Quantum dynamics of relativistic systems

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POTOR 8
Warszawa, 19 - 23 Sept, 2022

Basic remarks

- Diffeomorphisms $\Rightarrow$ Hamiltonian constraint $\mathbf{H} \approx 0 \Rightarrow$ plenty of internal times and their equivalence
- Quantum level: different internal time theories unitarily inequivalent
- Standpoints:
  Different internal times cannot be compared
  Differences cannot be large otherwise nonsensical
  Differences can be unfortunately significant
- In this talk: comparison method +

> *What are the dynamical predictions of QG models (if any) that do not depend on the choice of internal time?*

- Simple example: generation of primordial GWs

Comparison method

Classical level: clock transformations $(q, p, T) \mapsto (\bar{q}, \bar{p}, \bar{T})$:

$$\omega_{\mathcal{C}} = \mathrm{d}q\mathrm{d}p - \mathrm{d}T\mathrm{d}H = \mathrm{d}\bar{q}\mathrm{d}\bar{p} - \mathrm{d}\bar{T}\mathrm{d}\bar{H}$$

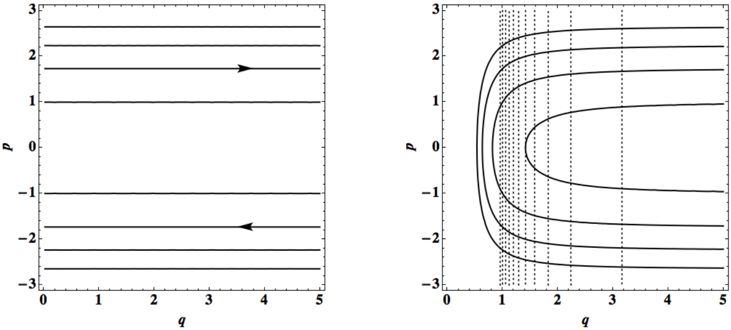
Quantum level: different internal time theories must be comparable and any discrepancies due to the usual quantization ambiguities excluded
Quantization: constants of motion (= Dirac observables),

$$C_J(q, p, T) \to \hat{C}_J : \mathcal{H} \mapsto \mathcal{H}$$

should be given the SAME quantum representation in a fixed $\mathcal{H}$ for ALL choices of internal time T, and (q, p).
Properties: (1) Self-adjoint $\hat{C}_J$'s provide unambiguous interpretation of quantum states in $\mathcal{H}$, $\langle c_J|\Psi\rangle$. (2) All the dynamical variables are defined in a fixed $\mathcal{H}$ for all internal times and CAN be compared.

FLRW quantum dynamics

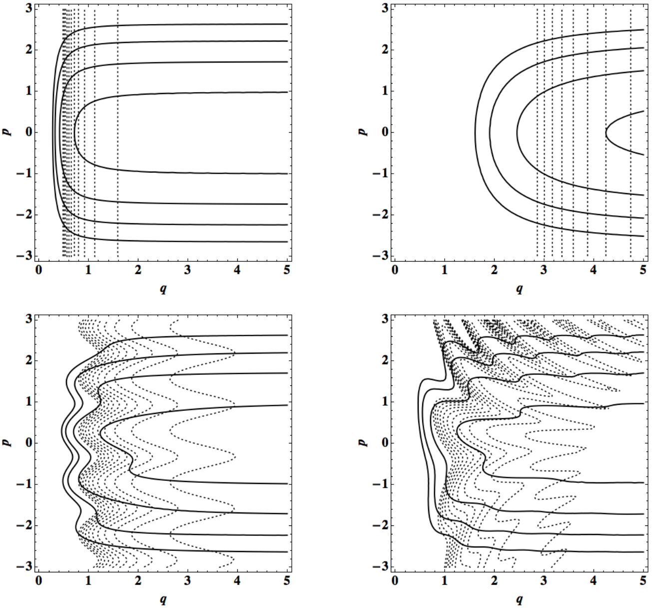
$$q = \text{SIZE}, \quad p = \text{EXPANSION RATE}$$



Left: classical trajectories, $\mathrm{H} = p^2$.

