Quantum simulations of loop quantum gravity

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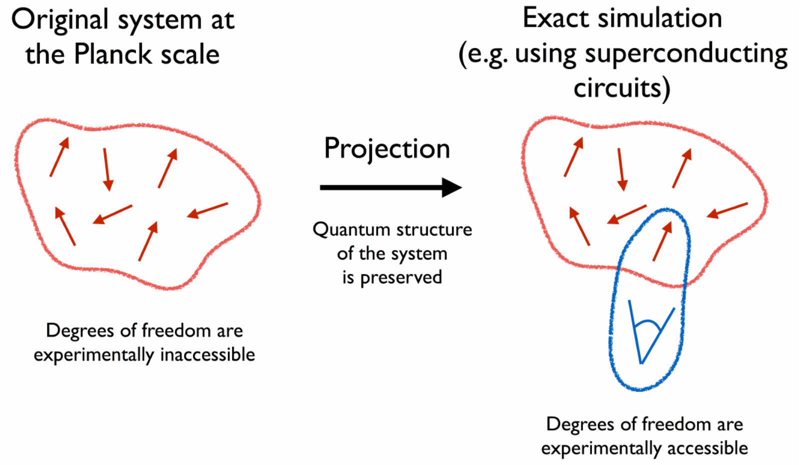
Quantum simulations of physics

Feynman, R. P. **Simulating physics with computers.** Int. J. Theor. Phys. 21, 467–488 (1982)



Quantum computers and quantum systems

for one qubit

$$\dim(\mathcal{H}) = 2$$

for N qubits

$$\dim\left(\mathcal{H}^{\otimes N}\right) = 2^N$$

Quantum computers and quantum systems

for one qubit

$$\dim(\mathcal{H}) = 2$$

for N qubits

$$\dim\left(\mathcal{H}^{\otimes N}\right) = 2^N$$

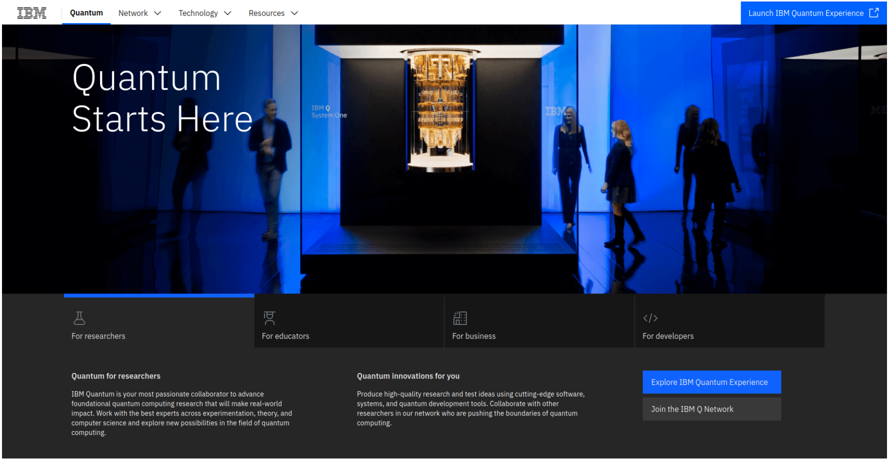
for $N = 50$

$$\dim\left(\mathcal{H}^{\otimes 50}\right) \simeq 10^{15}$$

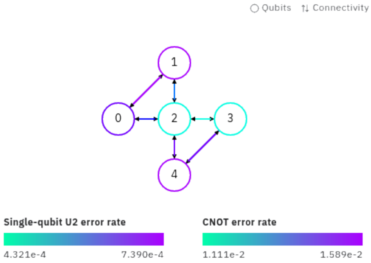
Simulations of LQG

- Cohen, L., Brady, A. J., Huang, Z., Liu, H., Qu, D., Dowling, J. P., & Han, M. (2021). **Efficient simulation of loop quantum gravity: A scalable linear-optical approach.** Physical Review Letters, 126(2), 020501.
- Li, K., Li, Y., Han, M., Lu, S., Zhou, J., Ruan, D., ... & Laflamme, R. (2019). **Quantum spacetime on a quantum simulator.** Communications Physics, 2(1), 1-6.
- Zhang, P., Huang, Z., Song, C., Guo, Q., Song, Z., Dong, H., ... & Wan, Y. (2020). **Observation of two-vertex four-dimensional spin foam amplitudes with a 10-qubit superconducting quantum processor.** arXiv preprint arXiv:2007.13682.
- van der Meer, R., Huang, Z., Anguita, M. C., Qu, D., Hooijschuur, P., Liu, H., ... & Cohen, L. (2022). **Experimental Simulation of Loop Quantum Gravity on a Photonic Chip.** arXiv preprint arXiv:2207.00557.

