Structure forming plane symmetric dust inhomogeneous cosmological model

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Motivation

- We consider a model whose space-time belongs to the G_3/S_2-symmetric class of space-times and is plane symmetric. We assume that this space-time solves the Einstein equations without the cosmological constant for a dust source.
- This model is a planar counterpart of the Lemaitre-Tolman model which is spherically symmetric.
- We will consider an infinite regular arrangement of inhomogeneities in the form of stacked planes of over and underdensities.
- We are going to determine the dependence of the temporal evolution of the energy density contrast with regard to the specifics of the inhomogeneities.
- Such formula could be useful for the modeling of the large-scale structure formation.
- There exist some formulas, which describe the evolution of the energy density contrast for general profile of inhomogeneities (Goode, Wainwright, 1982; Kasai, 1993), but they are limited to the cases with small contrast.

- The space-time metric and the matter four-velocity of the model are given in coordinates $t \in \mathbb{R}_+$, $x, y, z \in \mathbb{R}$ as follows

$$g_{mn} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & q^2 & 0 & 0 \\ 0 & 0 & p^2 & 0 \\ 0 & 0 & 0 & p^2 \end{pmatrix}, \qquad u_n = \begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

where

$$q = \frac{p'}{o}, \qquad \dot{p}^2 = \frac{n}{p} + o^2.$$

The quantity p is a function of coordinates t and x, quantities o and n are functions of x. The dot and prim denote the derivative with respect to the coordinate t and x respectively.

The model properties

- The matter in the model is defined as a dust fluid whose flow is geodesic and irrotational. The magnetic part of the Weyl tensor with respect to this flow also vanishes.
- The basic nonzero scalar quantities in the model are
  - the expansion rate
    $$\theta = \frac{\partial_x (p^2 \dot{p})}{p^2 p'},$$
  - the Ricci scalar of the spatial hypersurfaces
    $${}^3R = -\frac{2\partial_x (o^2 p)}{p^2 p'},$$
  - the energy density
    $$\varkappa \rho = \frac{n'}{p^2 p'}.$$

The metric functions

- The equation for the function p can be integrated parametrically to give the solution in the form

$$p = \frac{n}{2o^2}(\cosh\eta - 1), \qquad \frac{n}{2o^3}(\sinh\eta - \eta) = t - m,$$

  where m is an arbitrary function of x.

- For practical reasons, the derivative of the function p with respect to the coordinate x is very useful and it can be given (Barnes, 1970)

$$p' = \left(\frac{n'}{n} - \frac{\partial_x(o^2)}{o^2}\right)p - \left(\left(\frac{n'}{n} - \frac{3\partial_x(o^2)}{2o^2}\right)(t-m) + m'\right)\dot{p}.$$

- The model is fully determined by the three functions of the x coordinate, m, n and o. However, since we have freedom to choose the x coordinate, effectively we have only two free functions to restrict physical properties of the model.

The limiting cases

- The homogeneous limit of the considered model can be achieved by the following choice

$$m' = 0, \qquad n = \frac{\Omega_M}{(1-\Omega_M)^{\frac{3}{2}}}\, o^3.$$

  Then, it is the spatially open Friedmann-Lemaitre model characterized by the energy density parameter $\Omega_M \in (0, 1)$.
- When only $n \propto o^3$ then the model becomes asymptotically homogeneous in the infinite future. Such a model allows inhomogeneities only to decay.
- When only $m' = 0$ then the model becomes asymptotically homogeneous in the past, near the initial singularity. Such a model allows inhomogeneities only to grow.

The asymptotic behavior

- The inhomogeneous nature of the considered model manifests in the asymptotic behavior of the Ricci scalar of the spatial hypersurfaces and the energy density.
- The asymptotic profile of the Ricci scalar of the spatial hypersurfaces in the past depends on the x coordinate and equals, when $m \neq 0$

$$\lim_{t\to 0} {}^3R = -\frac{2\left(\frac{2}{3}\right)^{\frac{4}{3}} o^2}{n^{\frac{2}{3}}}(t-m)^{-\frac{4}{3}},$$

and when $m = 0$

$$\lim_{t\to 0} {}^3R = -\frac{2\left(\frac{2}{3}\right)^{\frac{4}{3}}}{n^{\frac{2}{3}}}\left(o^2 + \frac{3n\partial_x(o^2)}{n'}\right)t^{-\frac{4}{3}}.$$

- Similarly, the asymptotic profile of the energy density in the future depends on the x coordinate and equals (Hellaby, Krasiński, 2006)

$$\lim_{t\to\infty} \varkappa\rho = \frac{n'}{o^2 o'}(t-m)^{-3}.$$

- It is seen that the initial inhomogeneities in the Ricci scalar of the spatial hypersurfaces develop into the final inhomogeneities in the energy density.

The choice of the metric functions

- We are going to construct a model which comprises only growing inhomogeneities so we choose

$$m = 0.$$

  This also means that the initial singularity is simultaneous in the whole space-time.

- Furthermore, we want the model not to exhibit shell crossings and to be always expanding. This could be achieved with the general assumption that $n, o > 0$ and in particular we assume

$$o = \exp x.$$

  In such a model we do not expect the structure to virialize or to collapse, eventually. During the evolution, the energy density is decreasing everywhere, but its profile becomes frozen and stands into infinite future.

