Scalar-tensor cosmology in a minisuperspace formulation with Chaplygin gas

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23.09.2022

The 8th Conference of the Polish Society on Relativity

Hybrid metric-Palatini theories

- $f(R)$: $R = R(g)$ – Ricci; $\hat{R} = g^{\mu\nu}\hat{R}_{\mu\nu}(\Gamma)$ – Palatini-Ricci

$$S\left[g_{\mu\nu},\Gamma^{\alpha}_{\mu\nu}\right] = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[\Omega_A R(g) + F\left(\hat{R}(g,\Gamma)\right)\right] + S_m\left[g_{\mu\nu},\chi\right]; \quad (1)$$

- Scalar-tensor:

$$S\left[g_{\mu\nu},\Phi,\chi\right] = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[(\Omega_A+\Phi)R(g) + \frac{3}{2\Phi}\partial_\mu\Phi\partial^\mu\Phi - U(\Phi)\right] + S_m\left[g_{\mu\nu},\chi\right]; \quad (2)$$

- Limits:
  $\Omega_A \to 0$ **Palatini** $F(\hat{R})$;
  $\Omega_A \to \infty$ **GR**;
- Action:

$$S\left[g_{\mu\nu},\Phi\right] = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[\mathcal{A}(\Phi)\mathcal{R}(g) - \mathcal{B}(\Phi)g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \mathcal{V}(\Phi)\right] + S_m\left[e^{2\alpha(\Phi)}g_{\mu\nu},\chi\right]; \qquad \mathcal{R}(g) = \hat{R}(g,\Gamma); \quad (3)$$

Hybrid metric-Palatini generalization

- Field equations:

$$\mathcal{A}(\Phi)\mathcal{G}_{\mu\nu}(g) - \left(\nabla^g_\mu\nabla^g_\nu - g_{\mu\nu}\Box^g\right)\mathcal{A}(\Phi) = T^\Phi_{\mu\nu} + \kappa^2 T_{\mu\nu}; \tag{4}$$

$$\mathcal{A}'(\Phi)\mathcal{R}(g) + \mathcal{B}'(\Phi)\left(\partial\Phi\right)^2 + 2\mathcal{B}(\Phi)\Box^g\Phi - \mathcal{V}'(\Phi) = -2\kappa^2\alpha'(\Phi)T; \tag{5}$$

- Energy-momentum tensor for Φ:

$$\begin{aligned} T^\Phi_{\mu\nu} &= \mathcal{B}(\Phi)\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}\left[\mathcal{B}(\Phi)(\partial\Phi)^2 + \mathcal{V}(\Phi)\right]g_{\mu\nu} \\ &= -(\partial\Phi)^2\mathcal{B}(\Phi)u_\mu u_\nu - \frac{1}{2}\left[\mathcal{B}(\Phi)(\partial\Phi)^2 + \mathcal{V}(\Phi)\right]g_{\mu\nu}; \end{aligned} \tag{6}$$

where: $u_\mu = \dfrac{\partial_\mu\Phi}{\sqrt{-(\partial\Phi)^2}}$.

	$\mathcal{A}(\Phi)$	$\mathcal{B}(\Phi)$	$\mathcal{V}(\Phi)$	$\alpha(\Phi)$
metric	Φ	0	$U_F(\Phi-1)$	0
Palatini	Φ	$-\frac{3}{2\Phi}$	$U_F(\Phi-1)$	0
hybrid	$\Omega_A+\Phi$	$-\frac{3}{2\Phi}$	$U_F(\Phi)$	0

[3]

Table 1: The corresponding metric ST frames for three cases of $R+F(R)$ gravity.

