Metric-affine gravity effects on terrestrial (exo)planets' profiles

Aleksander Kozak

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Uniwersytet Wrocławski

Motivation

The effects on (exo)planets and the Earth that modified gravity will have, are expected to be small.

However, Earth's structure is well known. We know its composition and properties of matter the planet is built of to a high degree, unlike e.g. neutron stars.

Our knowledge of the Earth's interior will be even more accurate when the neutrino tomography methods become more reliable (they do not depend on the theory of gravity) [1].

Thus, we plan to devise methods of comparing modified gravity predictions with available seismic data.

[1] A. Donini, S. Palomares-Ruiz, J. Salvado, Nature Physics 15 (2019) 37

Invariant quantities in scalar-tensor theory of gravity

The theory is described by the following action functional[2]:

$$S[\bar{g}_{\mu\nu}, \bar{\Gamma}^{\alpha}_{\mu\nu}, \bar{\Phi}, \psi_m] = \frac{1}{2\kappa^2} \int_{\Omega} \sqrt{-\bar{g}} \left[\bar{\mathcal{A}}(\bar{\Phi}) R - \bar{\mathcal{B}}(\bar{\Phi})(\partial \bar{\Phi})^2 - \bar{\mathcal{V}}(\bar{\Phi}) \right] d^4 x \\ + S_{\text{matter}}[e^{2\bar{\sigma}(\bar{\Phi})} \bar{g}_{\mu\nu}, \psi_m]. \tag{1}$$

It is form-invariant under the Weyl rescaling of the metric tensor and redefinition of the scalar field[3]:

$$\bar{\bar{g}}_{\mu\nu} = e^{2\gamma(\bar{\Phi})} \bar{g}_{\mu\nu}, \quad \bar{\bar{\Phi}} = f(\bar{\Phi}). \tag{2}$$

One can introduce the following conformal invariants[4]:

$$\mathcal{I}_1 = \frac{\bar{\mathcal{A}}(\bar{\Phi})}{e^{2\bar{\sigma}(\bar{\Phi})}}, \quad \mathcal{I}_2 = \frac{\bar{\mathcal{V}}(\bar{\Phi})}{\bar{\mathcal{A}}^2(\bar{\Phi})}, \tag{3}$$

$$\frac{d\mathcal{I}}{d\bar{\Phi}} = \sqrt{\frac{\bar{\mathcal{B}}}{\bar{\mathcal{A}}} + \delta_{\Gamma} \left(\frac{3\bar{\mathcal{A}}'}{2\bar{\mathcal{A}}} \right)^2}, p = e^{-4\bar{\sigma}} \bar{p}, \rho = e^{-4\bar{\sigma}} \bar{\rho}. \tag{4}$$

[2] E. E. Flanagan, Class. Quant. Grav. 21 (2004) 3817

[3] L. Järv, P. Kuusk, M. Saal, O. Vilson, Phys. Rev. D 91 (2015) 2, 024041

[4] L. Järv *et al.*, Phys. Rev. D 102 (2020) 4, 044029

Palatini $f(\mathcal{R})$ theory

Action functional[5]:

$$S[\bar{g}_{\mu\nu}, \bar{\Gamma}^{\alpha}_{\mu\nu}, \psi_m] = \frac{1}{2\kappa^2} \int \sqrt{-\bar{g}} f(\mathcal{R}) d^4x + S_{\text{matter}}[\bar{g}_{\mu\nu}, \psi_m], \quad (5)$$

Connection turns out to be auxiliary field, can be integrated out. Palatini $f(\mathcal{R})$ gravity has an equivalent metric scalar-tensor representation:

$$S[\bar{g}_{\mu\nu}, \bar{\Gamma}^{\alpha}_{\mu\nu}, \bar{\Phi}, \psi_m] = \frac{1}{2\kappa^2} \int \sqrt{-\bar{g}} \left[\bar{\Phi}\mathcal{R}(\bar{g}, \bar{\Gamma}) - V(\bar{\Phi}) \right] d^4x + S_{\text{matter}}[\bar{g}_{\mu\nu}, \psi_m], \quad (6)$$

where $\bar{\Phi} = df/d\mathcal{R}$ and $V(\bar{\Phi}) = f'(\mathcal{R}(\bar{\Phi}))\mathcal{R}(\bar{\Phi}) - f(\mathcal{R}(\bar{\Phi}))$.

