Mixmaster universe: semiclassical dynamics and inflation from bouncing

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Current work in collaboration with:
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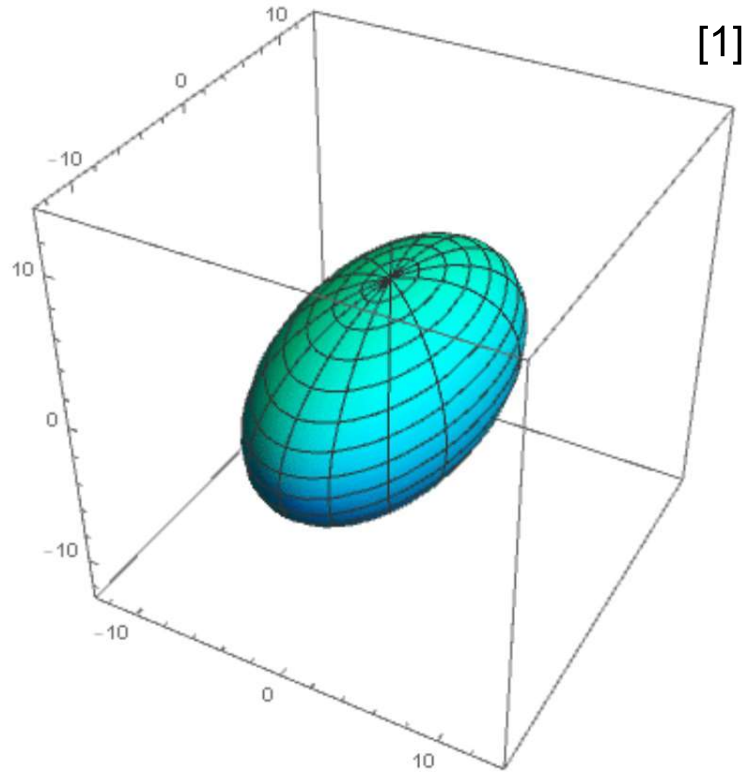
Introduction: Mixmaster Universe

- Homogeneous model of Early Universe.
  Studied by Belinski, Khalatnikov and Lifshitz (BKL) and independently by Misner.

- Oscillatory and Chaotic behaviour close to the initial singularity (Big Bang).

- Random and repeated squeezing and blowing up of spatial directions.
  3D Mixing→ Anisotropy.



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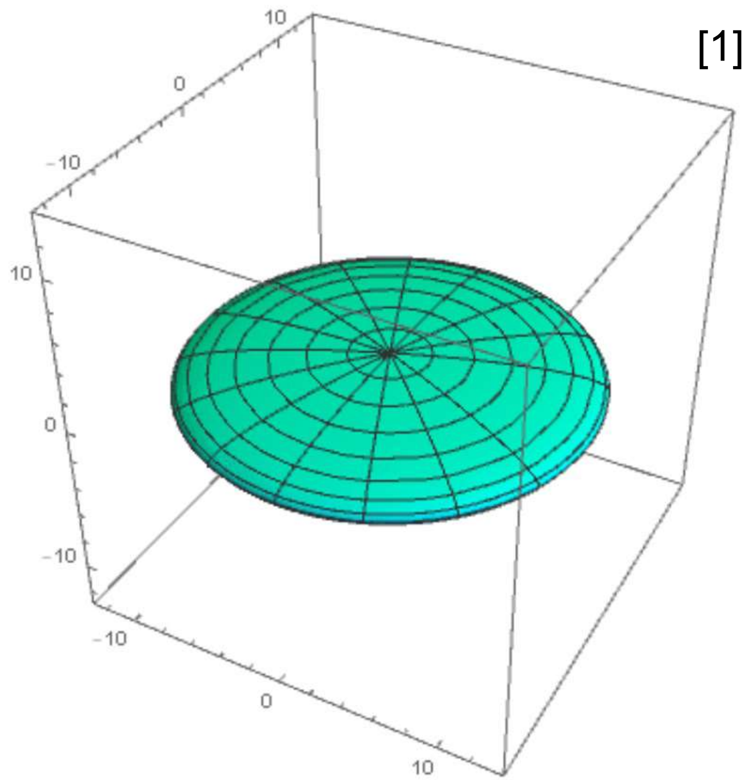

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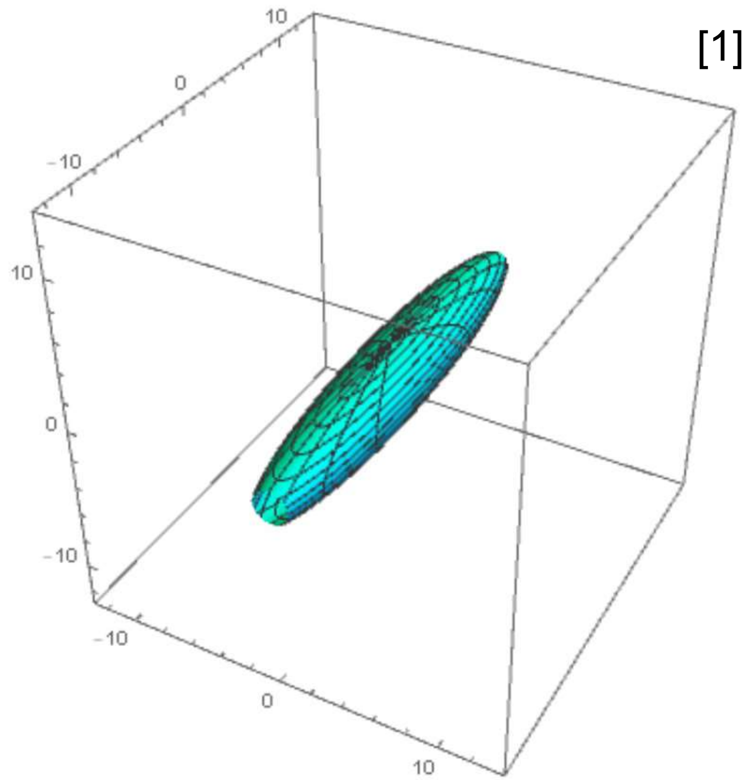

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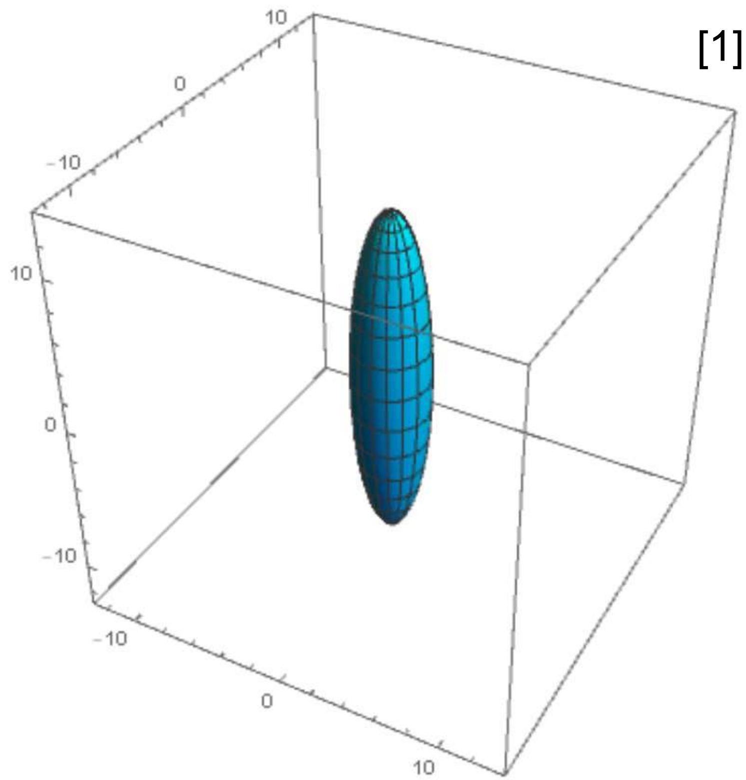

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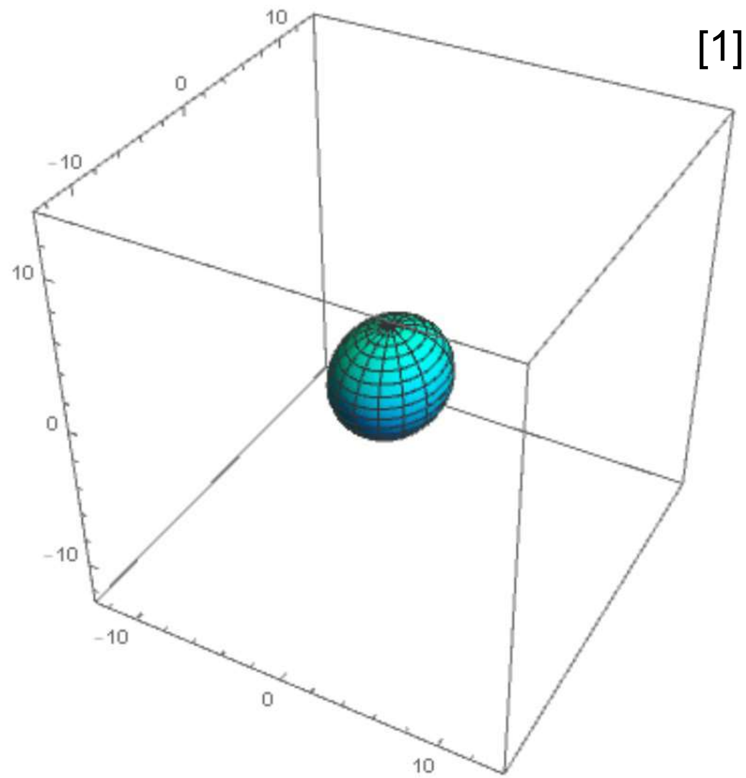

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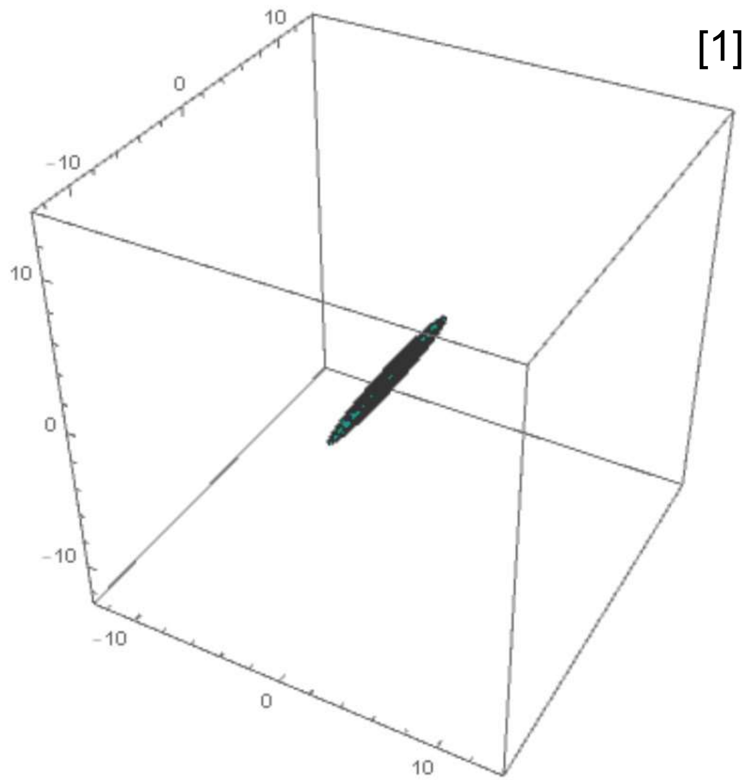