STT and FLRW cosmology with lapse function

- FLRW metric:

$$g_{\mu\nu} = \left(-N^2(t), \frac{a^2(t)}{1-kr^2}, a^2(t)r^2, a^2(t)r^2 \sin^2\theta \right) \tag{7}$$

- Ricci scalar:

$$R = \frac{6k}{a^2} + \frac{6}{N^2}\left(\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}\frac{\dot{N}}{N} \right); \tag{8}$$

- **Diffeomorphism invariance** $\Longrightarrow$ $N(t)$ **can be eliminated by setting** $N = 1$;
- Stress-energy tensor for perfect fluid:

$$T_{\mu\nu} = (p+\rho)u_\mu u_\nu + pg_{\mu\nu}, \qquad u^\mu = \left(N^{-1}, 0, 0, 0\right); \tag{9}$$

- In the most general case, the stress-energy tensor is **not conserved**, unless there is no anomalous coupling between the matter part of the action and Φ:

$$\nabla_\mu T^{\mu\nu} = \alpha'(\Phi) T \partial^\nu \Phi; \tag{10}$$

- Continuity equation for (9) in the case of non-minimal coupling between matter and Φ:

$$\dot{\rho} + 3H(p+\rho) = -\dot{\alpha}(\phi)(3p-\rho) \tag{11}$$

Equations of state for Chaplygin gas models

- Chaplygin gas (CG) [5]:

$$p_{CG}(\rho) = -\frac{A}{\rho}, \qquad A > 0; \tag{12}$$

- Generalized Chaplygin gas (GCG) [2]:

$$p_{GCG}(\rho) = -\frac{A}{\rho^\beta}, \qquad 0 < \beta \leq 1; \tag{13}$$

- Modified Chaplygin gas (MCG) [1]:

$$p_{MCG}(\rho) = B\rho - \frac{A}{\rho^\beta}; \tag{14}$$

- New generalized Chaplygin gas (NGCG) [7]:

$$p_{NGCG}(\rho) = -\frac{\tilde{A}(a)}{\rho^\beta} = \frac{\omega A a^{-3(1+\omega)(1+\beta)}}{\rho^\beta}, \qquad -1.46 < \omega < -0.78; \tag{15}$$

- Viscous generalized Chaplygin gas (VGCG) [6]:

$$p_{VGCG}(\rho) = -\frac{A}{\rho^\beta} - \sqrt{3}\zeta_0 p, \qquad \zeta_0 \geq 10^{-5}. \tag{16}$$

Example - CG

$$\rho_{CG}(a,\Phi) = \frac{e^{\alpha(\Phi)}\sqrt{Aa^6 e^{6\alpha(\Phi)} + B_1}}{a^3}; \tag{17}$$

Behavior of energy density - CG

- $a \ll 1 \Longrightarrow \rho_{CG}(a,\Phi) \simeq \frac{e^{\alpha(\Phi)}\sqrt{B_1}}{a^3} \simeq e^{\alpha(\Phi)} a^{-3}$ **(Matter domination)**;
- $a \to 1 \Longrightarrow \rho_{CG}(a,\Phi) \simeq e^{\alpha(\Phi)}\sqrt{Ae^{6\alpha(\Phi)} + B_1}$ **(Dark Energy domination)**;
- $a \to \infty \Longrightarrow \rho_{CG}(a,\Phi) \simeq e^{\alpha(\Phi)}\sqrt{Ae^{6\alpha(\Phi)}}$ **(Dark Energy domination)**.

MSS formalism

- We need to obtain from the action (3) the effective Lagrangian employing only the time-dependent variables - minisuperspace (MSS) formalism;
- The effective MSS Lagrangian:

$$L_{MSS}\left(N, x, \dot{x}\right) = \frac{1}{2N} m_{jk}(x)\dot{x}^j \dot{x}^k - NV_{MSS}(x) \tag{18}$$

  lives in a 3D configuration space with $\left(x^j\right)\Big|_{j=1,2} = (a, \Phi)$;
- Nondynamical character of the variable N: $p_N \equiv \frac{\partial L_{MSS}}{\partial \dot{N}} = 0$
  $\Longrightarrow$ (18) is singular and can be reduced to the plane as a configuration space;
- The kinetic energy term of such a reduced system is determined by a metric [4]:

$$m_{ij} \equiv m_{ij}\left(a, \Phi\right) = \begin{pmatrix} -12a\mathcal{A}(\Phi) & -6a^2\mathcal{A}'(\Phi) \\ -6a^2\mathcal{A}'(\Phi) & 2a^3\mathcal{B}(\Phi) \end{pmatrix} \tag{19}$$

  providing the geometry to a 2D configuration (a, Φ)-plane $\subset \mathbb{R}_+ \times \mathbb{R}$ - **MSS**;

