Low scale inflation without (much of) fine-tuning

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Introduction to cosmic inflation

Let us assume, that the flat FRW Universe with the metric tensor

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2) \ ,$$

is filled with a homogeneous scalar field $\phi(t)$ with potential $V(\phi)$. The $a(t)$ is the scale factor. Then

$$3H^2 = \rho = \frac{1}{2}\dot{\phi}^2 + V \ , \qquad 2\dot{H} = -(\rho + P) = -\dot{\phi}^2 \ , \qquad (1)$$

where $H = \frac{\dot{a}}{a}$ is a Hubble parameter.

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where $H = \frac{\dot{a}}{a}$ is a Hubble parameter. Let us note that

$$\frac{\dot{H}}{H^2} = -\frac{3\dot{\phi}^2}{\dot{\phi}^2 + 2V} \quad \Rightarrow \quad \dot{H} \ll H^2 \ \text{ for } \ \dot{\phi}^2 \ll V \ . \qquad (2)$$

When $H \sim const$ one obtains $a \sim e^{Ht}$ $\rightarrow$ exponential expansion of the Universe! This is an example of the cosmic inflation.

ABC of slow-roll approximation

We have a scalar field with a potential $V(\phi)$ and a canonical kinetic term $\frac{1}{2}\dot{\phi}^2$. Let's define 2 slow-roll parameters

$$\epsilon = \frac{1}{2}\left(\frac{V_\phi}{V}\right)^2 , \qquad \eta = \frac{V_{\phi\phi}}{V} \tag{3}$$

$\epsilon \ll 1$ means that $\dot{H} \ll H^2$, which means that $H \simeq const$, which gives

$$a \simeq e^{Ht} \quad \leftarrow \quad \text{inflation} \tag{4}$$

$\eta \ll 1$ means that inflation will last for some time (but it's actually more complicated). Inflation consistent with the data requires both slow-roll parameters to be very small!

Comparison with the data

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How to translate that to observables?

During inflation one produces primordial inhomogeneities. Knowing

- what kind of gravity we have
- what's the kinetic term of the field
- what's the potential of the field

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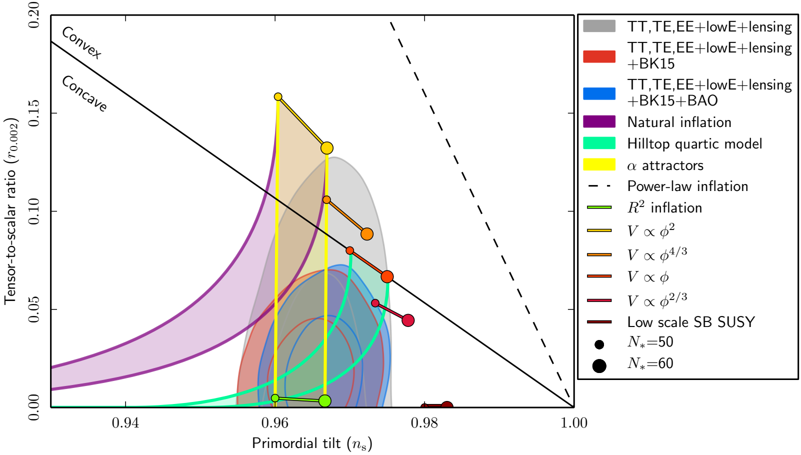
What can we compare to observations?

$$\mathcal{P}_\zeta \propto \frac{V}{\epsilon}\,, \quad r = \frac{\mathcal{P}_h}{\mathcal{P}_\zeta} = 16\epsilon\,, \quad n_s = 1 + \frac{d\log\mathcal{P}_\zeta}{d\log k} = 1 - 6\epsilon + 2\eta \quad (7)$$

All taken at some particular scale, which corresponds to $N \sim 50 - 60$ before the end of inflation

Experimental constraints



We have to get both η and ϵ very small! Small r translates to small V. Biggets possible V is $\sim (10^{15} GeV)^4$

Why low scale of inflation?

- Scale of inflation can be actually much lower that r would suggest. For instance in some theories of massive gravity one finds strong amplification of primordial GW and only theories with a very low scale of inflation may fit to the data
- Low scale inflation may be a part of the dynamical solution for Higgs hierarchy problem (Higgs + Axion)
- Low scale inflation is a must from the point of view of the lowest theoretically allowed physical scales.
- Allowed range of r is kind of wild. $\mathcal{O}(10^{-70}) < r < \mathcal{O}(10^{-2})$. What if we wont find any inflation in the highest possible scales?