[5] S. Capozziello, V. Faraoni, Beyond Einstein Gravity: A Survey of Gravitational Theories for Cosmology and Astrophysics, Springer (2011)

Tolman-Oppenheimer-Volkoff equation for quadratic model

Model:

$$f(\mathcal{R}) = \mathcal{R} + \beta\mathcal{R}^2. \tag{7}$$

Scalar field functions:

$$\bar{\mathcal{A}} = \bar{\Phi}, \quad \bar{\mathcal{B}} = 0, \quad \bar{\mathcal{V}} = \frac{(\bar{\Phi}-1)^2}{4\beta}, \quad \bar{\sigma} = 0. \tag{8}$$

The invariants[6]:

$$\mathcal{I}_1 = 1 + 4\beta\kappa^2(c^2\rho - 3p), \tag{9}$$

$$\mathcal{I}_2 = \frac{4\beta\kappa^4(c^2\rho - 3p)^2}{\left(1 + 4\beta\kappa^2(c^2\rho - 3p)\right)^2}, \tag{10}$$

$$\mathcal{I} = 0. \tag{11}$$

[6] A. Kozak, A. Wojnar, Eur. Phys. J. C, 81 6 (2021) 492

Tolman-Oppenheimer-Volkoff equation for quadratic model

Full relativistic hydrostatic equilibrium equation:

$$p' = \left[-\frac{G\mathcal{M}(r)}{c^2 r^2 \mathcal{I}_1^{1/2}}(c^2\rho + p)\left(1 - \frac{2G\mathcal{M}(r)}{c^2 r \mathcal{I}_1^{1/2}}\right)^{-1} \left(1 + \frac{4\pi \mathcal{I}_1^{\frac{3}{2}} r^3}{c^2 \mathcal{M}(r)}\left(\frac{p}{\mathcal{I}_1^2} + \frac{\mathcal{I}_2}{2\kappa^2}\right)\right)\right] \tag{12}$$

$$\times \left(\frac{r}{2}\partial_r \ln \mathcal{I}_1 + 1\right) + \left(-c^2\rho + 5p\right)\partial_r \ln \mathcal{I}_1,$$

and mass function:

$$\mathcal{M}' = 4\pi r^2 \frac{\rho - 2c^{-2}\beta\kappa^2(c^2\rho - 3p)^2}{(1 + 4\beta\kappa^2(c^2\rho - 3p))^{1/2}} \left[1 + \frac{r}{2}\partial_r \ln\left(1 + 4\beta\kappa^2(c^2\rho - 3p)\right)\right]. \tag{13}$$

In what follows, we will denote:

$$\alpha := 2c^2\kappa^2\beta. \tag{14}$$

Equation of state

Low-pressure EoS for solid materials ($p < 200$ GPa) - Birch-Murnagham:[7]

$$p = \frac{3}{2} K_0 (\eta^{7/3} - \eta^{5/3})[1 + \frac{3}{4}(K_0' - 4)(\eta^{2/3} - 1)] \tag{15}$$

where $\eta = \rho/\rho_0$, $K_0 = -V(\partial p/\partial V)_T$ is the bulk modulus of the material, K_0' is the first pressure derivative.
Pressure $< 10^7$ GPa EoS for solid materials - polytropic[8]:

$$p = \left(\frac{\rho - \rho_0}{c}\right)^{1/n} \tag{16}$$

where c, n depend on the material; fitted to the experimental data.

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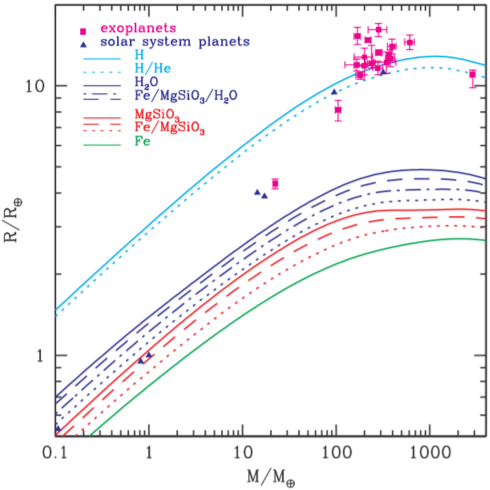
[7] J.-P. Poirier, Introduction to the Physics of the Earth's Interior, Cambridge University Press (2000)

[8] Seager *et al.*, ApJ 669 (2007) 1279

Mass-radius relationships



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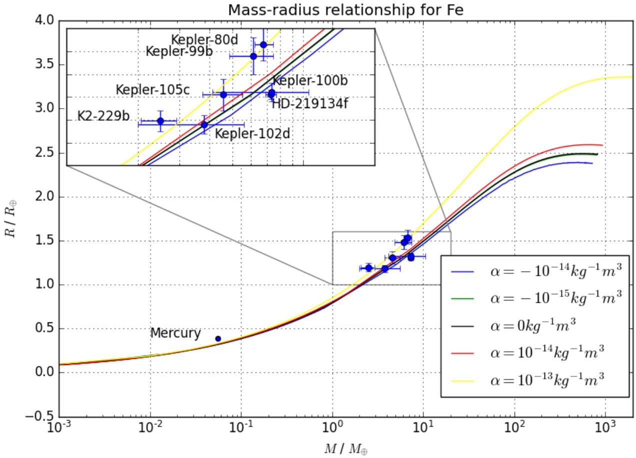
[9] S. Seager *et al.*, ApJ 669 (2007) 1279