The choice of the metric functions

- The homogeneous limit of the asymptotic profile of the energy density in the future reads $3\Omega_M(1-\Omega_M)^{-\frac{3}{2}}t^{-3}$. Thus, we choose to define the function n as a solution to the following asymptotic profile of the energy density in the future

$$\lim_{t\to\infty} \varkappa\rho = \frac{3\mu}{(1-\mu)^{\frac{3}{2}}} t^{-3}\left(1+\kappa\cos\left(\frac{x}{\lambda}\right)^{2\nu}\right),$$

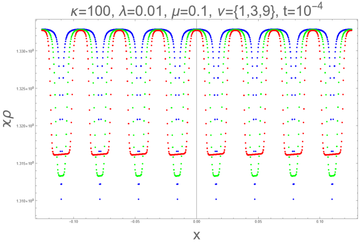
where for the constant of integration we assume that $\lim_{x\to-\infty} n = 0$. We have introduced here four parameters κ, λ, μ and ν which determine the properties of inhomogeneities. The assumed asymptotic profile of the energy density in the future has a form of an infinite regular chain of identical planar overdensities having a cosine-like shape.

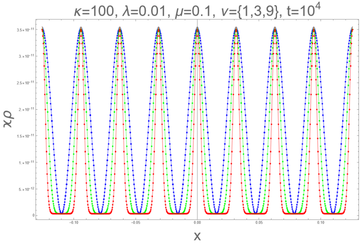
The range of the parameters

- The parameter μ is restricted to $\mu \in (0, 1)$. When there is no inhomogeneities it plays the role of the energy density parameter Ω_M. It controls the time of the structure formation, which is earlier for smaller values of μ.
- The parameter κ takes values $\kappa \in [0, \infty)$. It equals to the asymptotic value of the energy density contrast in the future.
- The parameter ν is considered to be a natural number, $\nu \in \mathbb{N}_+$. It controls the final width of the inhomogeneities.
- The parameter λ, $\lambda \in \mathbb{R}_+$, determines the distribution of the inhomogeneities. For example, for $\lambda = 10^{-2}$ the observer will count about 10 overdensities up to the redshift of about 10.
- The parameter λ is a natural small parameter in the model and thus we will consider only cases with $\lambda \ll 1$.
- In particular for such small values of λ, the function η very weakly depends on the x coordinate and then its formula reads

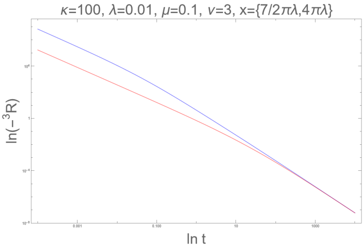
$$\frac{1}{2}(1 + f\kappa)\frac{\mu}{(1-\mu)^{\frac{3}{2}}}(\sinh\eta - \eta) = t, \qquad f = \frac{(2\nu)!}{2^{2\nu}(\nu!)^2}.$$

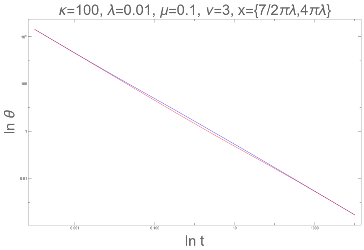
Spatial dependence

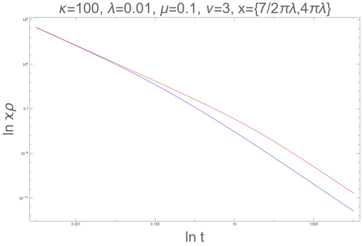




Temporal dependence







The energy density contrast

- We define the energy density contrast as a contrast between a local maximum and minimum of the energy density

$$\delta = \frac{\rho_{\text{max}} - \rho_{\text{min}}}{\rho_{\text{min}}}.$$

- From this definition we may find

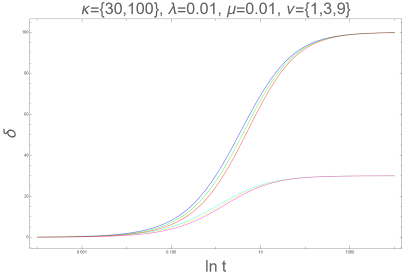
$$\delta = (1+\kappa)\frac{1 + f\kappa g}{1+\kappa-(1-f)\kappa g} - 1,$$

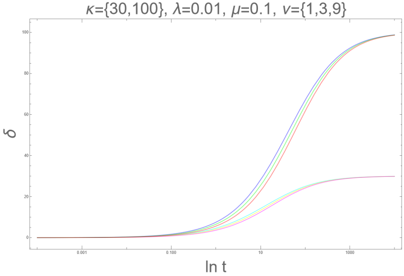
where

$$g = 1 + 3\frac{1-\frac{\eta}{2}\,\text{ctgh}\left(\frac{\eta}{2}\right)}{\sinh^2\left(\frac{\eta}{2}\right)}.$$

- We can see that the energy density contrast directly depends on the parameters κ and ν.

The energy density contrast evolution





Summary

- We have developed a simple inhomogeneous model for which we have calculated explicitly the evolution of the energy density contrast.
- We have shown that this evolution is not universal and depends on the properties of inhomogeneities.
- The model is considered to be a toy model. Its construction is not fully satisfactory. In particular, it has a planar symmetry and the distribution of inhomogeneities is not realistic.