MSS formalism

- $N(t)$ enters the action in a nondynamical way $\Longrightarrow$ constraint equation obtained from one of the Euler-Lagrange equations:

$$\frac{\partial L_{MSS}}{\partial N} - \frac{d}{dt}\frac{\partial L_{MSS}}{\partial \dot{N}} = \frac{\partial L_{MSS}}{\partial N} = 0 \Leftrightarrow \frac{1}{2N^2} m_{ij}\dot{x}^i\dot{x}^j + V_{MSS} = 0; \tag{20}$$

- Equations of motion for the remaining two variables [7]:

$$m_{il}\ddot{x}^l + \frac{1}{2}\delta_i^p\left(\partial_j m_{pk} + \partial_k m_{pj} - \partial_p m_{jk}\right)\dot{x}^j\dot{x}^k = m_{il}\frac{\dot{N}}{N}\dot{x}^l - N^2\partial_i V_{MSS}; \tag{21}$$

- $N(t)$ plays a role of an **additional gauge degree of freedom** which is responsible for a time reparametrization and suitably modifies the constraint equation (20);
- Newtonian mechanical system represented by the effective MSS Lagrangian (18) is **fully equivalent** to the one obtained from the Einstein field equations imposed on the FLRW metric (7);

Cauchy data

- Reformulation:

$$V_m(a,\Phi) = 3\mathcal{H}_0^2\Omega_0\rho(a,\Phi), \qquad \Omega_0 = \frac{2\kappa^2\rho_0}{3\mathcal{H}_0^2}; \tag{22}$$

- Normalization:

$$a_0 = 1 \Longrightarrow \dot{a}_0 = \mathcal{H}_0; \tag{23}$$

- Constraint on Φ and $\dot{\Phi}$:

$$3\mathcal{H}_0^2 = -3\frac{\mathcal{A}'}{\mathcal{A}}\bigg|_{\Phi=\Phi_0}\mathcal{H}\dot{\Phi}_0 + \frac{\mathcal{B}}{2\mathcal{A}}\bigg|_{\Phi=\Phi_0}\dot{\Phi}_0^2 - k + \frac{\mathcal{V}}{2\mathcal{A}}\bigg|_{\Phi=\Phi_0} + \frac{V_m}{2\mathcal{A}}\bigg|_{a=1,\Phi=\Phi_0}; \tag{24}$$

- Assuming that $\dot{\Phi}_0 = 0$, one gets a Λ-CDM type relation:

$$1 = \Omega_\Lambda + \Omega_k + \frac{1}{2\mathcal{A}(\Phi_0)}\rho(a_0, \alpha(\Phi_0)), \tag{25}$$

  where:

$$\Omega_k = -\frac{k}{3\mathcal{H}_0^2} \qquad \Omega_\Lambda = \frac{\mathcal{V}(\Phi_0)}{6\mathcal{H}_0^2\mathcal{A}(\Phi_0)} \tag{26}$$

- In such a scenario the observed matter could differ from “true” matter by an exponential factor with $\alpha(\Phi_0)$ in $\rho(a,\Phi)$.

	CG	GCG	MCG	NGCG	VGCGI	VGCGII
Metric-hybrid $\alpha = 0$ + Starobinsky	B, Λ	-	-	B, Λ	B, Λ	B, B
Metric $\alpha = 0$ + Starobinsky	Λ	-	-	Λ	Λ	Λ
Metric-hybrid $\alpha = 0$	Λ	-	-	Λ	Λ	B
Metric-hybrid $\alpha = \frac{1}{2}\ln\left(\frac{1}{\mathcal{A}(\Phi)}\right)$	CB	-	Λ	CB	X	Λ
Metric $\alpha = \frac{1}{2}\ln\left(\frac{1}{\mathcal{A}(\Phi)}\right)$ + Starobinsky	Λ	-	-	-	-	-
Metric-hybrid $\alpha = \frac{1}{2}\ln\left(\frac{1}{\mathcal{A}(\Phi)}\right)$ + Starobinsky	B, CB	-	Λ	CB	Λ	CB

Table 2: Types of cosmological models obtained for particular cases of Chaplygin gas. Models labels: B - bounce, CB - cyclic bounce, Λ - Λ-CDM, X - incompatible with Λ-CDM, - lack of numerical results.

