all particles)
- Experimental evidence of parton structure: constant form factor with the resolution power of the probe (photon)
- Differences between 1D PDFs and 3D GPDs: tomography and total angular momentum only accessible via GPDs
- Knowledge of PDFs is fundamental to make computations in the SM wherever hadrons play a role (LHC experiments, for instance)

Summary

- New analytical formulae has been derived for the electroproduction of a lepton pair
- DDVCS is already implemented in PARTONS (LO + leading-twist)
- We are interested in observables such as the beam spin asymmetry proportional to Im(BH x DDVCS*)
- Code will be included in EpIC MC generator to study feasibility of DDVCS