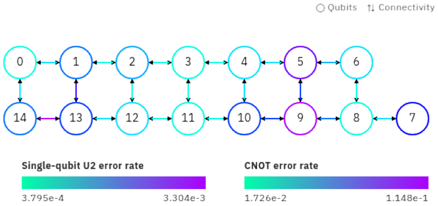
Quantum computers IBM Q



Quantum processors



Yorktown quantum processor



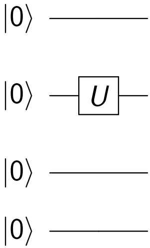
Melbourne quantum processor

Quantum circuits

Quantum register:

$$|0\rangle \ —$$
$$|0\rangle \ —$$
$$|0\rangle \ —$$
$$|0\rangle \ —$$

One-qubit gates:

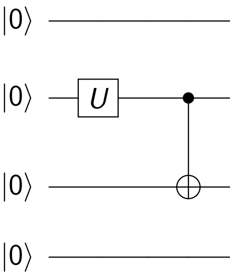


$$U_3 = \begin{pmatrix} \cos(\theta/2) & -e^{i\lambda}\sin(\theta/2) \\ e^{i\phi}\sin(\theta/2) & e^{i\lambda+i\phi}\cos(\theta/2)\,, \end{pmatrix}$$

$$U_3|0\rangle = \cos(\theta/2)\,|0\rangle + e^{i\phi}\sin(\theta/2)\,|1\rangle$$

Quantum circuits

Two-qubits gates:

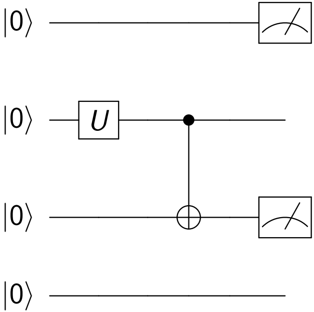


$$CX = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

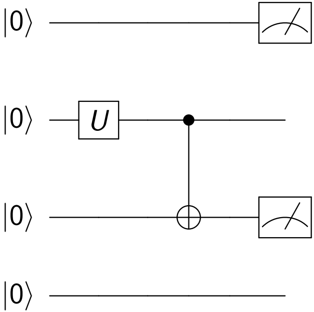
$$CX|00\rangle = |00\rangle$$
$$CX|01\rangle = |01\rangle$$
$$CX|10\rangle = |11\rangle$$
$$CX|11\rangle = |10\rangle$$

Quantum circuits

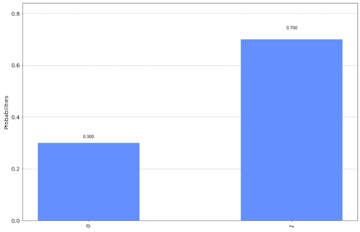
Measurements:



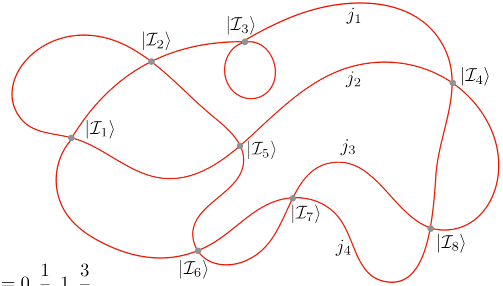
Measurements:



$\longrightarrow$



Spin-network states

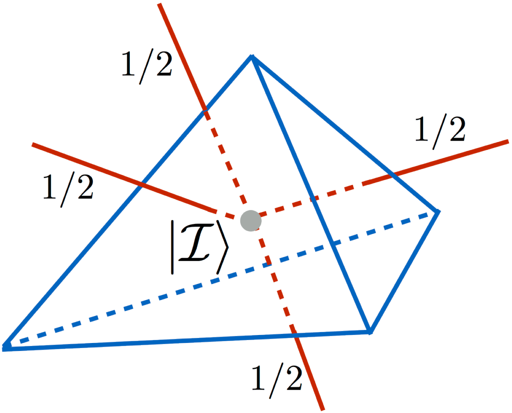


$j_i = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots$

$$|\mathcal{I}_n\rangle \in Inv_{SU(2)}\left(\mathcal{H}_{j_a} \otimes \mathcal{H}_{j_b} \otimes \mathcal{H}_{j_c} \otimes \mathcal{H}_{j_d}\right)$$

$$|\Gamma, j_l, \mathcal{I}_n\rangle = \bigotimes_n \mathcal{I}_n$$

Single node

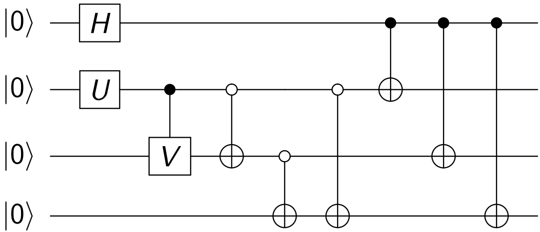


$$|\mathcal{I}\rangle \in Inv_{SU(2)}\left(\mathcal{H}_{1/2} \otimes \mathcal{H}_{1/2} \otimes \mathcal{H}_{1/2} \otimes \mathcal{H}_{1/2}\right)$$

$$|\mathcal{I}\rangle = \cos\frac{\theta}{2}|\iota_0\rangle + e^{i\phi}\sin\frac{\theta}{2}|\iota_1\rangle$$

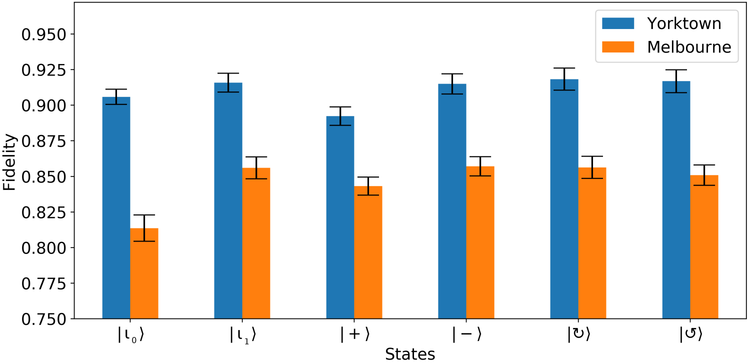
Single node on qubits

$$|\mathcal{I}\rangle = \frac{c_1}{\sqrt{2}}(|0011\rangle + |1100\rangle) + \frac{c_2}{\sqrt{2}}(|0101\rangle + |1010\rangle) + \frac{c_3}{\sqrt{2}}(|0110\rangle + |1001\rangle)$$