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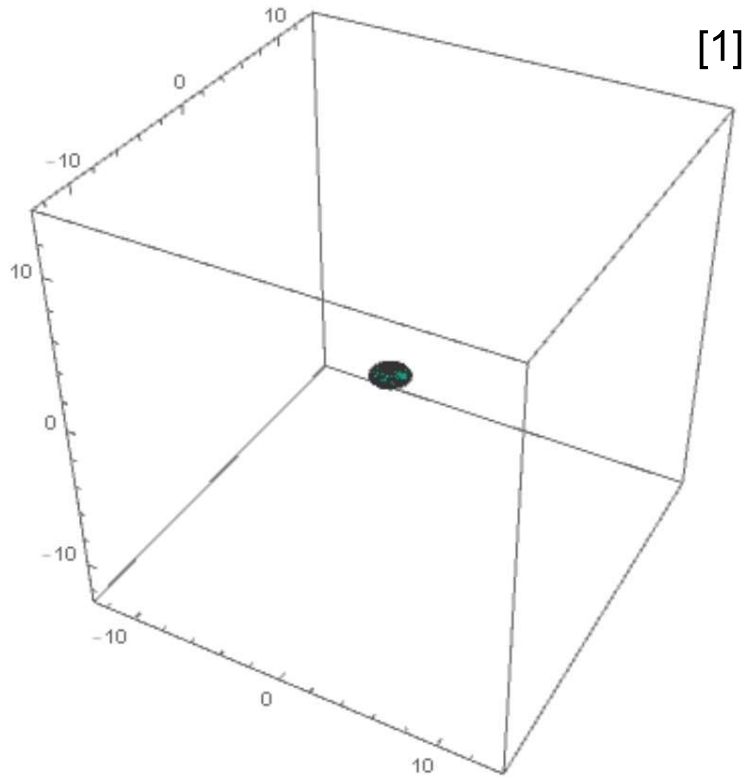

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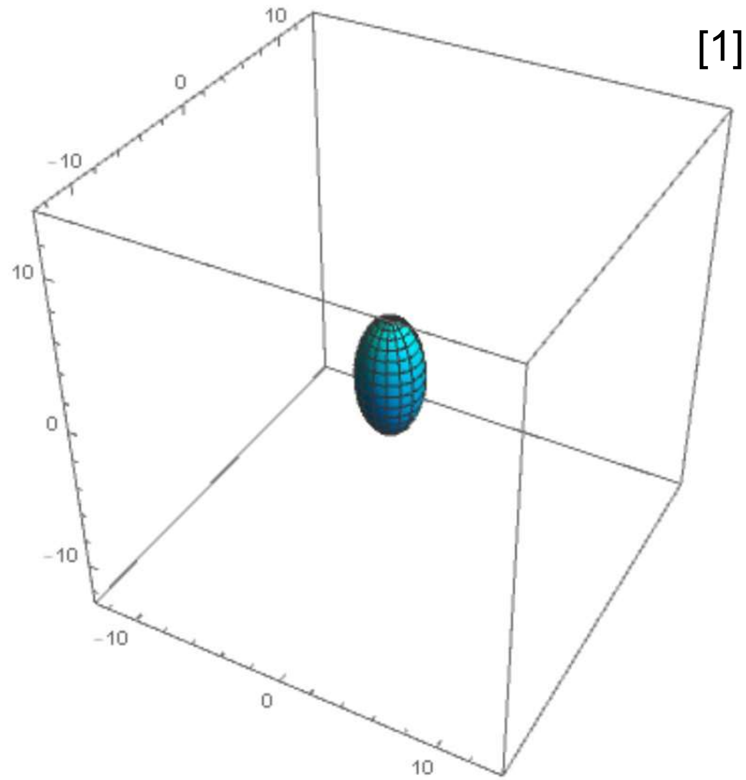

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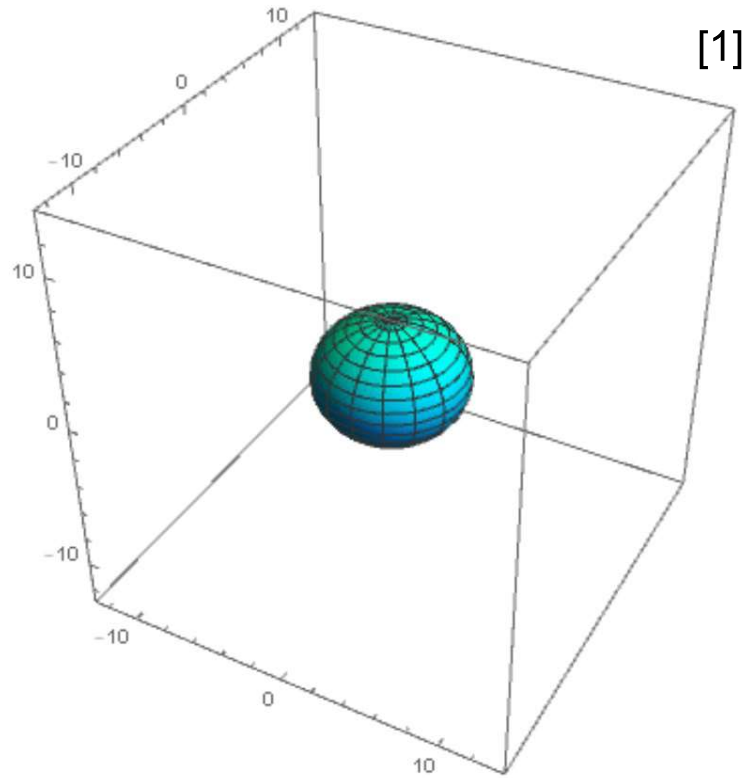

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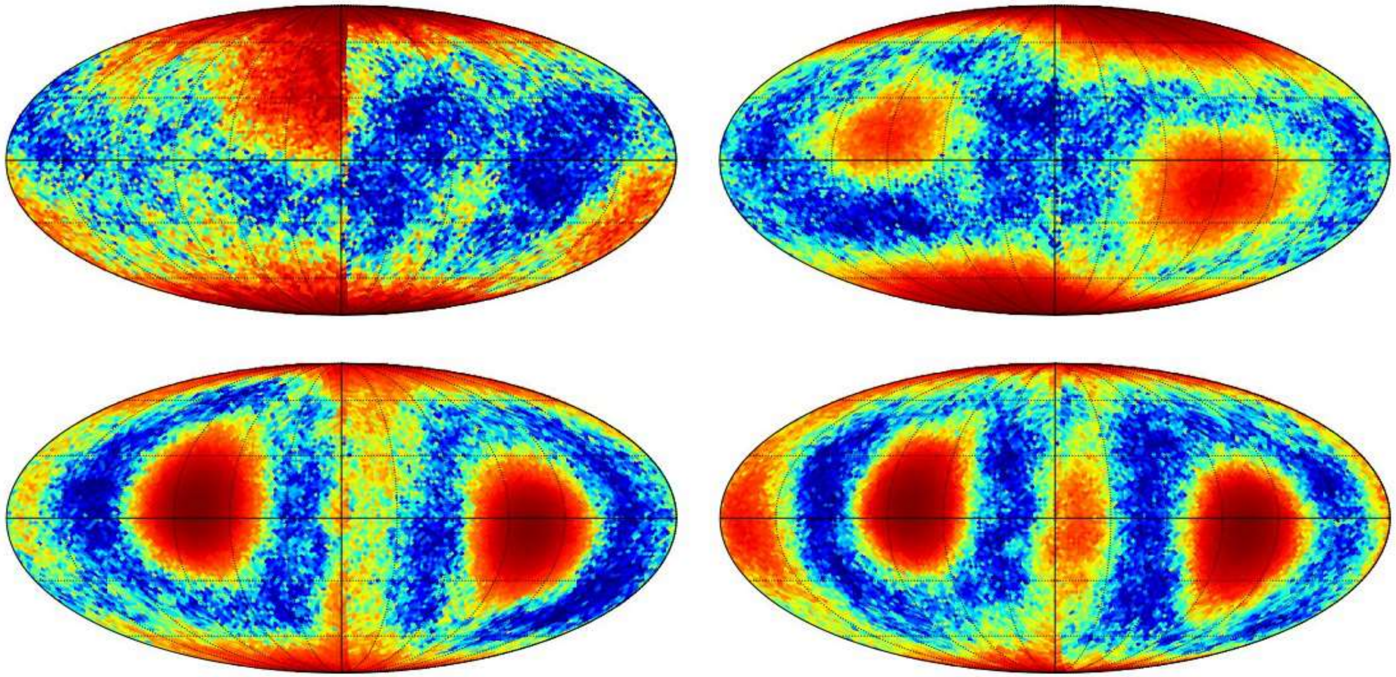
Motivation: Why Anisotropy?

- Anisotropic very early universe → Expansion flattens the effect → Isotropy
- More generic model, we need less primordial symmetries.
- Isotropic bouncing models → OK, but slightly blue-tilted spectrum. Can anisotropy improve that?
- Can anisotropy account for the origin of “depart from explanation” of primordial universe behaviour? Some observational data [2] suggest anomalies at large scales.



[2] A Durakovica, *et al*. (2018) *Reconstruction of a direction-dependent primordial power spectrum from Planck CMB data,* JCAP 1802

Quantum Primordial Universe

- End goal: Full description of primordial Universe → Quantum description
- Quantization: Replace singularity by quantum bounce.
- Interplay between isotropic and anisotropic variables → Complex quantum dynamics.