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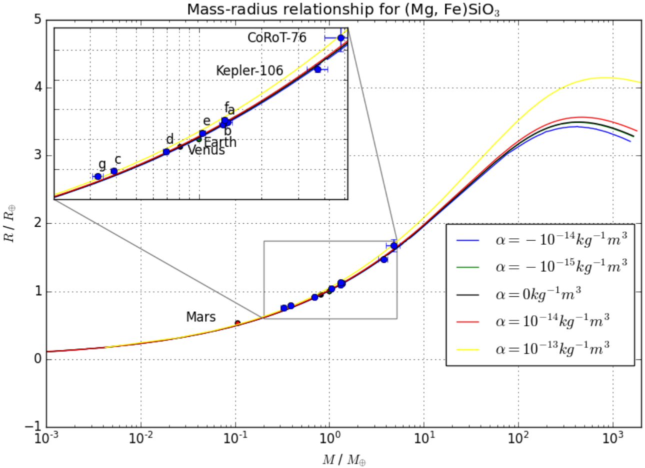
[10] A. Kozak, A. Wojnar, Phys. Rev. D 104 (2021) 084097
Exoplanets: B. Brugger, J. Lunine, O Mousis, M. Deleuil, EPSC Abstracts 13 (2019)



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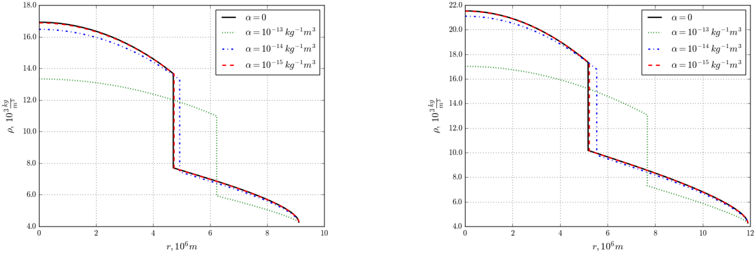
[11] A. Kozak, A. Wojnar, Phys. Rev. D 104 (2021) 084097
Exoplanets: B. Brugger, O. Mousis., M. Deleuil, F. Deschamps, ApJ 850 (2017) 93, E. Ago *et al.*, Planet. Sci. 2 (2021) 1



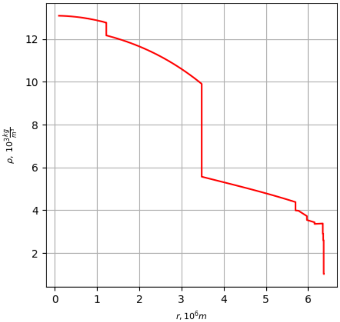
(a) K2-36 b, $M = 3.9M_\oplus$, $R = 1.43R_\oplus$

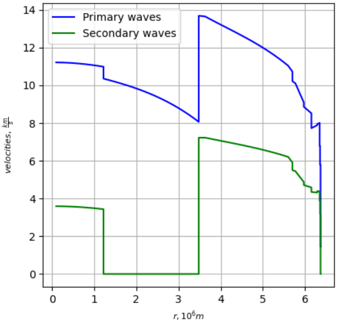
(b) Kepler-20 b, $M = 9.7M_\oplus$, $R = 1.87R_\oplus$

Figure: Density profiles for four different Earth-like exoplanets, for different values of the parameter $\alpha = c^2\kappa^2\beta$. The planets are assumed to be composed of two layers: iron core, and mantle made of (Fe, Mg)SiO_3.[12]

[12] A. Kozak, A. Wojnar, Universe 8 (2021) 3

Preliminary Reference Earth Model





[13]

[13] A. Dziewoński, D. Anderson, Phys. Earth Planet. Inter. 25.4 (1981) 297

Conclusions and outlook

- One can fomulate the TOV equation using conformal invariants.
- There exists extra degeneracy in the mass-radius relation providing different exoplanets properties.
- Modified gravity influences density profiles of planets.
- In the future, we plan to investigate the effect of Vainshtein mechanism breaking in DHOST theories on internal structure of the Earth.
- Data to be analyzed is either available[14] or will be collected during future missions, e.g. Seismic Experiment for Interior Structure[15].

[14] A.M. Dziewonski, D.L. Anderson, Phys. Earth Planet. Inter 25.4 (1981) 297

[15] https://mars.nasa.gov/insight/spacecraft/instruments/seis/