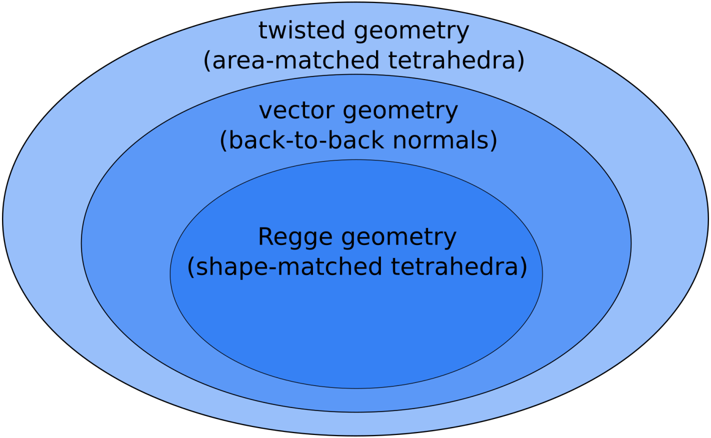
$$U = \begin{pmatrix} c_1 & \sqrt{|c_2|^2 + |c_3|^2} \\ -\sqrt{|c_2|^2 + |c_3|^2} & c_1^* \end{pmatrix} \qquad V = \begin{pmatrix} -\frac{c_2}{\sqrt{|c_2|^2+|c_3|^2}} & \frac{c_3^*}{\sqrt{|c_2|^2+|c_3|^2}} \\ -\frac{c_3}{\sqrt{|c_2|^2+|c_3|^2}} & -\frac{c_2^*}{\sqrt{|c_2|^2+|c_3|^2}} \end{pmatrix}$$

Simulation on quantum processor



G. Cz, Jakub Mielczarek, "**Quantum simulations of a qubit of space**" Phys. Rev. D 103, 046001 (2021)

Gluing tetrahedra



Spin-network basis states are un-entangled $|\Gamma, j_l, \mathcal{I}_n\rangle = \bigotimes_n \mathcal{I}_n$

Gluing tetrahedra

Baytaș, B., Bianchi, E., & Yokomizo, N. (2018). **Gluing polyhedra with entanglement in loop quantum gravity.** Physical Review D, 98(2), 026001.

squeezed states

$$|\mathcal{B}, \lambda_l\rangle = \left(1 - |\lambda_l|^2\right) \sum_j \sqrt{2j+1}\lambda_l^{2j} |\mathcal{B}, j\rangle \tag{1}$$

singlet state of spin j, which is maximally entangled

$$|\mathcal{B}, j\rangle = \frac{1}{\sqrt{2j+1}} \sum_{m=-j}^{j} (-1)^{j-m} |j, m\rangle_s |j, -m\rangle_t$$

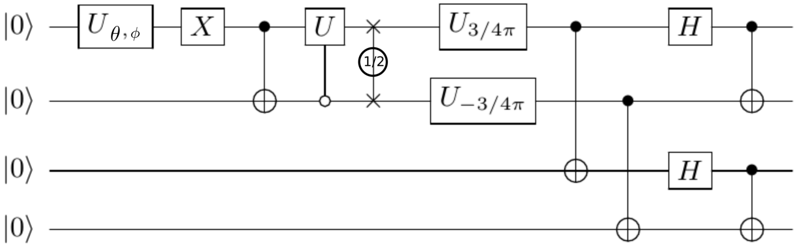
Gluing tetrahedra

projection on spin-network basis states

$$P_\Gamma = \sum_{j_l, \mathcal{I}_n} |\Gamma, j_l, \mathcal{I}_n\rangle \langle \Gamma, j_l, \mathcal{I}_n|$$

$$|\Gamma, \mathcal{B}, \lambda_l\rangle = P_\Gamma \bigotimes_l |\mathcal{B}, \lambda_l\rangle$$

New circuit for node



$$|\mathcal{I}\rangle = \cos\frac{\theta}{2}|\iota_0\rangle + e^{i\phi}\sin\frac{\theta}{2}|\iota_1\rangle =$$
$$\frac{c_1}{\sqrt{2}}(|0011\rangle + |1100\rangle) + \frac{c_2}{\sqrt{2}}(|0101\rangle + |1010\rangle) + \frac{c_3}{\sqrt{2}}(|0110\rangle + |1001\rangle)$$

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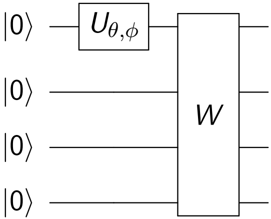

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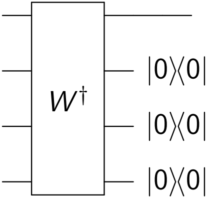
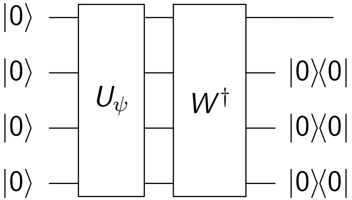
Projection



"Projection" operator on intertwiner subspace, expressed in one-qubit representation.

Projection



Projection of state $|\psi\rangle$ on intertwiner subspace, expressed in one-qubit representation.

