Rip Cosmological Models in Extended Symmetric Teleparallel Gravity

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Topic Outlines

- Introduction
- Overview of $f(Q,T)$ gravity
- Mathematical formalism
- Three rip cosmological models
- Analysis of the model
- Results and Conclusion

Introduction

- What is the requirement of modifying General Relativity?

- What is the advantages of considering modified theories of gravity?

- Why we are studying rip cosmological model in modified theories of gravity?
  - ▶phantom acceleration as transient phenomenon
  - ▶ Quantum effect
  - ▶ Modified theories of gravity
  - ▶ To couple dark energy with dark matter in the special way

Overview of $f(Q,T)$ gravity

• General relativity is basically a geometric theory, which is formulated in the Riemann metrical space and it has a great role within modified theories of gravity and also it helps to describe the gravitational field.[1]

• Even though Einstein's general relativity currently regarded as one of the most effective theories, there appear to be some limitations on standard GR in describing those phenomena in the wake of current observational advances in cosmology.

• We propose an extension of the symmetric teleparallel gravity in which the gravitational action L is given by an arbitrary function f, of the non-metricity Q and the trace of the matter-energy momentum tensor T, so that [2] $L = f(Q,T)$.

• We imposed the cosmological model which is the functional form of $f(Q,T)$:

$$f(Q,T) = aQ^m + bT$$

[1] Y. Xu, T. Harko, et al., *Eur. Phys. J. C*, **80**, 449 (2020).
[2] Y. Xu, G. Li, T. Harko, S. Liang, *Eur. Phys. J. C*, **79**, 708 (2019).

Mathematical Formalism

The gravitational action is:

$$S = \int \left[\frac{1}{16\pi} f(Q,T) + L_M \right] \sqrt{-g}\, d^4x \tag{1}$$

By varying the above gravitational action:

$$-\frac{2}{\sqrt{-g}} \nabla_\alpha (f_Q \sqrt{-g} P^\alpha{}_{\mu\nu}) - \frac{1}{2} f g_{\mu\nu} + f_T (T_{\mu\nu} + \Theta_{\mu\nu}) - f_Q (P_{\mu\alpha\beta} Q_\nu{}^{\alpha\beta} - 2Q^{\alpha\beta}{}_\mu P_{\alpha\beta\nu}) = 8\pi T_{\mu\nu} \tag{2}$$

With connection:

$$\nabla_\mu \nabla_\nu \left(\sqrt{-g} f_Q p^{\mu\nu}{}_\alpha + 4\pi H_\alpha{}^{\mu\nu} \right) = 0 \tag{3}$$

The traces of the non-metricity:

$$Q_\alpha = Q_\alpha{}^\mu{}_\mu \qquad \tilde{Q}_\alpha = Q^\mu{}_{\alpha\mu} \tag{4}$$

Where,

$$p^\alpha{}_{\mu\nu} = -\frac{1}{2} L^\alpha{}_{\mu\nu} + \frac{1}{4} (Q^\alpha - \tilde{Q}^\alpha) g_{\mu\nu} - \frac{1}{4} \delta^\alpha_{(\mu} Q_{\nu)} \tag{5}$$

General affine connection:

$$\Gamma^\lambda_{\mu\nu} = \mathring{\Gamma}^\lambda_{\mu\nu} + C^\lambda_{\mu\nu} + L^\lambda_{\mu\nu} \tag{6}$$

Also given

$$Q_{\lambda\mu\nu} = -\frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} + g_{\nu\sigma}\hat{\Gamma}^{\sigma}_{\mu\lambda} + g_{\sigma\mu}\hat{\sigma}^{\sigma}_{\nu\lambda} \quad (8)$$

$$\Gamma^{\lambda}_{\mu\nu} = -L^{\lambda}_{\mu\nu} \quad (9)$$

$$Q \equiv -g^{\mu\nu}(L^{\alpha}_{\beta\mu}L^{\beta}_{\nu\alpha} - L^{\alpha}_{\beta\alpha}L^{\beta}_{\mu\nu}) \quad (10)$$