What's the scale of inflation?

OK, getting very low scale of inflation should be really easy. We just need

$$\epsilon \to 0\,, \qquad \eta \to\sim -0.02 \tag{8}$$

It's actually really hard, because η and ϵ are not really independent!

The easiest way to see it? Let's consider $\epsilon = \epsilon(N)$

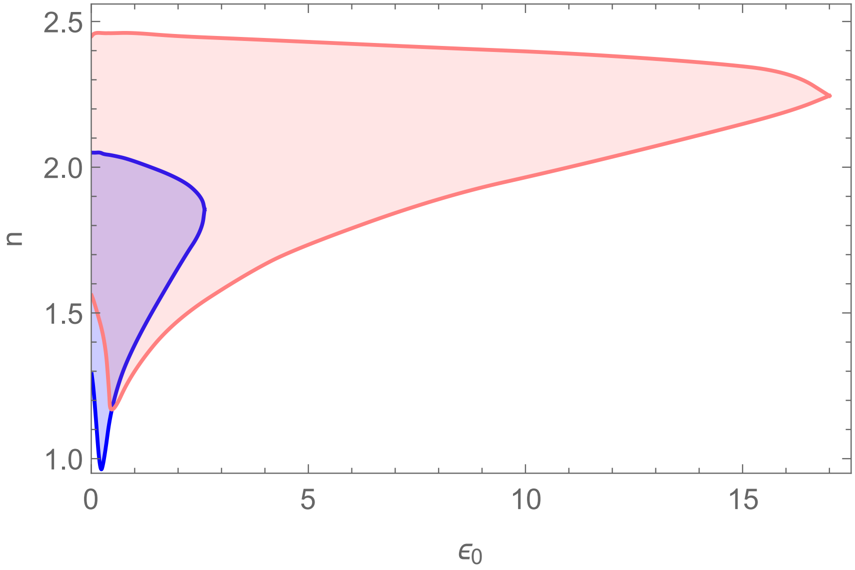
$$\eta = 2\epsilon + \frac{\epsilon_N}{2\epsilon}\,, \qquad n_s = 1 - 6\epsilon + 2\eta = 1 + \frac{\epsilon_N}{2\epsilon} - 2\epsilon\,, \tag{9}$$

which for $\epsilon = \epsilon_0 N^{-n}$ gives

$$\eta \sim -\frac{n}{2N} \tag{10}$$

Since $N \sim 50$ one cannot get n much bigger than 2

Allowed range of n and ϵ_0



The best ϵ

Using

$$\epsilon = \frac{\epsilon_0}{N^n} \,, \tag{11}$$

One can parameterize vast majority of used models

- $n = 1$ - a typical case of power-law potentials, like $V \propto \phi$, $V \propto \phi^2$, $V \propto \phi^4$ etc.
- $n = 2$ - Starobinsky-like models, Higgs inflation, α-attractors. These guys typically predict $r \sim 10^{-3}$. Anything significantly lower than that requires fine tuning.

The very best option is big n or $n \to \infty$

$$\epsilon = e^{-4\frac{N_\star}{M^2}} \,, \qquad n_s = 1 - \frac{4}{M^2} - 2\, e^{-4\frac{N_\star}{M^2}} \,, \qquad V \propto e^{-\phi^2/M^2} \tag{12}$$

Warm inflation

What's the true source of the problem? Big η parameter. How could we decrease it? By increasing the cosmic friction

$$\begin{aligned} \ddot{\phi} + 3H\dot{\phi} + V_{\phi} &= 0 \qquad \text{cold inflation} && (13) \\ \dot{\rho}_r + 4H\rho_r &= 0 && (14) \end{aligned}$$

vs

$$\begin{aligned} \ddot{\phi} + (3H + \Gamma)\dot{\phi} + V_{\phi} = \ddot{\phi} + 3H(1+Q)\dot{\phi} + V_{\phi} = 0\,, && (15) \\ \dot{\rho}_r + 4H\rho_r = \dot{\phi}^2\Gamma = 3HQ\dot{\phi}^2\,. \quad \text{warm inflation} && (16) \end{aligned}$$