Mathematical description: Hamiltonian formulation

· **Bianchi IX metric**:

$$\mathrm{d}s^2 = -\mathcal{N}^2 \mathrm{d}\tau^2 + \sum_i a_i^2 \underbrace{(\omega^i)^2}$$

$\mathcal{N}$ → Lapse function

$a_i(t)$: 3 different principal direction scale factors (anisotropy)

(ω^i) → Spatial hypersurface: S^3 topology, Closed universe

Usual parametrization:

$$\omega^1 = -\sin(\varphi)d\theta + \cos(\varphi)\sin(\theta)d\phi$$
$$\omega^2 = \cos(\varphi)d\theta + \sin(\varphi)\sin(\theta)d\phi$$
$$\omega^3 = \sin(\theta)d\phi + d\varphi$$

with: $\xi_i \cdot \omega^j = \delta_i^j$

→ Killing vector: $SO(3,\mathbb{R})$ isometry group generator

The basis 1-forms satisfy: $\mathrm{d}\omega_i = \frac{1}{2}\varepsilon_i^{\;jk}\omega_j \wedge \omega_k$

Mathematical description: Hamiltonian formulation

Extrinsic curvature:

$$K^{ij} = \frac{1}{2\mathcal{N}}(2\nabla^{(i}N^{j)} - \dot{\gamma}^{ij})$$

3+1 ADM formalism: $$\mathrm{d}s^2 = -\mathcal{N}^2\mathrm{d}\tau^2 + \sum_i \gamma_{ij}(\omega^i + N^i\mathrm{d}\tau)(\omega^j + N^j\mathrm{d}\tau)$$

Phase space: Bianchi IX 3-metric: a_i + 3-momemtum: $\pi^{ij} = \sqrt{\gamma(K^{ij} - K\gamma^{ij})}$

Hamiltonian constraint: $$H = \mathcal{N}\sqrt{\gamma}\left((-^{(3)}R + \gamma^{-1}(\pi_{ij}\pi^{ij} - \frac{1}{2}\pi^2)\right)$$

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Using Misner Variables: $\Omega = \frac{1}{3}\ln a_1a_2a_3, \quad \beta_+ = \frac{1}{6}\ln\frac{a_1a_2}{a_3^2}, \quad \beta_- = \frac{1}{2\sqrt{3}}\ln\frac{a_1}{a_2}$

+ Conjugate momenums:

$$\begin{pmatrix} a_1p_1 \\ a_2p_2 \\ a_3p_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{6} & \frac{1}{12} & \frac{\sqrt{3}}{12} \\ \frac{1}{6} & \frac{1}{12} & -\frac{\sqrt{3}}{12} \\ \frac{1}{6} & -\frac{1}{6} & 0 \end{pmatrix} \begin{pmatrix} p_\Omega \\ p_+ \\ p_- \end{pmatrix}$$

Isotropic geometry

Anisotropic variables

Canonical transformation for isotropic variables: $q = e^{\frac{3}{2}\Omega}, \quad p = \frac{2}{3}e^{-\frac{3}{2}\Omega}p_\Omega$

Mathematical description: Hamiltonian formulation

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Isotropic geometry (Ω, p_Ω)

Anisotropic variables (β_+, β_-, p_+, p_-)

Canonical transformation for isotropic variables: $q = e^{\frac{3}{2}\Omega}, \quad p = \frac{2}{3}e^{-\frac{3}{2}\Omega}p_\Omega$

($q > 0$ always)

Hamiltonian Constraint in Misner variables (natural units and $\mathcal{N} = -24$):

$$C = -\frac{9}{4}p^2 - 36q^{\frac{2}{3}} + \frac{\mathbf{p}_\pm^2}{q^2} + 36q^{\frac{2}{3}}V(\boldsymbol{\beta}_\pm)$$

→ Particle in 3D Minkowski s-t. with time dependent potential

Where: $V(\boldsymbol{\beta}) = \frac{e^{4\beta_+}}{3}\left[\left(2\cosh(2\sqrt{3}\beta_-) - e^{-6\beta_+}\right)^2 - 4\right] + 1$

$$(q, p, \boldsymbol{\beta}_\pm, \mathbf{p}_\pm) \in \mathbb{R}^6$$

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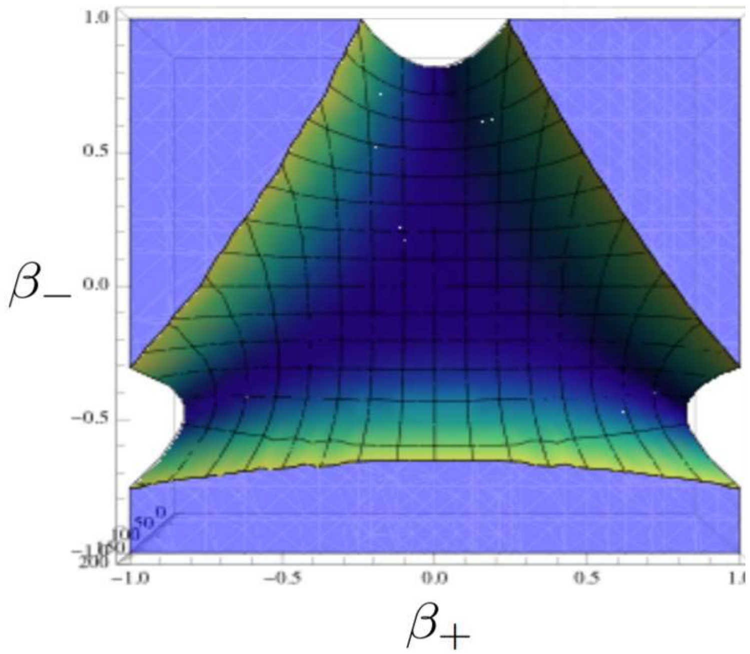

Quantization and semicassical portrait

- Isotropic canonical variables: $(q\,,p)\in\mathbb{R}_+^*\times\mathbb{R}$ → Positive half-plane

Affine group structure $\mathrm{Aff}_+(\mathbb{R})$:

$$(q,p)(q_0,p_0)=\left(qq_0,\frac{p_0}{q}+p\right)$$

unity: $(1,0)$

$$(U(q,p)\psi)(x)=\frac{e^{ipx}}{\sqrt{q}}\psi\left(\frac{x}{q}\right)$$

Covariant affine group quantization,[3,4]
and semiclassical portrait
using affine coherent states:

$$A_f=\int_{\Pi_+}f(q,p)\,|q,p\rangle\langle q,p|\,\frac{\mathrm{d}q\mathrm{d}p}{2\pi c_{-1}}$$

Bounce

$$p^2\rightarrow\widetilde{(p^2)}=p^2+\frac{K(\mu,\nu)}{q^2},\qquad q^\alpha\rightarrow\widetilde{(q^\alpha)}=Q_\alpha(\mu,\nu)q^\alpha\,,$$

$$K(\mu,\nu)=e^{\frac{3}{2\mu}}\left(\frac{\mu+\nu}{2}+\frac{1}{4}\right),\qquad Q_\alpha(\mu,\nu)=e^{\frac{\alpha(\alpha-1)}{4\mu}}e^{\frac{\alpha(\alpha-1)}{4\nu}}$$

[3] H. Bergeron, *et al,* (2020), *Quantum Mixmaster as a Model of the Primordial Universe,* Universe **6, 7**.
[4] J.-P. Gazeau, *et al.* (2016) *Covariant affine integral quantization(s)*, J. Math. Phys.**57,** 052102

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$$p^2 \to \widecheck{(p^2)} = p^2 + \frac{K(\mu,\nu)}{q^2}, \qquad q^\alpha \to \widecheck{(q^\alpha)} = Q_\alpha(\mu,\nu) q^\alpha\,,$$

$$K(\mu,\nu) = e^{\frac{3}{2\mu}}\left(\frac{\mu+\nu}{2} + \frac{1}{4}\right), \qquad Q_\alpha(\mu,\nu) = e^{\frac{\alpha(\alpha-1)}{4\mu}} e^{\frac{\alpha(\alpha-1)}{4\nu}}$$

- Anisotropic variables: $\mathbf{r}_\pm = (\beta_\pm, p_\pm) \in \mathbb{R}^2$ → Full plane

→ Weyl-Heisenberg group:

$$[Q,P] = \mathrm{i}\hbar\mathbb{1}$$

rep: $U(\mathbf{r}) = e^{\mathrm{i}(pQ-\beta P)}$

Covariant Weyl-Heisenberg integral quantization:

$$f(\mathbf{r}) \mapsto A_f := \int_{\mathbb{R}^2} f(\mathbf{r})\mathcal{Q}(\mathbf{r})\frac{\mathrm{d}^2\mathbf{r}}{2\pi}, \leftarrow \begin{cases} \mathcal{Q}(\mathbf{r}) = U(\mathbf{r})\mathcal{Q}_0 U(\mathbf{r})^\dagger \\ \mathcal{Q}_0 = \int_{\mathbb{R}^2} U(\mathbf{r})\Pi(\mathbf{r})\frac{\mathrm{d}^2\mathbf{r}}{2\pi} \end{cases}$$

Gaussian weight function

Semiclassical portrait: $\check{f}(\mathbf{r}) = \int_{\mathbb{R}^2} \mathcal{F}[\Pi] * \mathcal{F}[\widetilde{\Pi}](\mathbf{r}'-\mathbf{r}) f(\mathbf{r}')\frac{\mathrm{d}^2\mathbf{r}'}{4\pi^2}$

$$p_\pm^2 \to \widecheck{(p_\pm^2)} = p_\pm^2 + \frac{8}{\sigma_\pm^2}, \qquad V(\beta_\pm) \to \check{V}(\beta_\pm) = \frac{1}{3}\left(D(4\sqrt{3},4)e^{4\sqrt{3}\beta_- + 4\beta_+} + D(4\sqrt{3},4)e^{-4\sqrt{3}\beta_- + 4\beta_+} + D(0,8)e^{-8\beta_+}\right)$$
$$- \frac{2}{3}\left(D(2\sqrt{3},2)e^{-2\sqrt{3}\beta_- - 2\beta_+} + D(2\sqrt{3},2)e^{2\sqrt{3}\beta_- - 2\beta_+} + D(0,4)e^{4\beta_+}\right) + 1$$

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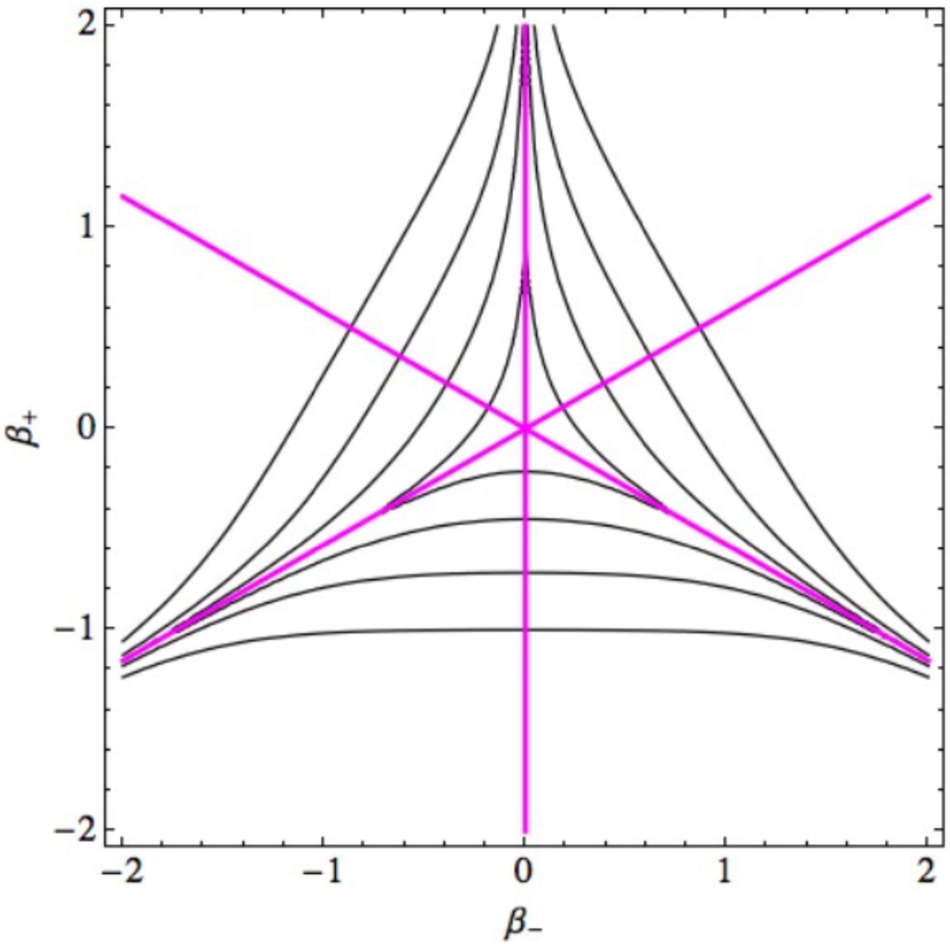