Assume a flat FLRW space time:

$$ds^2 = a^2(t)(dx^2 + dy^2 + dz^2) - N^2(t)dt^2$$

$$H = \frac{\dot{a}}{a}, \qquad \tilde{T} \equiv \frac{\dot{N}}{N}$$

By solving equation (10) we get

$$Q = 6\frac{H^2}{N^2}$$

The energy momentum tensor is given by

$$T^{\mu}_{\nu} = diag(-\rho, p, p, p)$$

Also,

$$\Theta^{\mu}_{\nu} = diag(2\rho + p, -p, -p, -p)$$

By using FLRW metric from the field equation we can easily find

$$\frac{f}{2} - 6F\frac{H^2}{N^2} = 8\pi\tilde{G}(\rho + p) \tag{11}$$

$$\frac{f}{2} - \frac{2}{N^2}\left[\left(\dot{F} - F\tilde{T}\right)H + F\left(\dot{H} + 3H^2\right)\right] = -8\pi p \tag{12}$$

Next we consider the standard case when $N = 1$ which is the case of standard FLRW geometry.Thus we get

$$Q = 6H^2 \tag{13}$$

and the generalized Friedmann equations reduces to

$$\rho = \frac{1}{8\pi}\left[\frac{f}{2} - 6FH^2 - 2\frac{\tilde{G}}{1+\tilde{G}}(\dot{F}H + F\dot{H})\right] \tag{14}$$

$$p = -\frac{1}{8\pi}\left[\frac{f}{2} + 6FH^2 + 2(\dot{F}H + F\dot{H})\right] \tag{15}$$

$$F \equiv f_Q \qquad \text{and} \quad 8\pi\tilde{G} \equiv f_T \tag{16}$$

Three Rip Cosmological Models

Our considered model is, $f(Q,T) = aQ^m + bT$

$$p = \frac{-(1-2m)aQ^m + 2\dot{\chi}[2+\kappa-\kappa\kappa_1]}{4\pi[(2+\kappa)(2+3\kappa)-3\kappa^2]} \tag{17}$$

$$\rho = \frac{(1-2m)aQ^m + 2\dot{\chi}[3\kappa-(2+3\kappa)\kappa_1]}{4\pi[(2+\kappa)(2+3\kappa)-3\kappa^2]} \tag{18}$$

$$\omega = \frac{-(1-2m)aQ^m + 2\dot{\chi}[2+\kappa-\kappa\kappa_1]}{(1-2m)aQ^m + 2\dot{\chi}[3\kappa-(2+3\kappa)\kappa_1]} \tag{19}$$

Little Rip

• In 2011, Frampton, Ludwick and Sherrer has given some crucial concept about little rip along with some description and conditions of future singularities.

• The little rip is a cosmological abrupt event predicted by some phantom dark energy models that could describe the future evolution of our Universe.

• Only phantom energy with improbable physical attributes is capable of experiencing a sudden rip singularity. Physically, in the little rip, the scale factor and the density are never infinite at a finite time.

- The little rip scale factor[3,4] is taken as:

$$R = R_0 Exp^{\left[\frac{A}{\lambda}(e^{\lambda t} - e^{-\lambda t_0})\right]}$$

- $H = H_0 e^{\lambda t}$
- $q = -1 - \frac{\lambda e^{-\lambda t}}{A}$

The dynamical parameters are represented as:

$$\begin{aligned} p &= -\frac{a(6)^{m-1}(1-2m)(Ae^{\lambda t})^{2m-1}}{8\pi(1+2\kappa)}\left[3Ae^{\lambda t} + m\lambda(2+\kappa-\kappa\kappa_1)\right], \\ \rho &= \frac{a(6)^{m-1}(1-2m)(Ae^{\lambda t})^{2m-1}}{8\pi(1+2\kappa)}\left[3Ae^{\lambda t} - m\lambda(3\kappa-2\kappa_1-3\kappa\kappa_1)\right], \\ \omega &= -1 - \frac{2m\lambda(1-\kappa_1+2\kappa-2\kappa\kappa_1)}{3Ae^{\lambda t} - m\lambda(3\kappa-2\kappa_1-3\kappa\kappa_1)} \end{aligned}$$

[3] P. H. Frampton, K. J. Ludwick, R. J. Scherrer, *Phys. Rev. D*, **84**, 063003 (2011).
[4] P. H. Frampton, K. J. Ludwick, R. J. Scherrer, *Phys. Rev. D*, **85**, 083001 (2012).