We define $Q = \Gamma/3H$. Now our slow-roll parameters look like

$$\epsilon_{eff} = \frac{\epsilon}{1+Q}\,, \qquad \eta_{eff} = \frac{\eta}{1+Q} \tag{17}$$

Big Q decreases the value of η and should enable good n_s

ABC of warm inflation

Assuming temperature-independent form of Γ one finds

$$\mathcal{P}_h = 8\left(\frac{H}{2\pi}\right)^2, \tag{18}$$

$$\mathcal{P}_\mathcal{R} = \left(\frac{H}{2\pi}\right)^2 \left(\frac{H}{\dot{\phi}}\right)^2 \left(1 + \frac{T}{H}\frac{2\sqrt{3}\,\pi\, Q}{\sqrt{3 + 4\pi\, Q}}\right), \tag{19}$$

where all of the functions are taken at the moment of the horizon crossing. In the cold inflationary scenario, i.e. for $Q \to 0$, one recovers $\mathcal{P}_\mathcal{R} \propto H^4/\dot{\phi}^2$.

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EASY to calculate, since H, ϕ and T satisfy slow-roll equations

Strong dissipation regime

In the $Q \gg 1$ limit one finds

$$\mathcal{P}_{\mathcal{R}} \simeq \frac{\sqrt{3\pi}}{2} \frac{H^3 Q\, T}{\dot{\phi}^2}\,, \qquad n_s \simeq 1 - \frac{1}{Q}\left(\frac{9}{4}(\epsilon + \beta) - \frac{3}{2}\eta\right)\,, \quad (20)$$

where

$$\epsilon = \frac{V_\phi^2}{2V^2}\,, \qquad \eta = \frac{V_{\phi\phi}}{V}\,, \qquad \beta = \frac{\Gamma_\phi V_\phi}{\Gamma V} \quad (21)$$

and $V_\phi = \frac{dV}{d\phi}$. The simplest scenario to consider is $Q = const$, which gives $\Gamma \propto \sqrt{H}$. In such a case one finds $\Gamma_\phi/\Gamma \propto \frac{1}{2} V_\phi/V$, which gives $\beta = \epsilon$ and in consequence $n_s \simeq 1 - (9\epsilon/2 - 3\eta/2)/Q$.

Example of the bell-curve potential

Our starting point was the $n \to \infty$ limit, i.e. the bell-curve potential! Dreadful, due to its huge η parameter. For finite n it all works really well!

$$n_s \simeq 1 - \frac{1}{4Q}\left(6\epsilon_0 N^{-n} + \frac{3n}{N}\right) \tag{22}$$

Typical situation: $\epsilon_0 \sim \mathcal{O}(1)$, hence $3n/N \gg \epsilon_0 N^{-n}$.

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Typical situation: $\epsilon_0 \sim \mathcal{O}(1)$, hence $3n/N \gg \epsilon_0 N^{-n}$. Otherwise - massive fine tuning!

Note: $Q \sim n$ always saves the day

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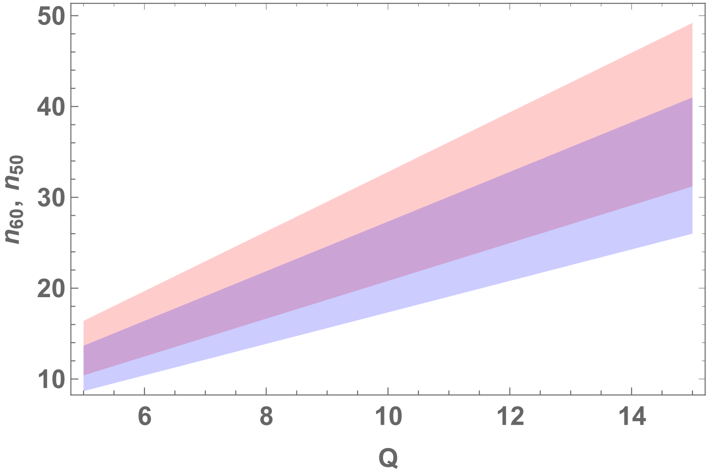

Figure: Allowed range of n for $N = 60$ (red area) and $N = 50$ (blue area), for which n_s from the Eq. (22) is consistent with the Planck data.

Problems? Advantages?

- Temperature-dependence of Γ would make everything super complicated
- No need for reheating! No need for thermalization! BBN could start right away after inflation
- At the end of the day ANY scale of inflation would do. We can stop worrying about the nearest vicinity of the GUT scale