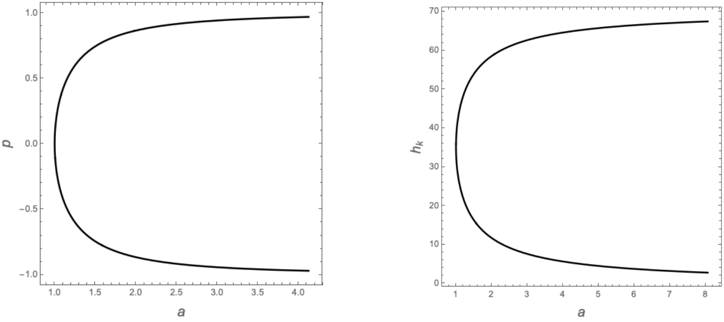
Right: semiclassical trajectories, $\mathrm{H}^{sem} = p^2 + \hbar^2 \frac{\mathcal{K}}{q^2}$.

FLRW in various internal times $\bar{T} = T + D(q, p)$



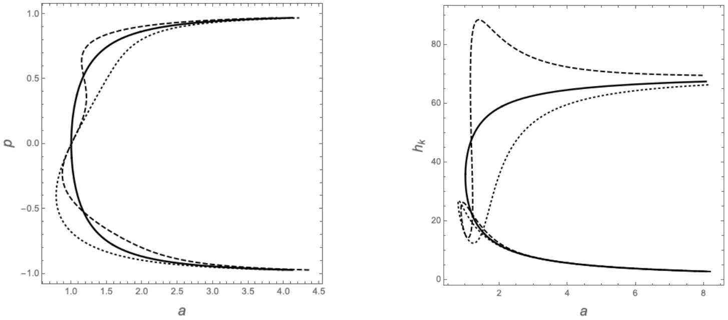
Gravitational waves in FLRW

$$H_{FRW,GWs} = p^2 - \frac{1}{2}|\pi_\pm(\vec{k})|^2 - \frac{1}{2}\left(k^2 - \frac{\ddot{a}}{a}\right)|\mu_\pm(\vec{k})|^2, \quad h_\pm(\vec{k}) = \frac{\mu_\pm(\vec{k})}{a}$$



"Evolution" of the FLRW background and of a GW amplitude.

Gravitational waves in FLRW



"Evolution" of the FLRW background and of GW amplitude for different internal times.

Recover ordinary QM

System $=$ **internal system** $\times$ **internal observer**:

$$(q_S, p_S) \times (q_O, p_O) \in \mathbb{R}^4, \quad T \in \mathbb{R},$$

Clock transformations that involve **internal observer** only:

$$T \mapsto \bar{T} = T + D(q_O, p_O).$$

$$\Delta(T) = D(q_O(T), p_O(T))$$

Quantum **internal system** is "shifted in time" by $U = e^{-\frac{i\Delta}{2}P^2}$:

Clock T	Clock $\bar{T} = T + \Delta(T)$
$p_s \mapsto \hat{P}$	$p_s \mapsto \hat{P}$
$q_s \mapsto \hat{Q}$	$q_s \mapsto \hat{Q} - \Delta(T)\hat{P}$
$\vert\Psi\rangle \mapsto \Psi(q) = \langle q\vert\Psi\rangle$	$\vert\Psi\rangle \mapsto \varphi(q) = \langle q\vert U^\dagger\vert\Psi\rangle$
$i\partial_T\psi(q) = \hat{H}\psi(q)$	$i\partial_{\bar{T}}\varphi(q) = \hat{H}\varphi(q)$

DISCUSSION

Different internal clocks lead to different quantum dynamics for any single state (motion) in a fixed $\mathcal{H}$.

Dynamical predictions for asymptotic classical states are unambiguous BUT is "asymptotic" really enough?

If some DOFs are classical then Schrödinger's eq. is invariant under clock transformations involving those DOFs.

Is the clock effect "in principle" measurable?

Is there a quantum state for which a certain DOF behaves classically in all internal clocks?