Analysis of the LR graph

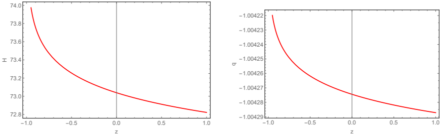

Figure 1: Behaviour of Hubble parameter (left panel) and deceleration parameter (right panel) in redshift, (A = 25.11, $\lambda = 0.3122$, $t_0 = 3.42$).

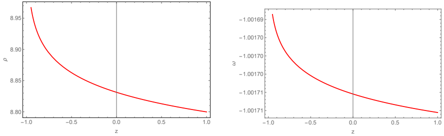

Figure 2: Behaviour of energy density (left panel) and EoS parameter (right panel) in redshift, (a = -4.4, b = 0.01, m = 0.6, A = 25.11, $\lambda = 0.3122$, $t_0 = 3.42$).

- The EoS parameter value from observational sources, Supernova data[5]. $\omega = -1.084 \pm 0.063$, WMAP[6]. $\omega = -1.073 \pm^{0.090}_{0.089}$ favours ΛCDM.

[5] R. Amanullah*et al.*, *Astrophy. J.*, **712**, 716 (2010)

[6] G. Hinshaw *et al*, *Astrophys. J. Suppl. Ser.*, **19**, 208 (2013)

Big Rip

- The most well-known sort of finite time future singularity is the Big Rip singularity, which is linked to phantom evolution.
- The density of the dark energy increases with increasing scale factor, and both the scale factor and the phantom energy density can become infinite at a finite time t, is known as big rip.
- In the big rip, the scale factor and density diverge in a singularity at a finite future time.
- The big rip scale factor is

$$R(t) = R_0 + \frac{1}{(t_s - t)^{\alpha}}$$

- $H(t) = \frac{\alpha}{(t_s - t)^{\alpha}}$
- $q = -\frac{(\alpha+1)(1+R_0(t_s-t)^{\alpha})}{\alpha}$

$$\begin{aligned}
p &= -\frac{a(6)^{m-1}(1-2m)\left(\frac{\alpha}{t_s-t}\right)^{2m-2}}{8\pi(1+2\kappa)}\left[3\left(\frac{\alpha}{t_s-t}\right)^2 + m(2+\kappa-\kappa\kappa_1)\frac{\alpha}{(t_s-t)^2}\right], \\
\rho &= \frac{a(6)^{m-1}(1-2m)\left(\frac{\alpha}{t_s-t}\right)^{2m-2}}{8\pi(1+2\kappa)}\left[3\left(\frac{\alpha}{t_s-t}\right)^2 - m(3\kappa-2\kappa_1-3\kappa\kappa_1)\frac{\alpha}{(t_s-t)^2}\right], \\
\omega &= -1 - \frac{2m(1-\kappa_1+2\kappa-2\kappa\kappa_1)\frac{\alpha}{(t_s-t)^2}}{3\left(\frac{\alpha}{t_s-t}\right)^2 - m(3\kappa-2\kappa_1-3\kappa\kappa_1)\frac{\alpha}{(t_s-t)^2}}
\end{aligned}$$

Analysis of the BR graph

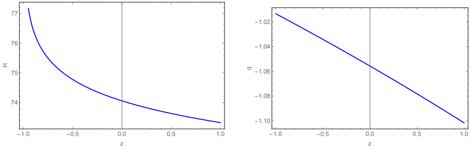

Figure 3: The behaviour of Hubble parameter (left panel) and deceleration parameter (right panel) vs redshift, H_0=74.31, t_s=13.8, α=12.7

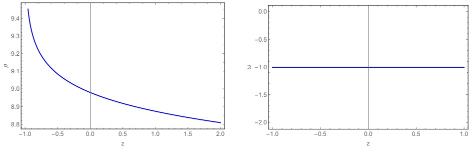

Figure 4: Behaviour of energy density (left panel) and EoS parameter (right panel) vs redshift, a=-4.4, b=0.01, m=0.6, t_s=13.8, α=12.7

- According to the present observational value of the deceleration parameter q = -1.08, the present value of the Hubble parameter H_0 for the BR model can be $74.33 Kms^{-1}Mpc^{-1}$. [7]

[7] D Camarena, V. Marra, *Phys. Rev. D*, **2**, 013028 (2020).