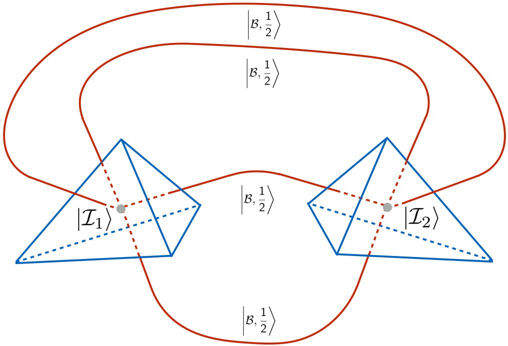
$$\left(\sum_k |\iota_k\rangle \langle \iota_k|\right) |\psi\rangle$$

Gluing tetrahedra

for $j = \frac{1}{2}$ in qubit notations:

$$\left|\mathcal{B}, \frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

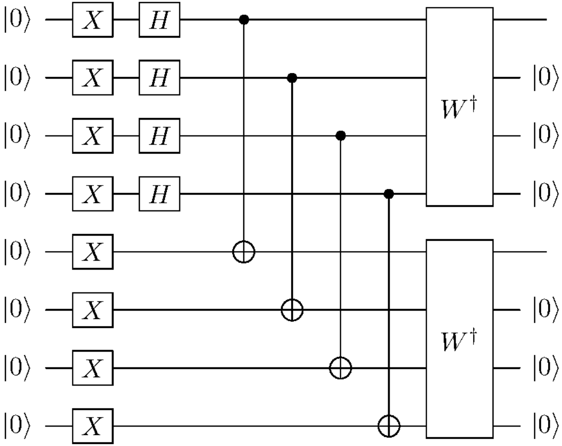
Dipole



$$\left|\Gamma_2, \mathcal{B}, \frac{1}{2}\right\rangle = P_\Gamma \bigotimes_l \left|\mathcal{B}, \frac{1}{2}\right\rangle = \sum_{k,l} \iota_{(k)}^{m_1 m_2 m_3 m_4} \iota_{(l) m_1 m_2 m_3 m_4} \left|\iota_k \iota_l\right\rangle$$

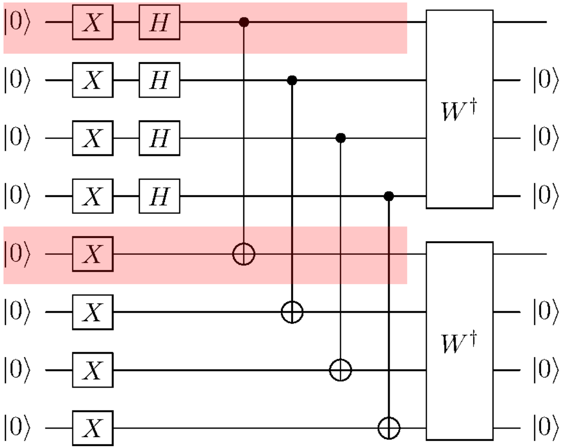
$$= \frac{1}{\sqrt{2}} \left(\left|\iota_0 \iota_0\right\rangle + \left|\iota_1 \iota_1\right\rangle\right)$$

Dipole



$$\left|\Gamma_2, \mathcal{B}, \frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}\left(|\iota_0\iota_0\rangle + |\iota_1\iota_1\rangle\right)$$

Dipole



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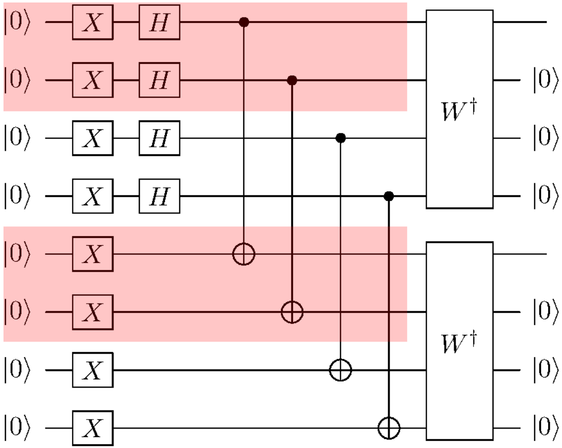
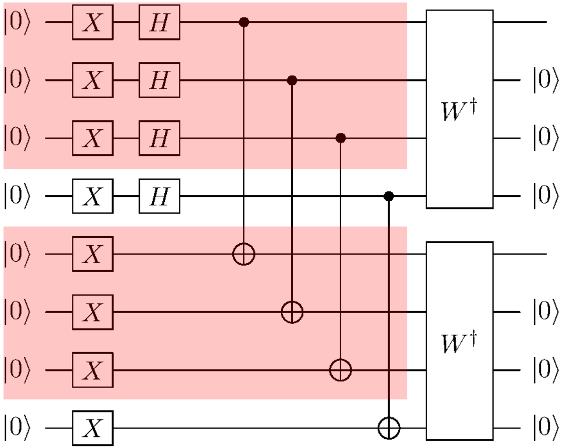
Dipole



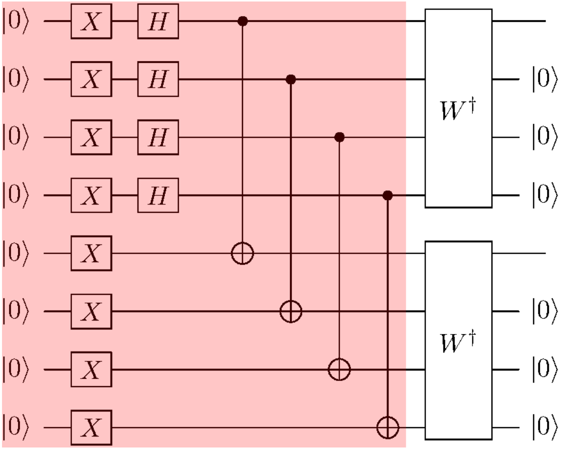
$$\left|\Gamma_2, \mathcal{B}, \frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}\left(|\iota_0\iota_0\rangle + |\iota_1\iota_1\rangle\right)$$