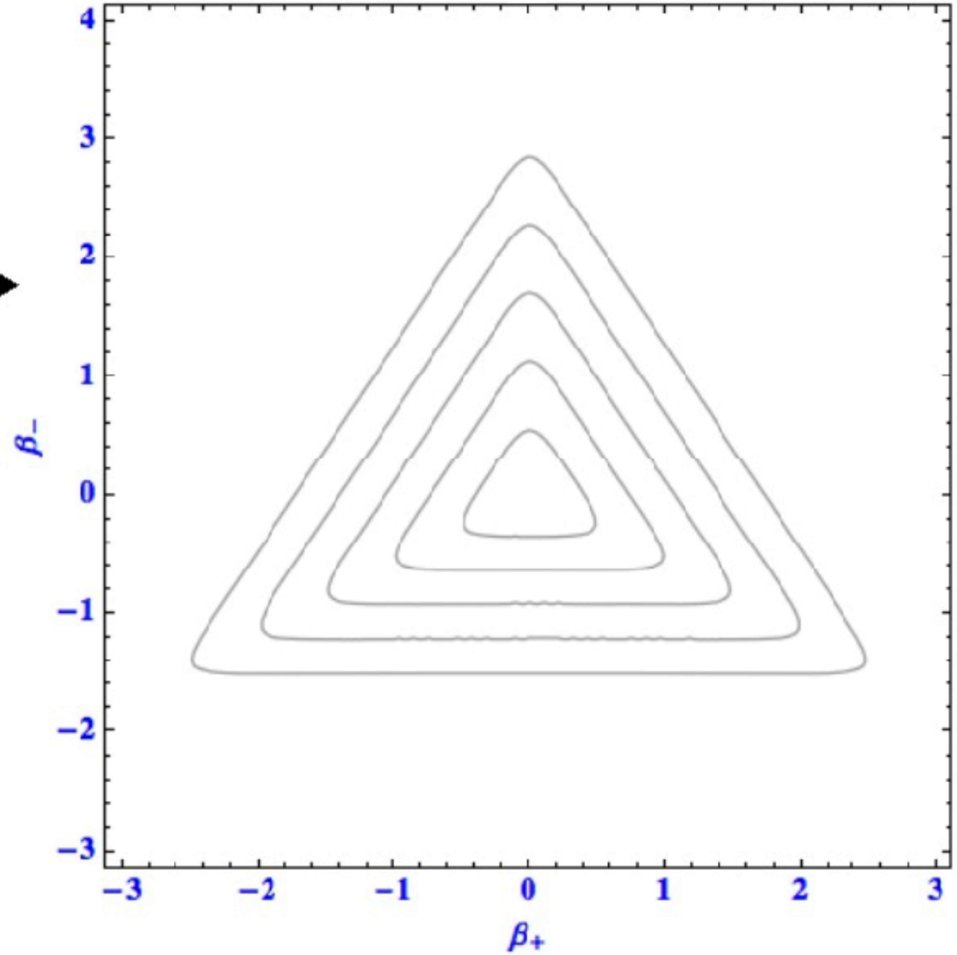

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$p_\pm^2 \longrightarrow \widecheck{(p_\pm^2)} = p_\pm^2 + \frac{8}{\sigma_\pm^2},$ $\quad V(\beta_\pm) \longrightarrow \check{V}(\beta_\pm) = \frac{1}{3}\left(D(4\sqrt{3},4)e^{4\sqrt{3}\beta_- + 4\beta_+} + D(4\sqrt{3},4)e^{-4\sqrt{3}\beta_- + 4\beta_+} + D(0,8)e^{-8\beta_+}\right)$

$$- \frac{2}{3}\left(D(2\sqrt{3},2)e^{-2\sqrt{3}\beta_- - 2\beta_+} + D(2\sqrt{3},2)e^{2\sqrt{3}\beta_- - 2\beta_+} + D(0,4)e^{4\beta_+}\right) + 1$$

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Semiclassical portrait of full Hamiltonian constraint

$$\breve{\mathrm{C}} = \frac{9}{4}\left(p^2 + \frac{K(\mu,\nu)}{q^2}\right) - Q_{-2}(\mu,\nu)\frac{p_\pm^2 + \frac{8}{\sigma_\pm^2}}{q^2} - 36 Q_{\frac{2}{3}}(\mu,\nu) q^{\frac{2}{3}}[\breve{V}(\boldsymbol{\beta}) - 1] - \frac{R}{q^{2/3}}$$

$\mu, \nu, \sigma_\pm, \omega_\pm$ → 6 quantization + semiclassical parameters

Hamilton equations:

$$\dot{q} = \frac{9}{2}p\,,$$

$$\dot{p} = \frac{9}{2}\frac{K}{q^3} - 2Q_{-2}\frac{p_\pm^2 + \frac{8}{\sigma_\pm^2}}{q^3} + 24 Q_{\frac{2}{3}} q^{-\frac{1}{3}}[\breve{V}(\boldsymbol{\beta}) - 1] - \frac{2}{3}R q^{-\frac{5}{3}}\,,$$

(we added Radiation)

$$\dot{\beta}_\pm = -2Q_{-2}\frac{p_\pm}{q^2}\,,$$

$$\dot{p}_\pm = 36 Q_{\frac{2}{3}} q^{\frac{2}{3}} \partial_\pm \breve{V}(\boldsymbol{\beta})\,,$$

- Very rich model:
  Let's investigate, numerically, the effects of interplay between anisotropy and quantum bounce.

Solution for Isotropic case: $\beta_\pm = 0 = p_\pm$

$$\check{C}_{isotropic} = \frac{9}{4}\left(p^2 + \frac{K(\mu,\nu) - \frac{4}{9}Q_{-2}(\mu,\nu)\frac{8}{\sigma_\pm^2}}{q^2}\right) + 36Q_{\frac{2}{3}}(\mu,\nu)q^{\frac{2}{3}} - \frac{R}{q^{2/3}}$$

Let's call:

$$L := 36Q_{\frac{2}{3}}$$

(intrinsic isotropic curvature)

$$K_{iso} := K - \frac{4}{9}Q_{-2}\frac{8}{\sigma_\pm^2}$$

(isotropic repulsive strength)

$$M := Q_{-2}$$

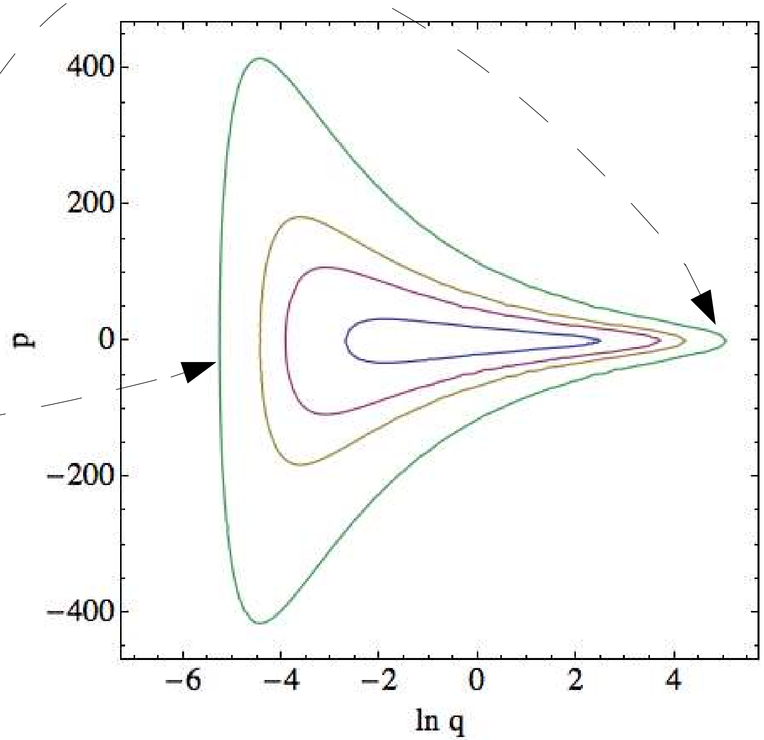
(for simplicity)

Analytical aproximations for
quantum bounce and classical recolapse:

$$q_{min} = \left(\frac{9K_{iso}}{4R}\right)^{\frac{3}{4}}, \qquad q_{max} = \left(\frac{R}{L}\right)^{\frac{3}{4}}$$

Phase-space solutions:

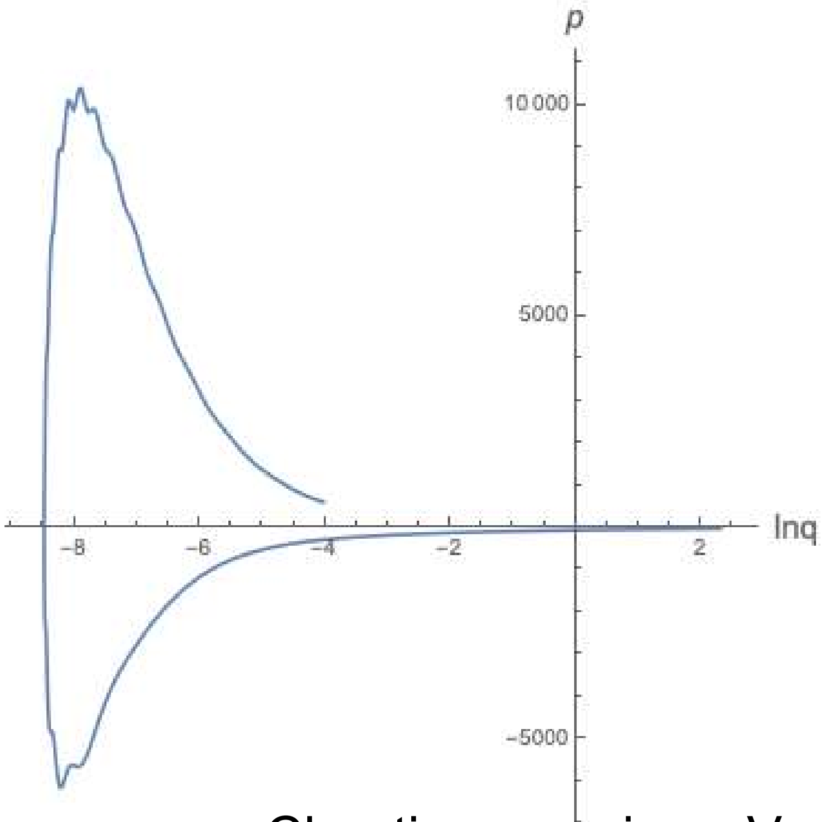
(for different R radiation contents)