- **Hybrid metric-Palatini with** $\alpha = \frac{1}{2}\left(\frac{1}{\mathcal{A}(\Phi)}\right)$**: MCG**

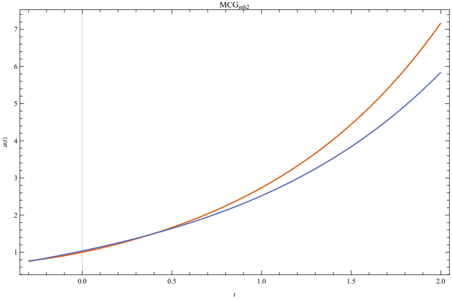

Figure 1: Time dependence of the scale factor (NGCG , Λ-CDM).

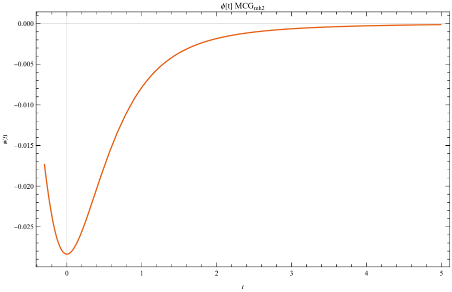

Figure 2: Time dependence of the scalar field for MCG.

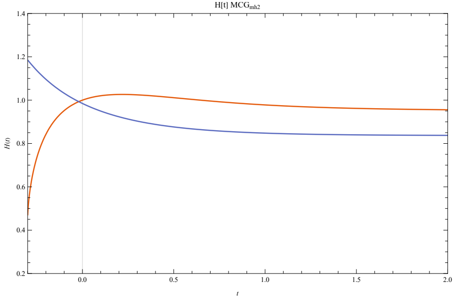

Figure 3: Time dependence of the Hubble parameter (NGCG , Λ-CDM).

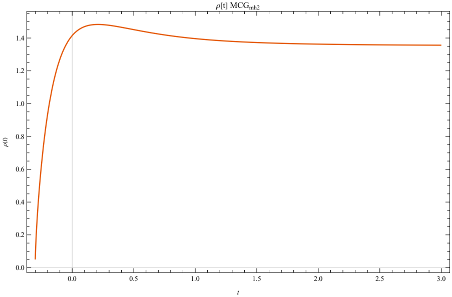

Figure 4: Time dependence of the energy density for MCG.

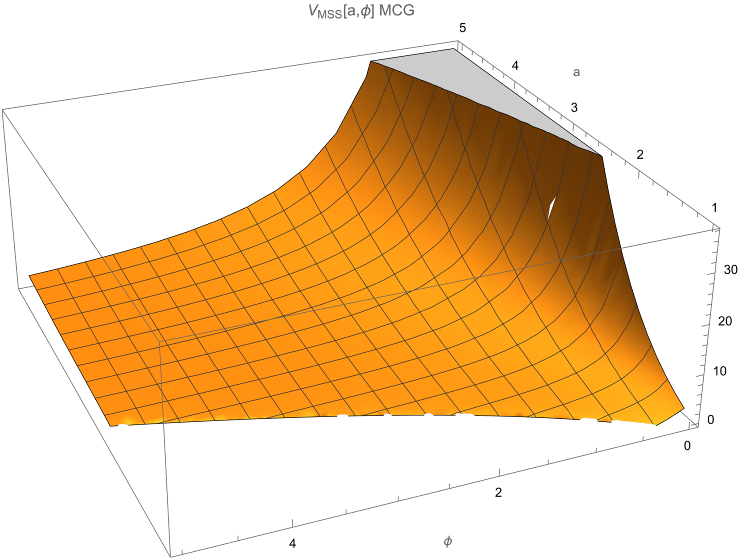

Figure 5: Effective MSS potential for MCG.