Pseudo Rip

- The cosmos begins in the infinite past from a phase where the scale factor was zero, but the Hubble parameter was a constant. This situation is known as the early phase Pseudo-bang as the characteristics of this are similar to the fate of Pseudo Rip(PR).
- The scale factor of PR model[8] is

$$\mathcal{R} = R_1 exp\left[H_0 t + H_1 \frac{1}{\eta} e^{\eta t}\right]$$

- $H = H_0 - \frac{H_1}{e^{\eta t}}$
- $q = -1 - \frac{\eta H_1 e^{-\eta t}}{(H_0 - H_1 e^{\eta t})^2}$

Now the dynamical parameters can be obtained as,

$$p = -\frac{a(6)^{m-1}(1-2m)\left(H_0 - H_1 e^{-\eta t}\right)^{2m-2}}{8\pi(1+2\kappa)}\left[3\left(H_0 - H_1 e^{-\eta t}\right)^2 + m(2+\kappa-\kappa\kappa_1)\{\eta H_1 e^{-\eta t}\}\right], \quad (20)$$

$$\rho = \frac{a(6)^{m-1}(1-2m)\left(H_0 - H_1 e^{-\eta t}\right)^{2m-2}}{8\pi(1+2\kappa)}\left[3\left(H_0 - H_1 e^{-\eta t}\right)^2 - m(3\kappa-2\kappa_1-3\kappa\kappa_1)\{\eta H_1 e^{-\eta t}\}\right], \quad (21)$$

$$\omega = -1 - \frac{2m(1-\kappa_1+2\kappa-2\kappa\kappa_1)\{\eta H_1 e^{-\eta t}\}}{3\left(H_0 - H_1 e^{-\eta t}\right)^2 - m(3\kappa-2\kappa_1-3\kappa\kappa_1)\{\eta H_1 e^{-\eta t}\}} \quad (22)$$

[8] W. EI. Hanafy, E. N. Saridakis, *Journal of Cosmology and Astroparticle Physics*, ***09***, *019 (2021)*.

Analysis of the PR graph

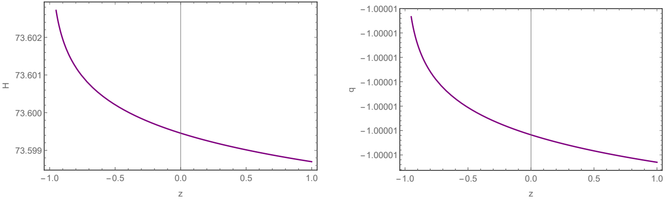

Figure 5: The behaviour of Hubble parameter (left panel) and deceleration parameter (right panel) vs redshift, H_0=74.31, H_1=1, η=0.3011

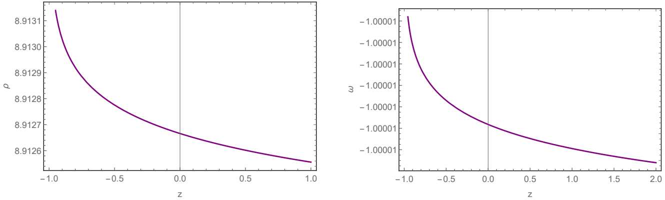

Figure 6: Behaviour of energy density (left panel) and EoS parameter (right panel) vs redshift, a=-4.4, b=0.01, m=0.6, H_0=74.31, H_1=1, η=0.3011

Energy Conditions for LR Model

$$\rho + p = -\frac{a6^{m-1}m(1-2m)\lambda(Ae^{\lambda t})^{2m-1}}{4\pi}[1-\kappa_1] \rightarrow \textbf{NEC}$$

$$\rho + 3p = -\frac{a6^{m-1}(1-2m)(Ae^{\lambda t})^{2m-1}}{4\pi(1+2\kappa)}[3Ae^{\lambda t} + m\lambda(3+3\kappa-\kappa_1-3\kappa\kappa_1)] \rightarrow \textbf{SEC}$$

$$\rho - p = \frac{a6^{m-1}(1-2m)(Ae^{\lambda t})^{2m-1}}{4\pi(1+2\kappa)}[3Ae^{\lambda t} + m\lambda(1-\kappa+\kappa_1+\kappa\kappa_1)] \rightarrow \textbf{DEC}$$

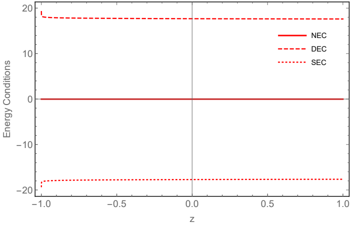

Figure 7: Behaviour of energy Conditions vs redshift in LR model, a=-4.4, b=0.01, m=0.6, $A = 25.11$, $\lambda = 0.3122$, $t_0 = 3.42$).