Dipole



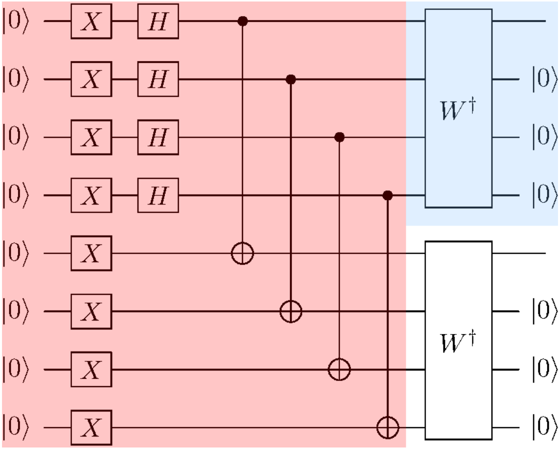
$$\left|\Gamma_2, \mathcal{B}, \frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}\left(|\iota_0\iota_0\rangle + |\iota_1\iota_1\rangle\right)$$

Dipole



$$\left|\Gamma_2, \mathcal{B}, \frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}\left(|\iota_0\iota_0\rangle + |\iota_1\iota_1\rangle\right)$$

Dipole



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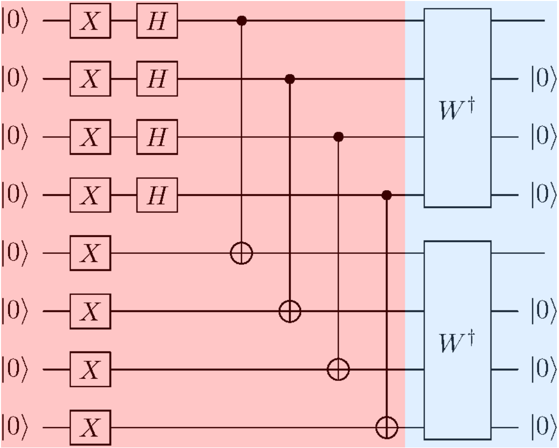
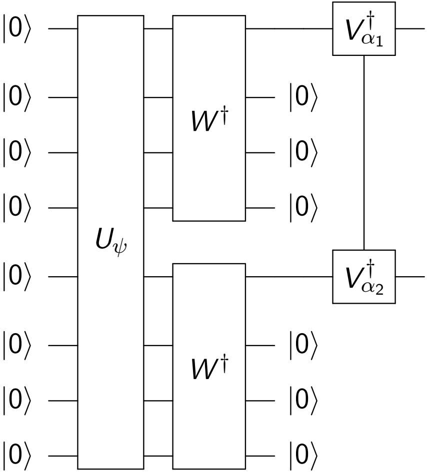
Dipole



$$\left|\Gamma_2, \mathcal{B}, \frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}\left(|\iota_0\iota_0\rangle + |\iota_1\iota_1\rangle\right)$$

Dipole

Transfer of projected state on ansatz:

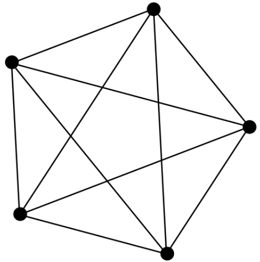


Pentagram

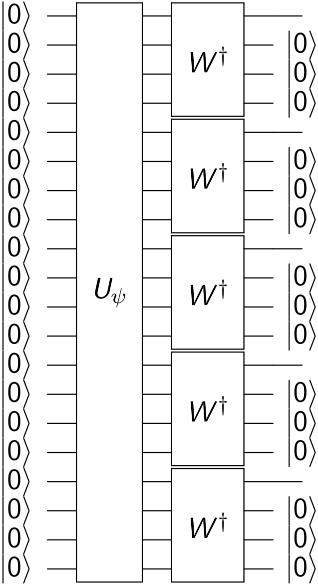
$$P_\Gamma |\psi\rangle = \sum_{\iota_{k_i}} \overline{\{15j\}} \, |\iota_{k_1}\iota_{k_2}\iota_{k_3}\iota_{k_4}\iota_{k_5}\rangle$$

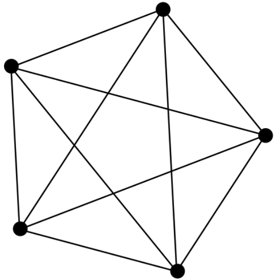
where

$$\{15j\} = \iota_1^{m_{12}m_{13}m_{14}m_{15}} \iota_{2;m_{12}}{}^{m_{13}m_{14}m_{15}} \iota_{3;m_{12}m_{13}}{}^{m_{14}m_{15}} \iota_{4;m_{12}m_{13}m_{14}}{}^{m_{15}} \iota_{5;m_{12}m_{13}m_{14}m_{15}}$$