- **Hybrid metric-Palatini with** $\alpha = \frac{1}{2}\left(\frac{1}{\mathcal{A}(\Phi)}\right)$ **and Starobinsky potential: CG**

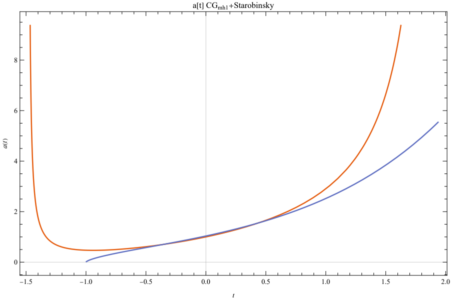

Figure 6: Time dependence of the scale factor (NGCG , Λ-CDM).

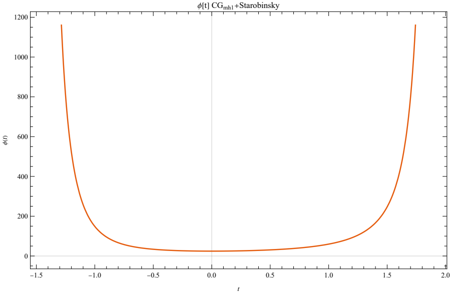

Figure 7: Time dependence of the scalar field for CG.

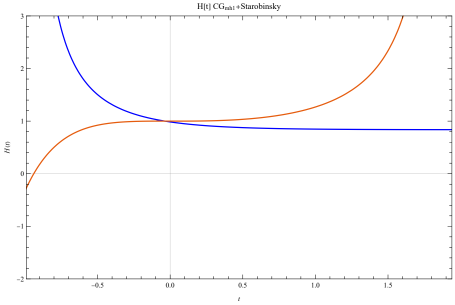

Figure 8: Time dependence of the Hubble parameter (NGCG , Λ-CDM).

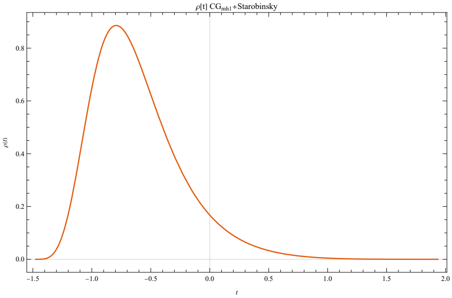

Figure 9: Time dependence of the energy density for CG.

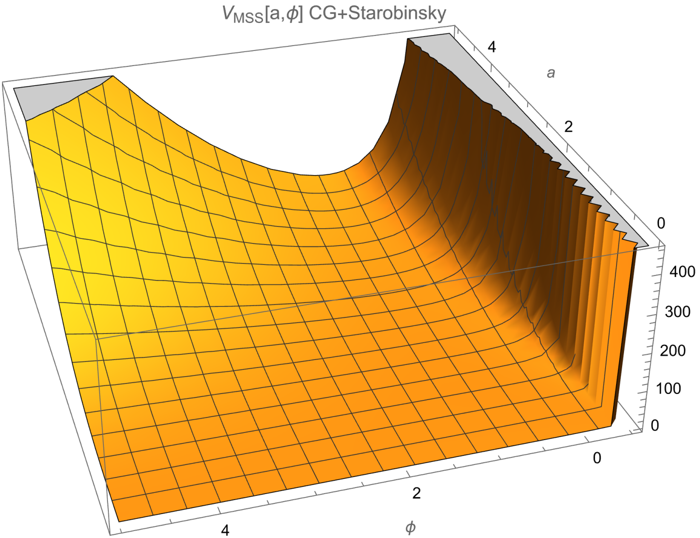

Figure 10: Effective MSS potential for CG.