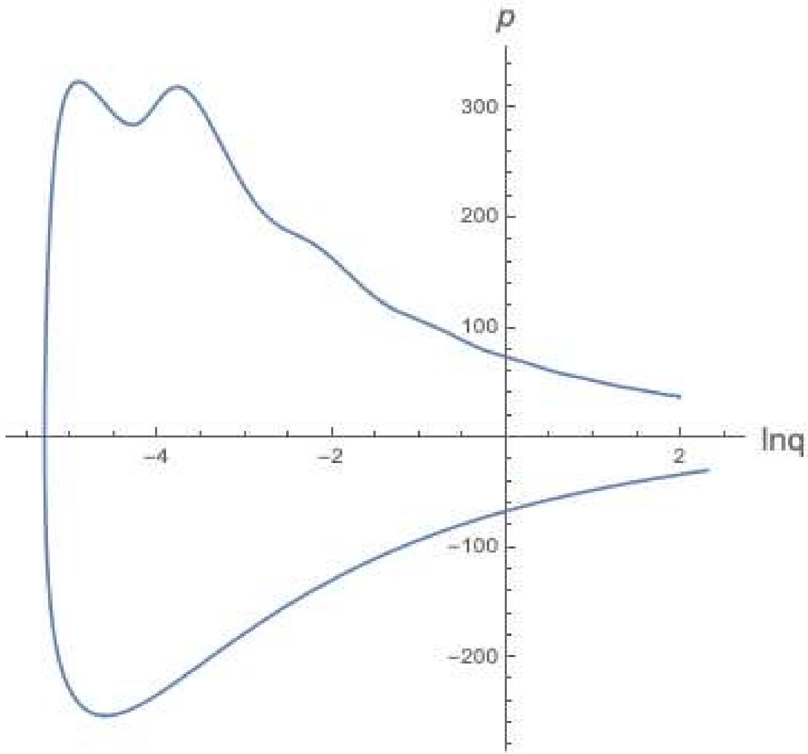


Full anisotropic solution:

$$\mathrm{C} = \frac{9}{4}\left(p^2 + \frac{K_{iso}}{q^2}\right) - M\frac{p_{\pm}^2}{q^2} - Lq^{\frac{2}{3}}[V-1] - \frac{R}{q^{\frac{2}{3}}}$$

Phase space of isotropic variables: <u>Asymmetric bounce</u> of the universe, due to increase of the role of anisotropy energy.
→ <u>Extra boost to the post-bounce expansion</u>





Chaotic scenario→ Very sensitive to initial conditions

(+ we have 6 quantization/semiclassical parameters)

A realistic initial scenario:

Density parameters of generalised-Friedmann equation:

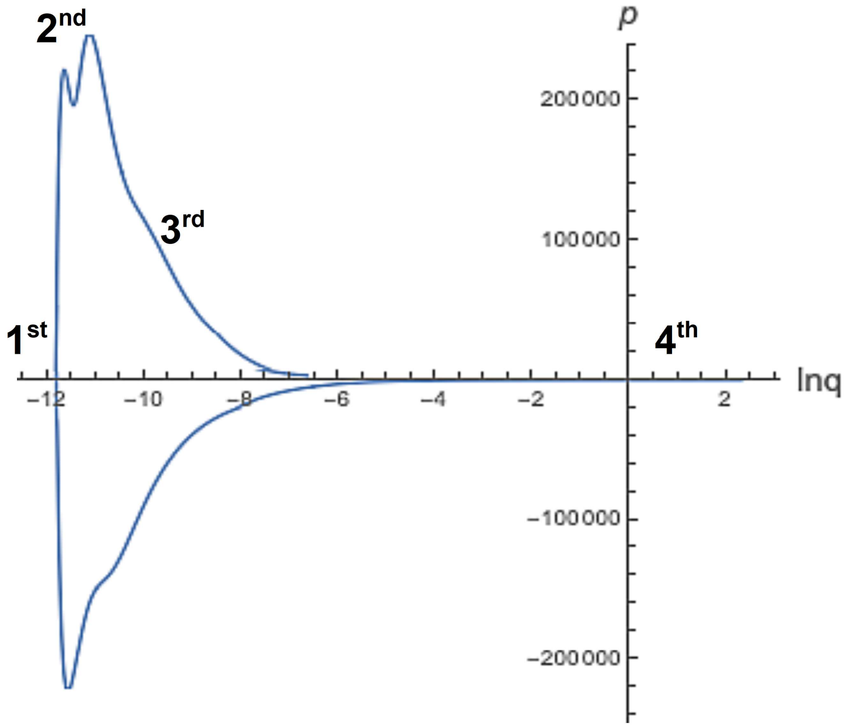
Anisotropic Kinetic term

Anisotropic Potential term

$$1 = -\frac{K_{iso}}{q^2 p^2} + \frac{4}{9p^2}\underbrace{\left(M\frac{p_\pm^2}{q^2} + Lq^{\frac{2}{3}}V\right)} + \frac{4}{9}\frac{R}{p^2 q^{\frac{2}{3}}} - \frac{4}{9}\frac{Lq^{\frac{2}{3}}}{p^2}$$

$$1 = -\Omega_{sem} + \Omega_{ani} + \Omega_{rel} - \Omega_K$$

Intially (close to the bounce) we want: $\Omega_{sem} > \Omega_{ani} > \Omega_{rel} >> \Omega_K$



1st: Semiclassical Repulsion dominates → Expansion

2nd: Anisotropy plays a significant role → non-trivial Asymmetry

3rd: Transition to radiation, matter dominates dynamics.

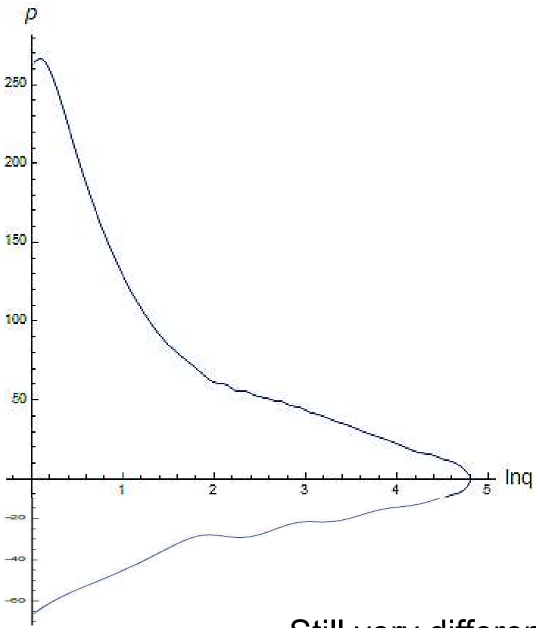
4th: Classical recolapse.

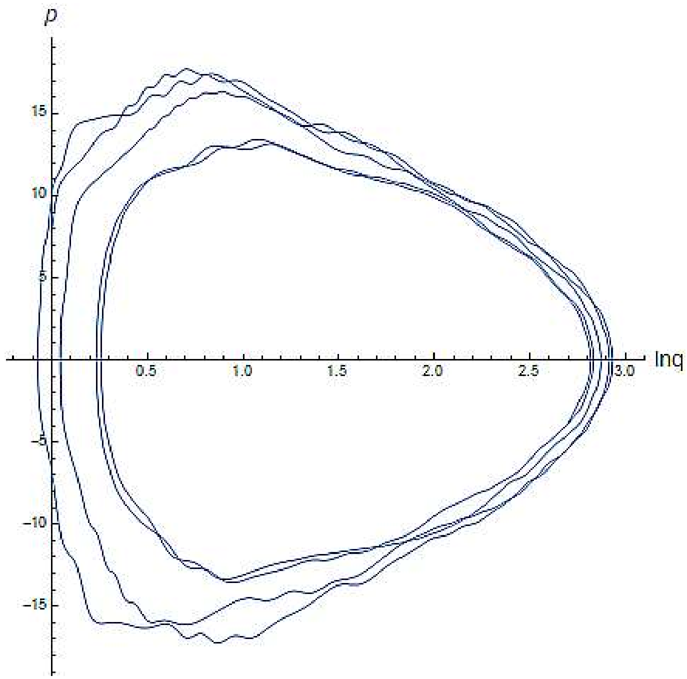
To be close to the bounce we require:

$$\frac{9K_{iso}}{4M} \geq p_\pm^2 \geq \frac{9K_{iso}}{16M}$$

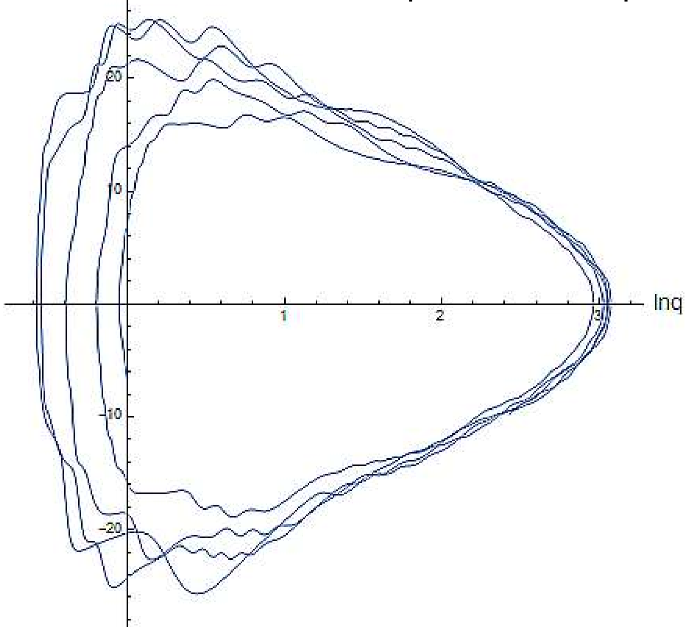
A realistic initial scenario:

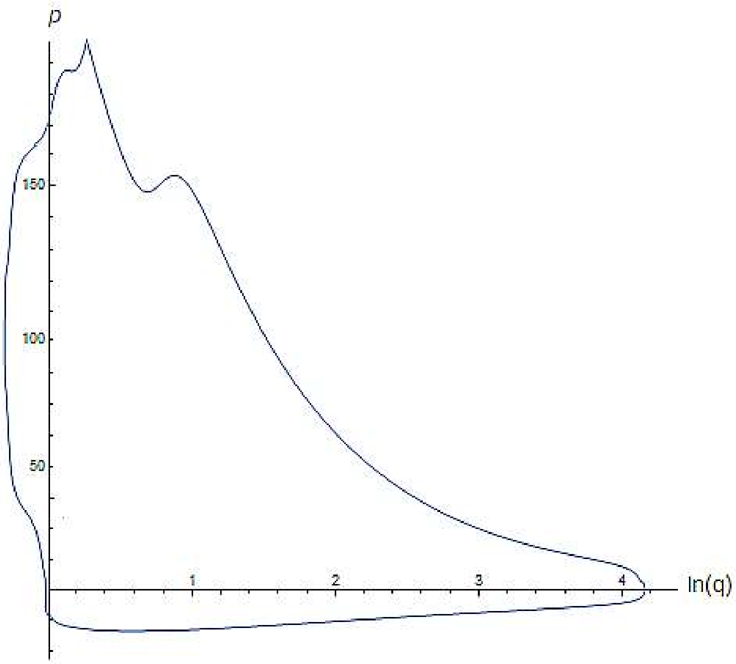
Different examples:





Still very different initial values of variables
and parameters reproduce this starting scenario





Inflationary-expansion behaviour?