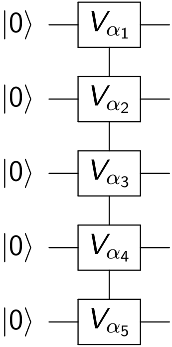
Pentagram

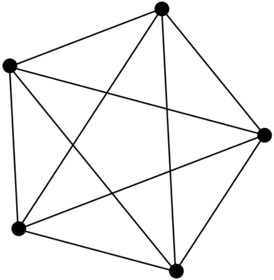




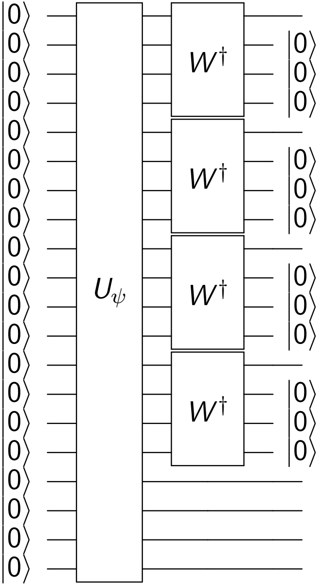
Pentagram

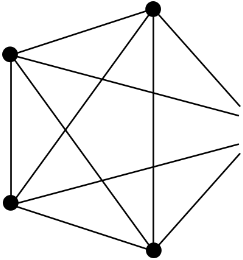
Ansatz:





Open pentagram





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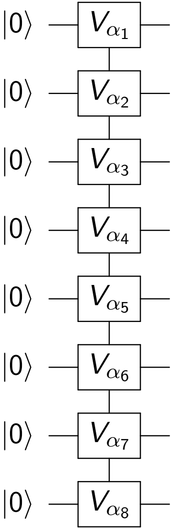
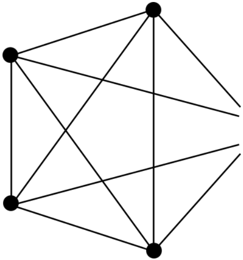
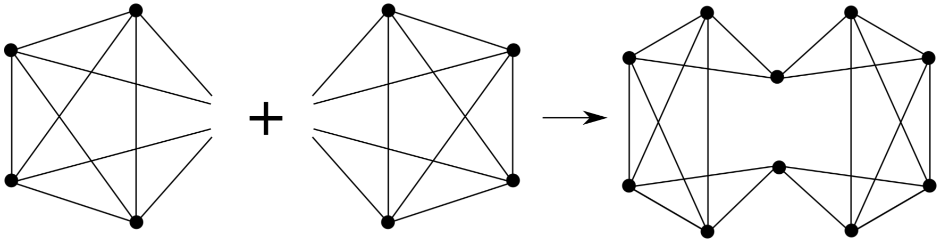




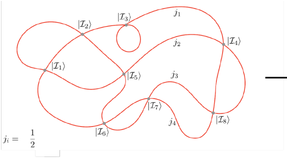
Decagram

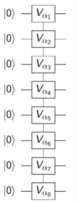


8 qubits + 8 qubits → 10 qubits

Simulations of spin networks

Spin network





Parametrized quantum circuit

Measurements of observables

$$\langle \hat{V} \rangle,\ S(\rho_i),\ \ldots$$

Execution on quantum device

Mutual information

Mutual information:

$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

Entropy:

$$S(\rho) = -Tr\rho \ln \rho$$

Mutual information

Mutual information:

$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

Entropy:

$$S(\rho) = -Tr\rho \ln \rho$$

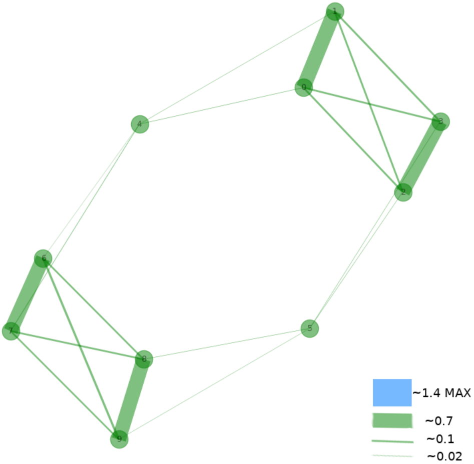
Correlation funtion:

$$\mathcal{C}(O_A, O_B) = \frac{1}{||O_A|| \, ||O_B||} \left(\langle O_A O_B \rangle - \langle O_A \rangle \langle O_B \rangle \right)$$

Bound on correlation function:

$$\frac{1}{2} \mathcal{C}(O_A, O_B)^2 \leq I(\rho_{AB})$$

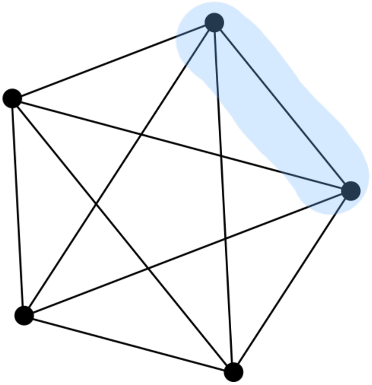
Mutual information: Decagram



$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

Page curve - random pentagram

Pairs of spins in random state on the links:



Entropy of subsystem:

$$N_A = 2$$

$$\rho = Tr_A |\Gamma_5\rangle$$

$$S(\rho) = -Tr \rho \log \rho$$

Page curve - random pentagram

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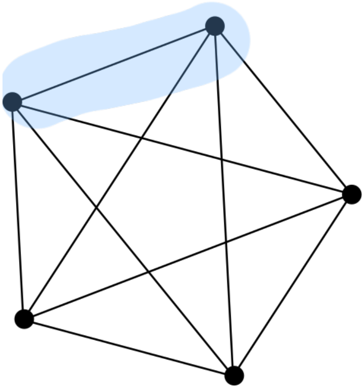