Energy Conditions for BR Model

$$\rho + p = -\frac{a6^{m-1}m(1-2m)\left(\frac{\alpha}{t_s-t}\right)^{2m-2}\frac{\alpha}{(t_s-t)^2}}{4\pi}[1-\kappa_1] \rightarrow \textbf{NEC}$$

$$\rho + 3p = -\frac{a6^{m-1}(1-2m)\left(\frac{\alpha}{t_s-t}\right)^{2m-2}}{4\pi(1+2\kappa)}\left[3\left(\frac{\alpha}{t_s-t}\right)^2 + m(3+3\kappa-\kappa_1-3\kappa\kappa_1)\frac{\alpha}{(t_s-t)^2}\right] \rightarrow \textbf{SEC}$$

$$\rho - p = \frac{a6^{m-1}(1-2m)\left(\frac{\alpha}{t_s-t}\right)^{2m-2}}{4\pi(1+2\kappa)}\left[3\left(\frac{\alpha}{t_s-t}\right)^2 + m(1-\kappa+\kappa_1+\kappa\kappa_1)\frac{\alpha}{(t_s-t)^2}\right] \rightarrow \textbf{DEC}$$

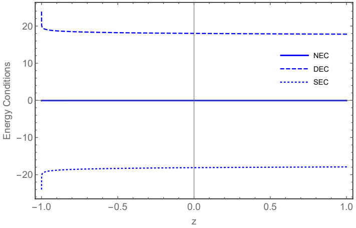

Figure 8: Behaviour of energy Conditions vs redshift in BR model, a=-4.4, b=0.01, m=0.6, t_s=13.8, α=12.7.

Energy Conditions for PR Model

NEC: $\rho + p$, **SEC:** $\rho + 3p$, **DEC:** $\rho - p$

$$\rho + p = -\frac{a6^{m-1}(1-2m)\left(H_0 - H_1 e^{\eta t}\right)^{2m-2}}{4\pi}[m(1-\kappa_1)(\eta H_1 e^{-\eta t})],$$

$$\rho + 3p = -\frac{a6^{m-1}(1-2m)\left(H_0 - H_1 e^{\eta t}\right)^{2m-2}}{4\pi(1+2\kappa)}[3\left(H_0 - H_1 e^{\eta t}\right)^2 + m(3+3\kappa-\kappa_1-3\kappa\kappa_1)(\eta H_1 e^{-\eta t})],$$

$$\rho - p = \frac{a6^{m-1}(1-2m)\left(H_0 - H_1 e^{\eta t}\right)^{2m-2}}{4\pi(1+2\kappa)}[3\left(H_0 - H_1 e^{\eta t}\right)^2 + m(1-\kappa+\kappa_1+\kappa\kappa_1)(\eta H_1 e^{-\eta t})]$$

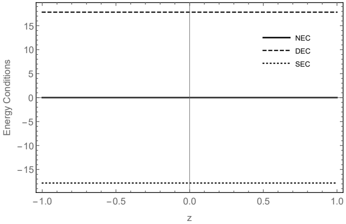

Figure 9: Behaviour of energy Conditions vs redshift in PR model, a=-4.4, b=0.01, m=0.6, H_0=74.31, H_1=1, η=0.3011.

Results and Conclusion

• It able to give rise to a nonsingular, little rip cosmology, which is considered to be a viable alternative to ΛCDM cosmology.

• The presence of numbers of free possible parameters gives enough space for fine-tuning the models which can be useful for fitting with observational data.

• As required in the modified theories of gravity, here also in all three models violation of SEC and satisfaction of DEC are obtained. A simple way to see this, if $\omega < 1$, which occurs for any rip, a boost is allowed with $\frac{v}{c^2} > \frac{-\omega}{c}$ to an inertial frame with negative energy density.

• Finally, based on our model we can conclude that no singularity scenario appear in the accelerating models, so the study in $f(Q,T)$ gravity may give new insight in to resolving the singularity issue.


Thank you