The Big Question: Can anisotropy make the phase of accelerated expansion to last long enough?

For inflationary scenario: $\ddot{a} > 0 \rightarrow \dot{\mathcal{H}} > 0$ Increasing # modes leaving the horizon (super-Hubble)

Friedmann equation: $\mathrm{H}^2 = \underbrace{E \cdot q^{\alpha}} + \ldots$

(Remember: $q = a^{\frac{3}{2}}$)

Driving density term during inflation (anisotropy in our model)

Where α : Power law of our "scale factor" variable q for inflationary term

This condition translates into: $\boxed{\alpha > -4/3}$ → For inflationary behaviour

Our Friedmann equation:

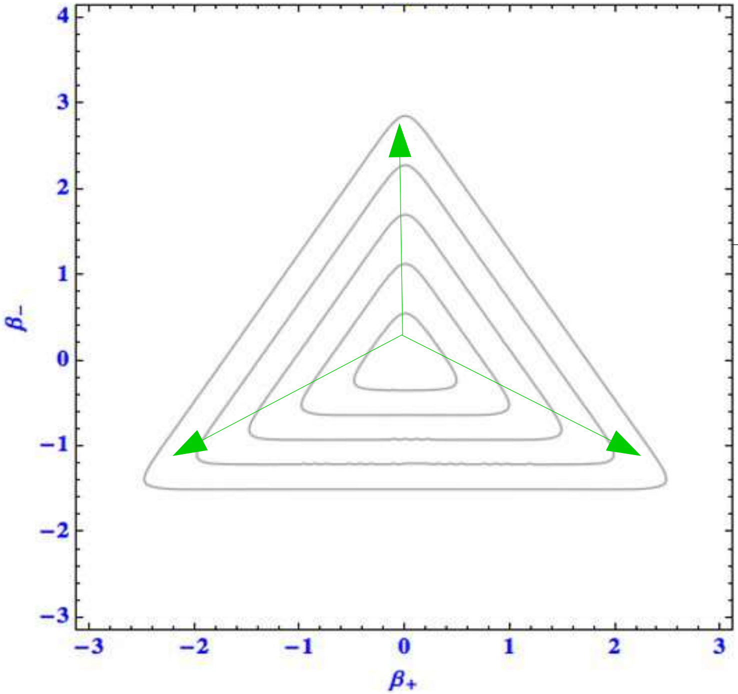
$$\mathrm{H}^2 = \frac{1}{64}\frac{p^2}{q^2} = -\frac{1}{64}\frac{K_{iso}}{q^4} + \frac{M}{144}\frac{p_{\pm}^2}{q^4} + \frac{L}{144}[V-1]\cdot q^{-\frac{4}{3}} + \frac{1}{144}\frac{R}{q^{\frac{8}{3}}}$$

$E_{\mathrm{ANI}} \cdot q^{\alpha}$

We have to make this potential V increase with time initially, close to the bounce, to make this term decrease slower than $q^{-4/3}$ during expansion

Specific situation for the system:
Extremal case for inflationary behaviour ←

How to make potential increase?



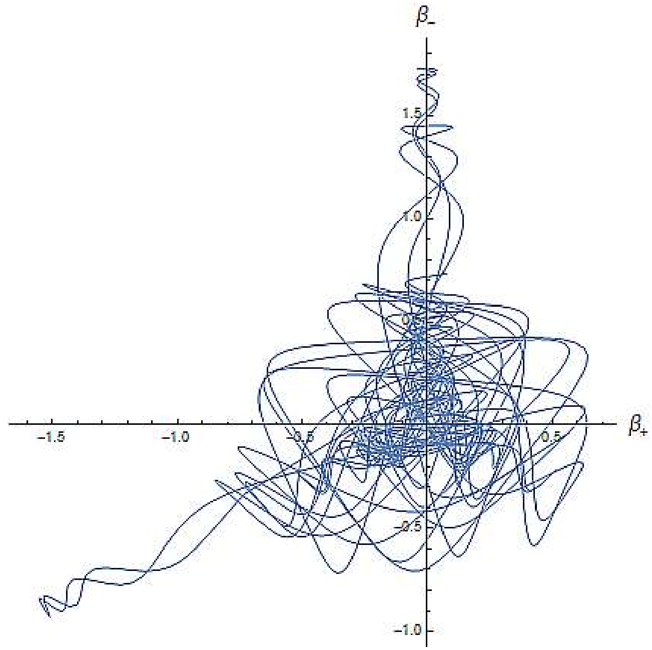
$V(\beta_{\pm}) \longrightarrow$ Very steep triangular walls, with flatter central part and 3 canyons.

→ Length of the (closed) canyons in the vertex modulated by semiclassical parameter $\boldsymbol{\omega}_{\pm}$

We throw the particle in the exact direction of the canyons to make the value of the potential increase for the longest time possible. Afterwards it will roll down in the opposite direction.

Two important things:

- Small initial q, for smaller Universe
  the walls are further away → flatter potential.
  With expansion the walls get closer, smaller $\beta_{\pm}$.



- Bigger initial $p_{\pm}$ longer time the particle rolling up.

  BUT : $p_{\pm}$ cannot be such big that it makes $M\dfrac{p_{\pm}^2}{q^4} > L\dfrac{V}{q^{4/3}}$

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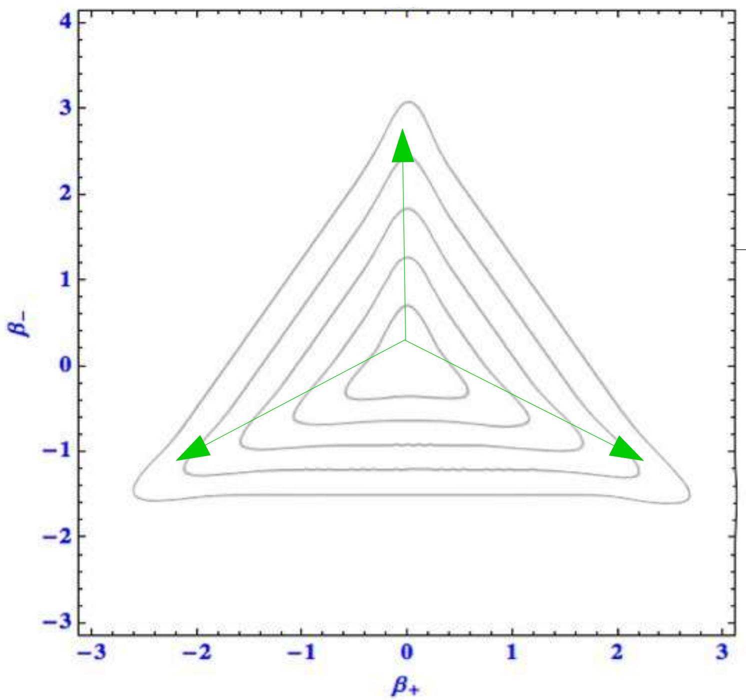

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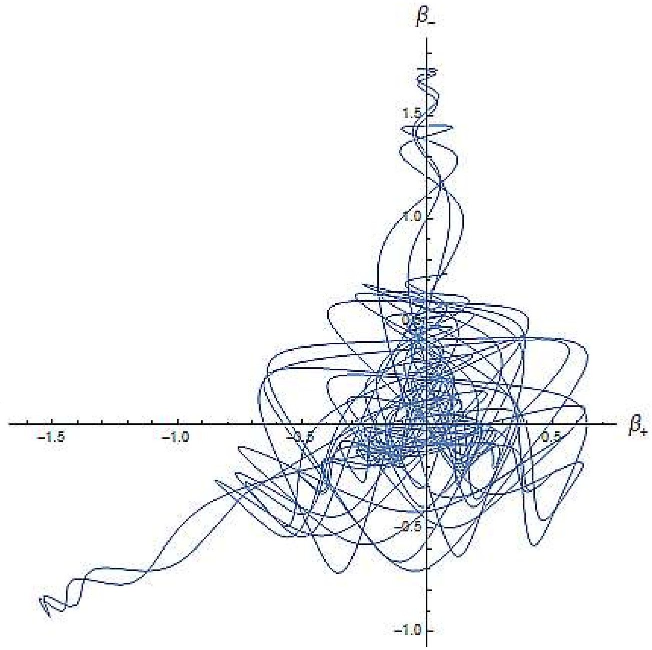

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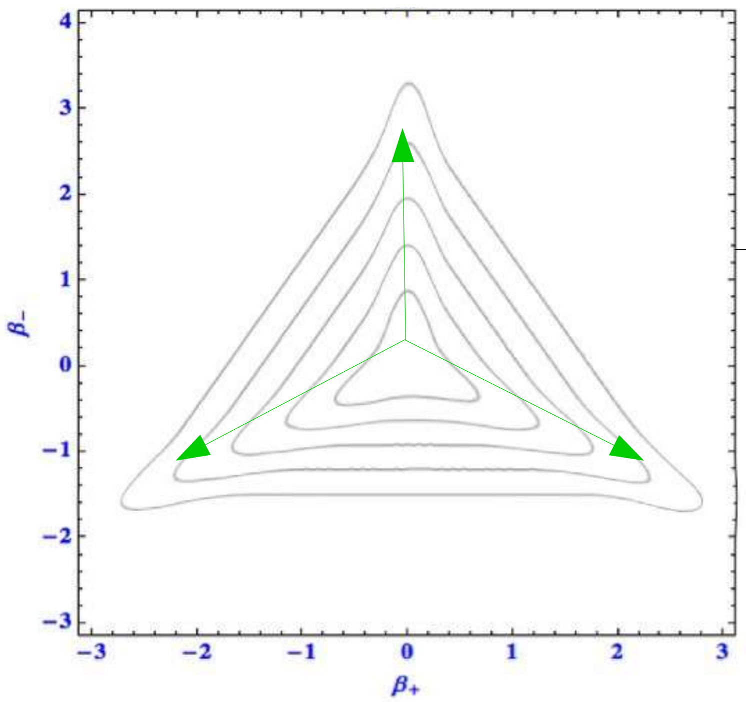