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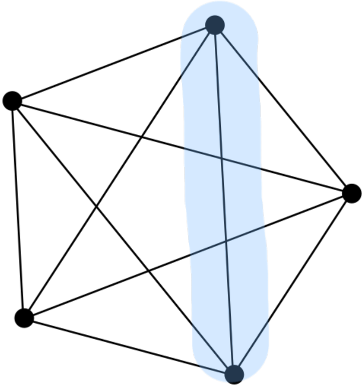

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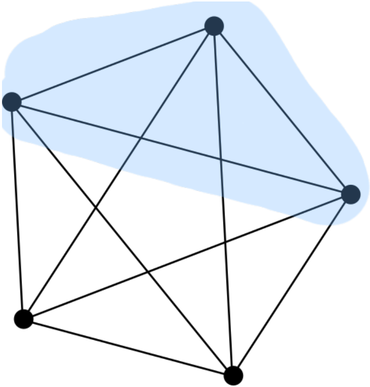


Entropy of subsystem:

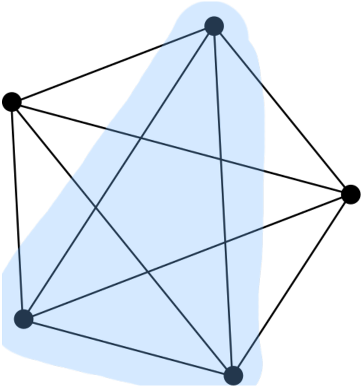
$$N_A = 3$$

$$\rho = Tr_A |\Gamma_5\rangle$$

$$S(\rho) = -Tr \rho \log \rho$$

Page curve - random pentagram

Pairs of spins in random state on the links:



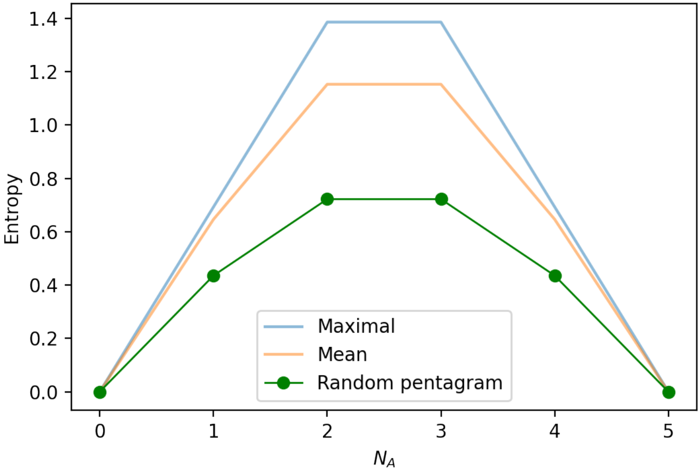
Entropy of subsystem:

$$N_A = 3$$

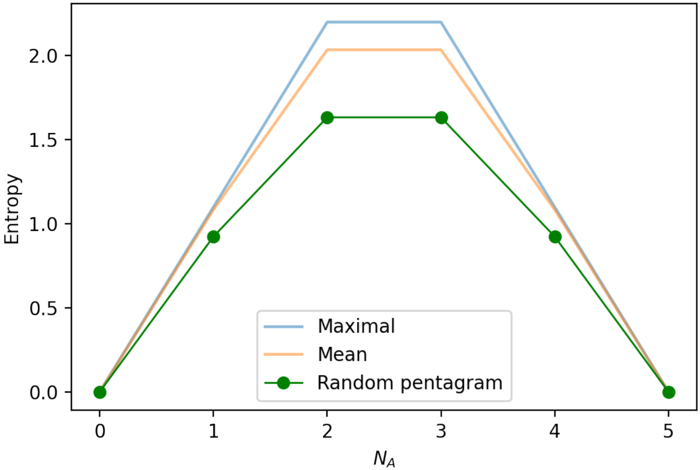
$$\rho = Tr_A|\Gamma_5\rangle$$

$$S(\rho) = -Tr \rho \log \rho$$

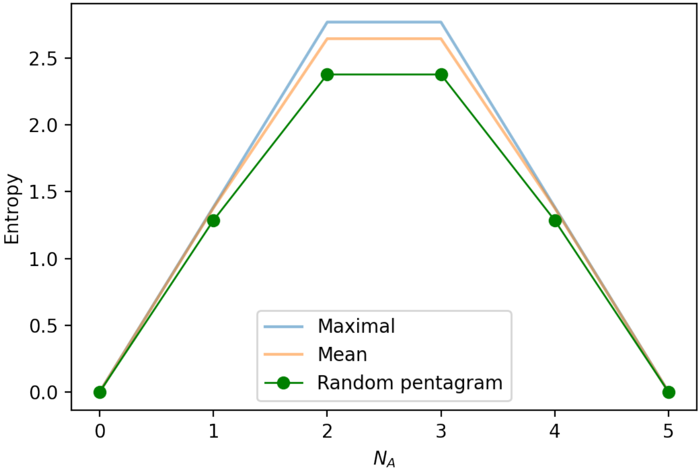
Random pentagram - spin $\frac{1}{2}$



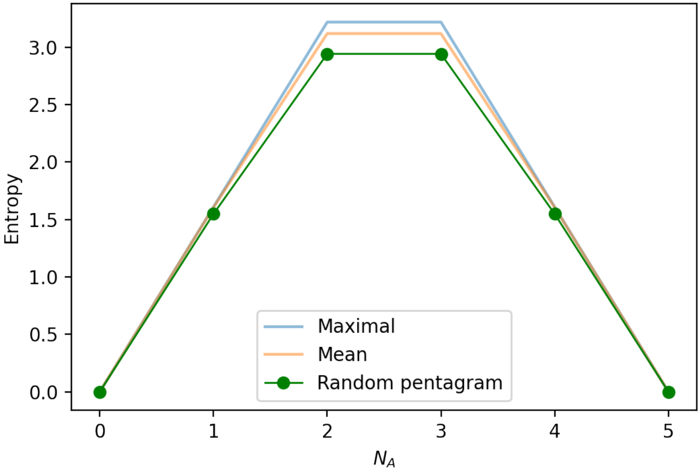
Random pentagram - spin 1



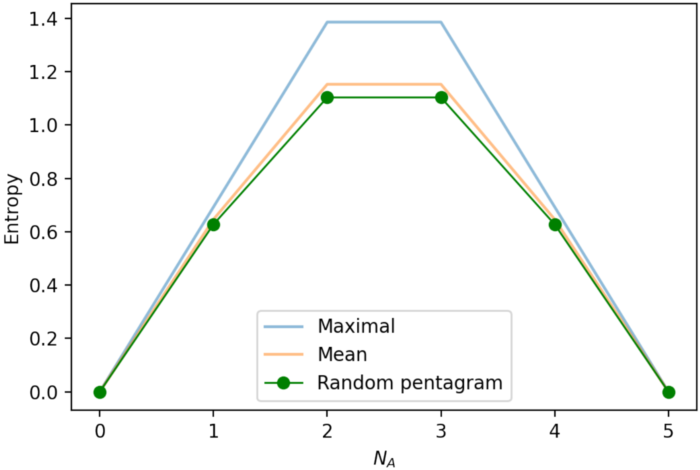
Random pentagram - spin $\frac{3}{2}$



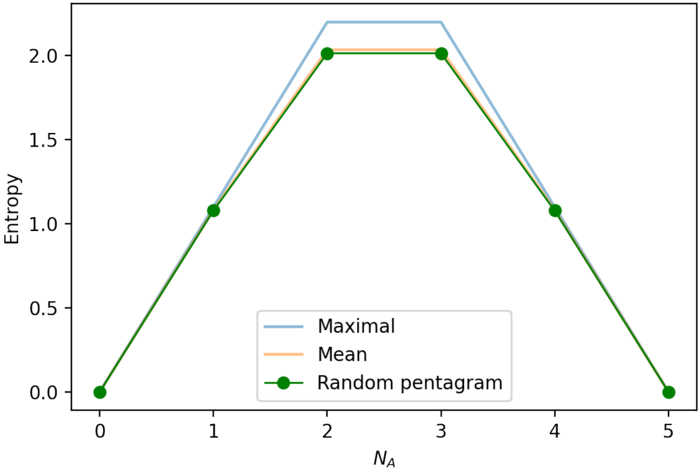
Random pentagram - spin 2



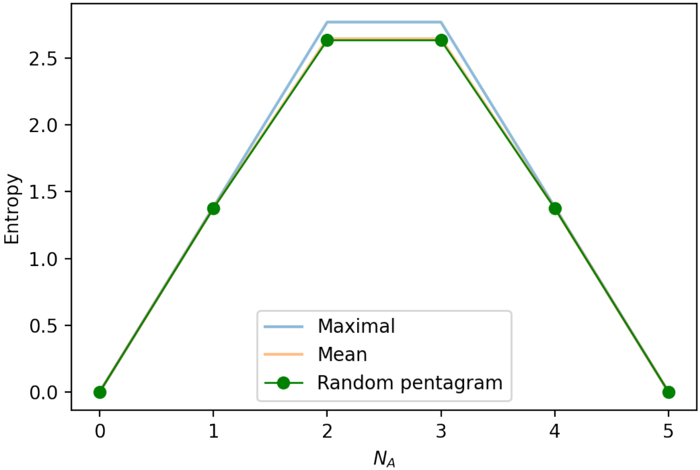
Random pentagram - maximally entangled links - spin $\frac{1}{2}$



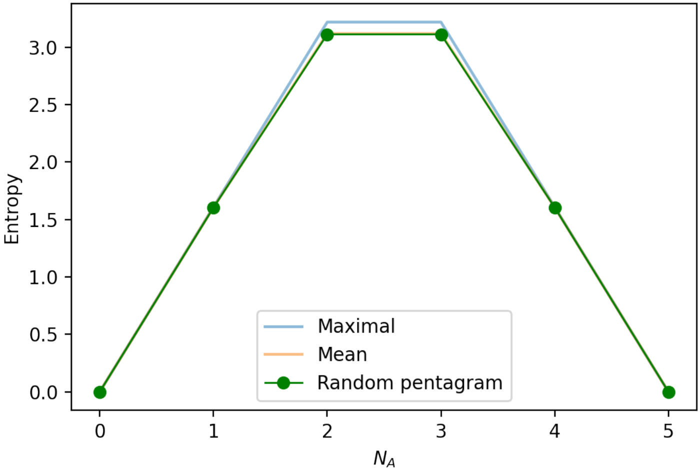
Random pentagram - maximally entangled links - spin 1



Random pentagram - maximally entangled links - spin $\frac{3}{2}$



Random pentagram - maximally entangled links - spin 2



Summary

- we can prepare a broad class of spin-network states on a quantum register
- measurements can be performed both on simulator and real quantum processor
- a number of required qubits is not much greater than the number of nodes
- entropies of subsystems and Page curve can be extracted
- for low spins, typical entropy of a subsytem in random pentagram is smaller than in random 5-qubit quantum system
- entropy of subsystems in pentagram grows up to typical entropy of a 5-qubit quantum system if maximally entangled links are used

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Thank you for your attention