- **Hybrid metric-Palatini with** $\alpha = \frac{1}{2}\left(\frac{1}{\mathcal{A}(\Phi)}\right)$**: NGCG**

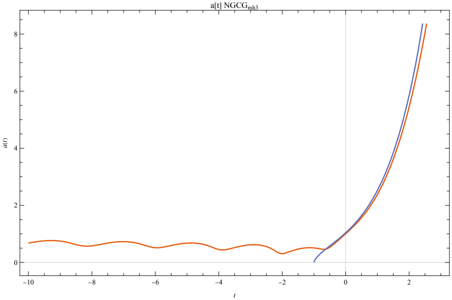

Figure 11: Time dependence of the scale factor (NGCG , Λ-CDM).

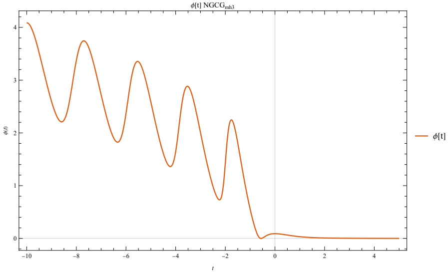

Figure 12: Time dependence of the scalar field for NGCG.

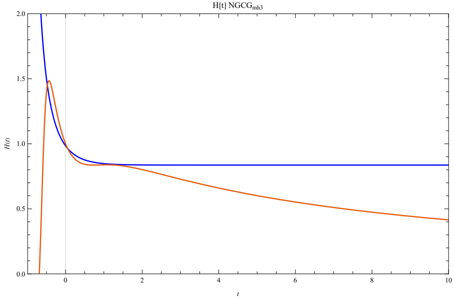

Figure 13: Time dependence of the Hubble parameter (NGCG , Λ-CDM).

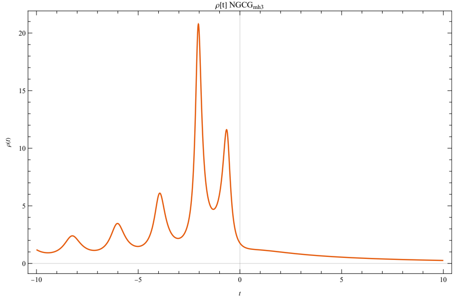

Figure 14: Time dependence of the energy density for NGCG.

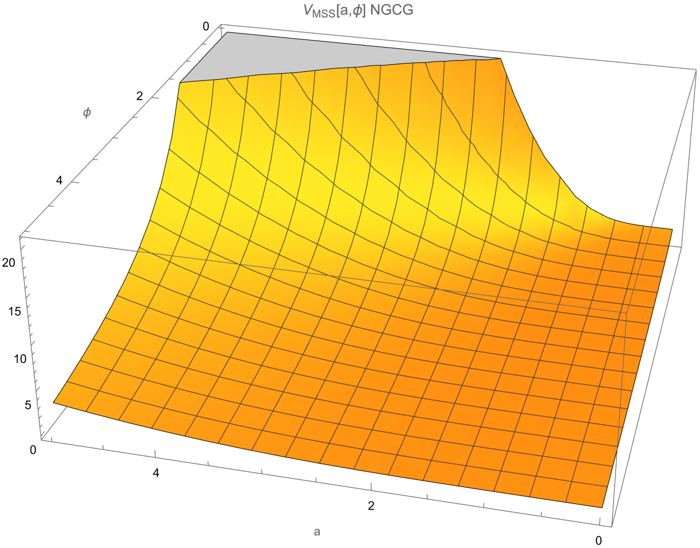

Figure 15: Effective MSS potential for NGCG.

Conclusions

- **Hybrid metric-Palatini models with nonzero** $\alpha(\Phi)$ **in the MSS formalism** seem to best fit the Λ-CDM model for the current observation range;
- Of all the cases considered, **NGCG reproduces the standard cosmological model in the best way** and it represents the Cyclic Bounce cosmological model;
- Models with **non-minimal coupling between** Φ **and** R **and simultaneous coupling between matter and** Φ in the case of the MSS formalism may describe general models of **Bounce Cosmology** and **New Bounce Cosmology**.

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