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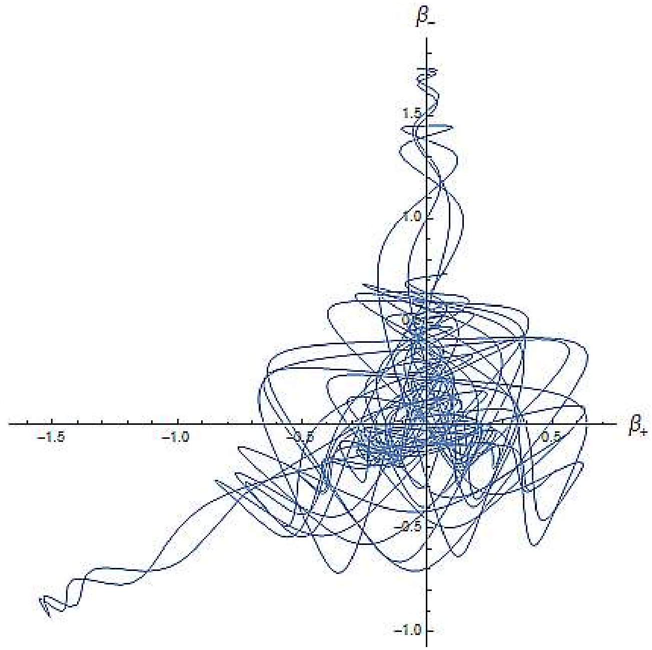

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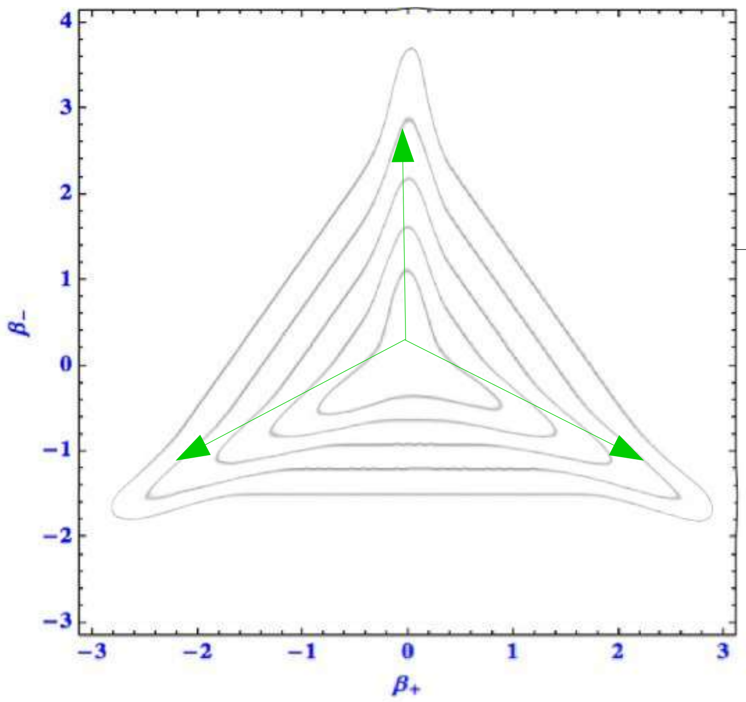

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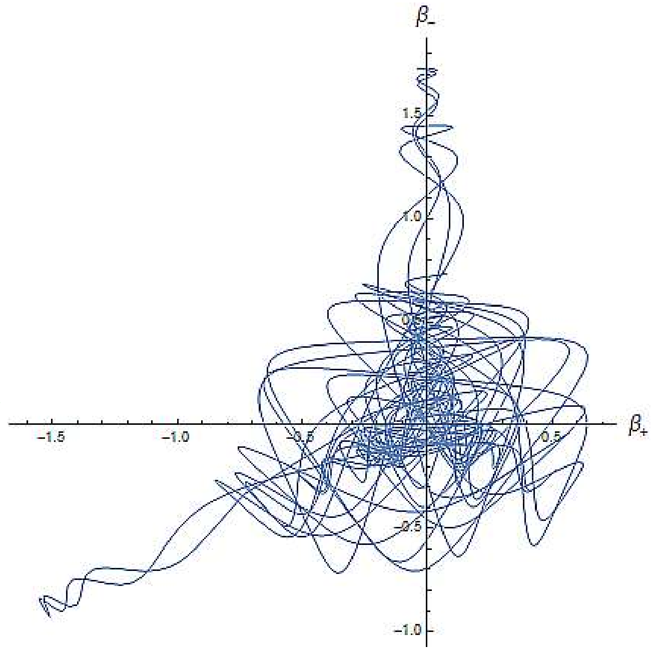

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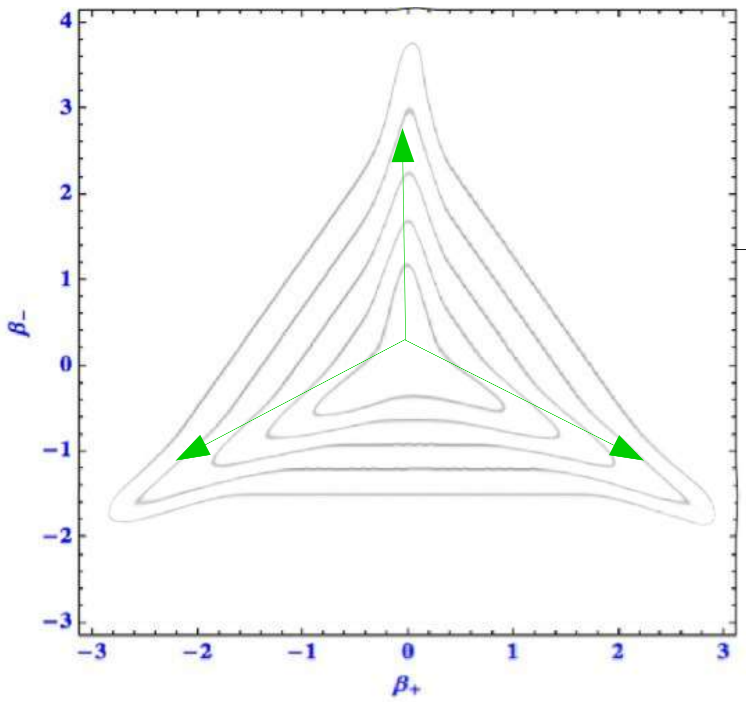

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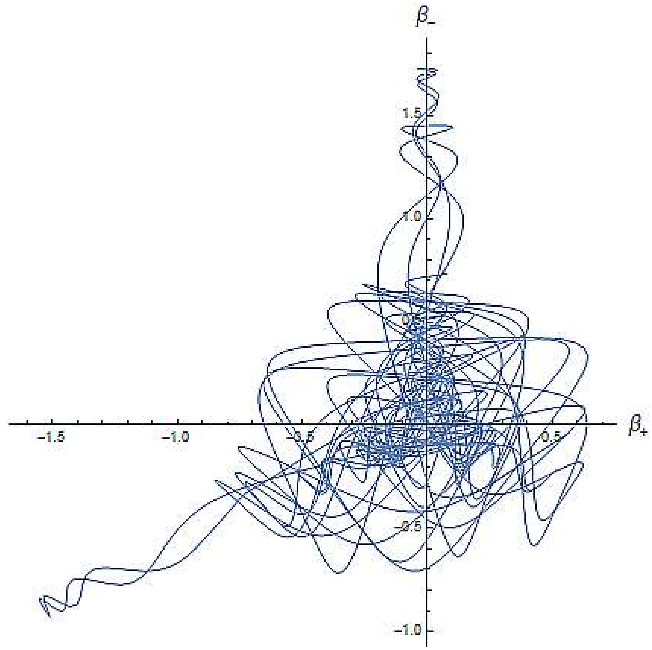

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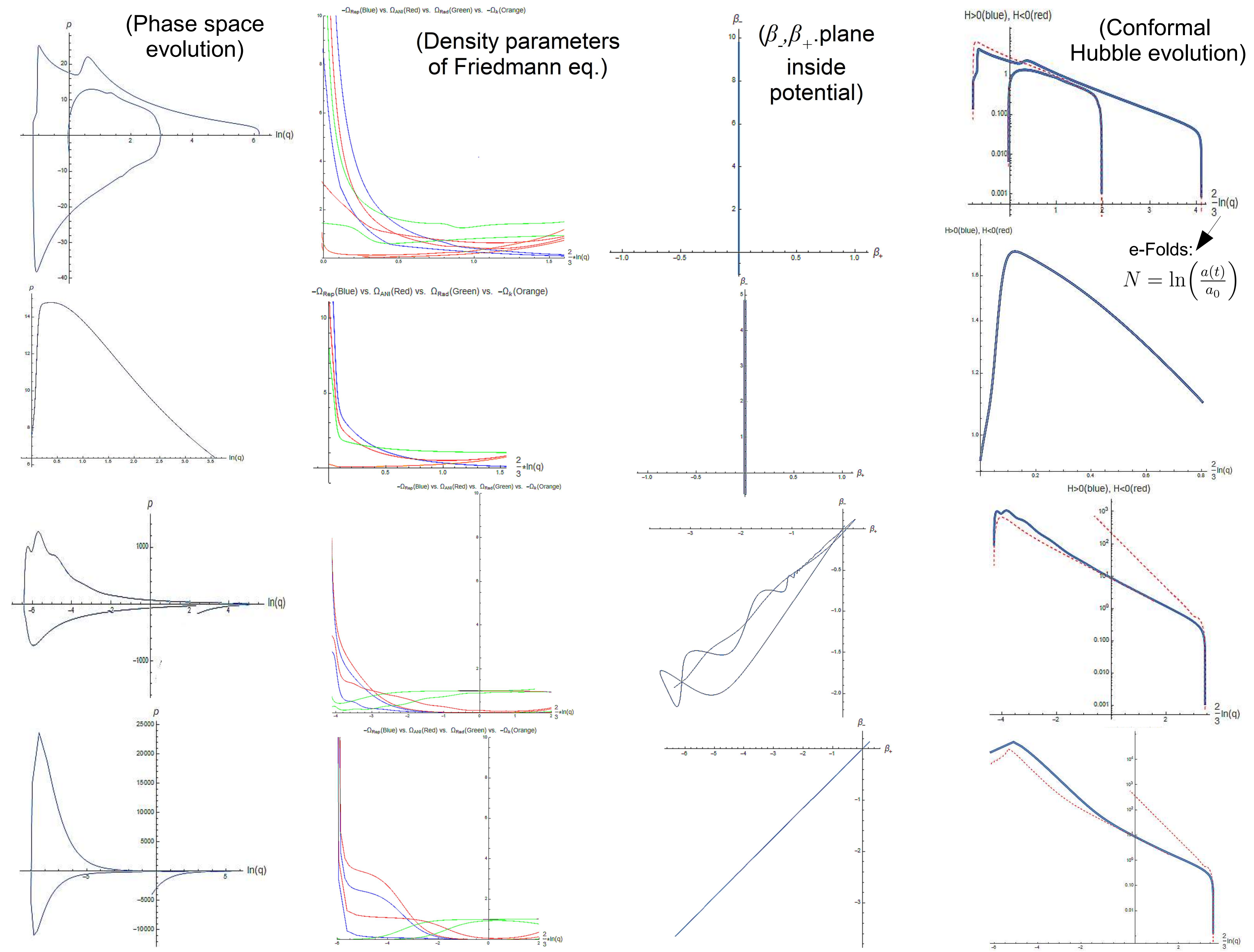

(Phase space evolution)
(Density parameters of Friedmann eq.)
(β_-,β_+.plane inside potential)
(Conformal Hubble evolution)
H>0(blue), H<0(red)
$-\Omega_{Rep}$(Blue) vs. Ω_{ANI}(Red) vs. Ω_{Rad}(Green) vs. $-\Omega_k$(Orange)
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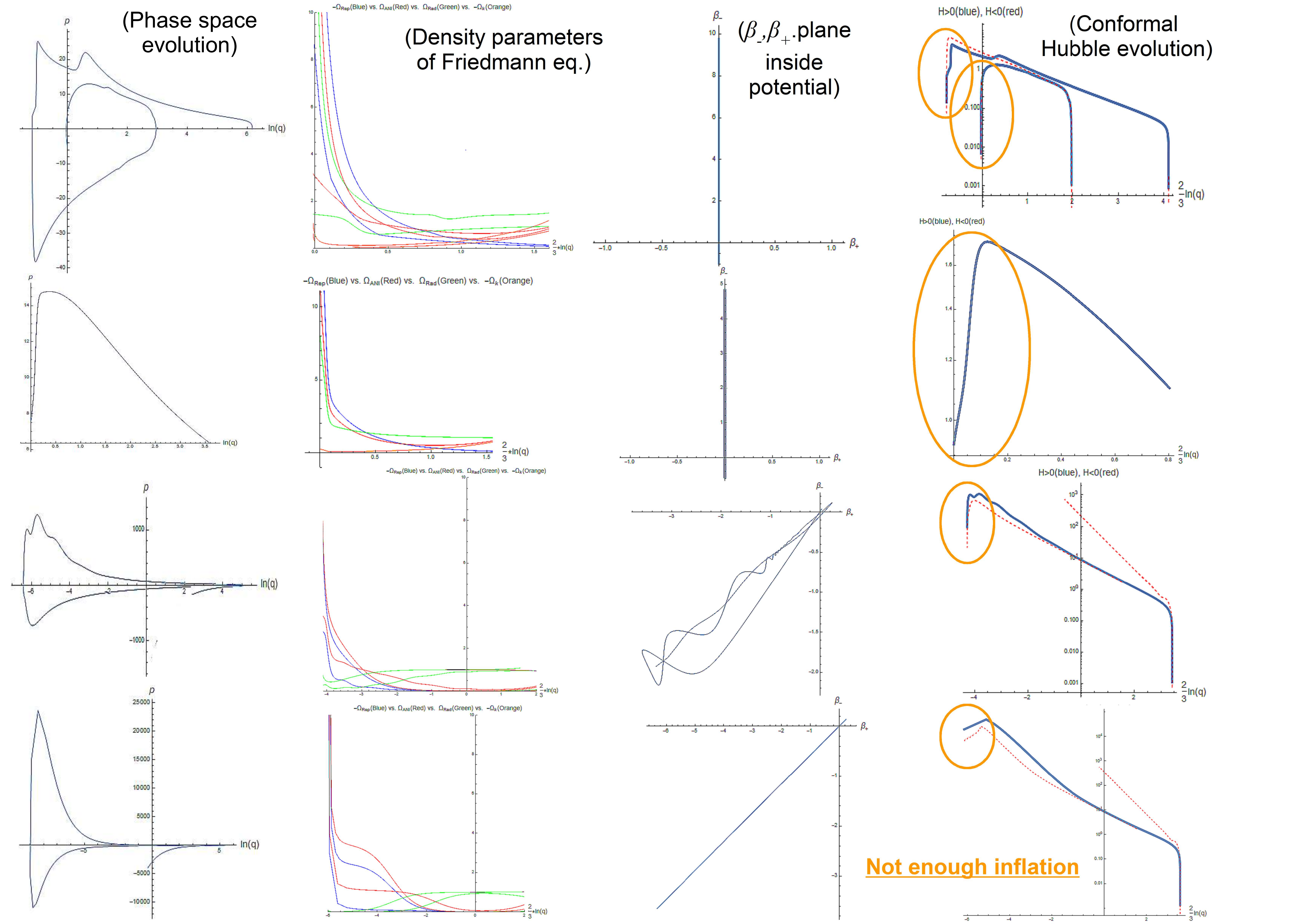
$\frac{2}{3}$ln(q)
β_-
β_+
e-Folds:
$N = \ln\left(\frac{a(t)}{a_0}\right)$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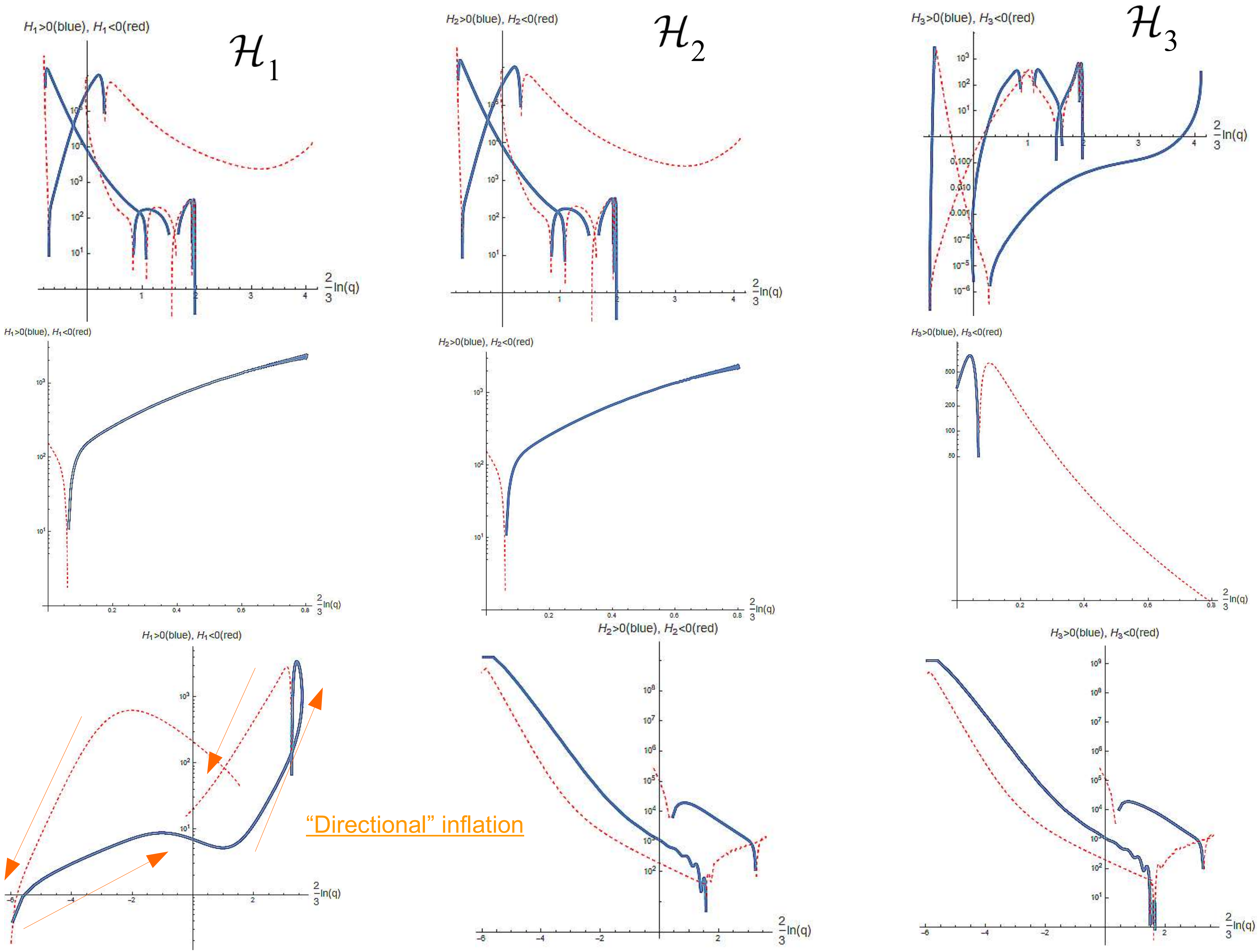
Not enough inflation

Directional Hubble rates: Inflation in principal directions?



Conclusions and Future Investigations:

- Very simple model → Rich dynamics, many possibilities.

- Solve singularity problem → Quantum Bounce

- Anisotropy + bounce by themselves do not generate sufficient inflationary dynamics.

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- Very simple model → Rich dynamics, many possibilities.

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- Anisotropy + bounce by themselves do not generate sufficient inflationary dynamics.

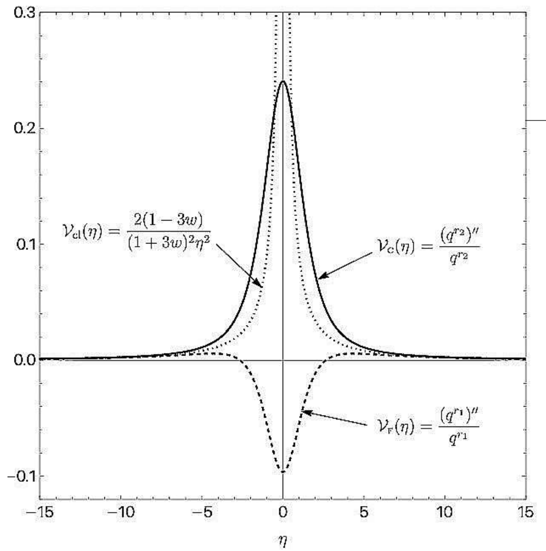
(Does not mean we cannot generate structures)

→ **BUT**: Might be the seed for future investigations:

- Generation of gravitational potential?
- Interplay with primordial perturbations?
- "Directional" inflation → Amplification by gravitational potential in each direction separately?

→ **Another approach:** Full quantum model → Maybe semiclassicality erase some features close to the bounce. Full quantum is more complicated.

Gravitational potentials $\frac{a''}{a}$ for other previously studied *isotropic* models:



→ Isotropic bouncing models + perturbations give this kind of gravitational potential → Generation of cosmological structures.

The primordial spectrum is nearly scale invariant but slightly blue-tilted → Can anisotropy improve this?

[5] J. de Cabo Martin, P. Małkiewicz, and P. Peter, (2021) [arXiv:2111.02963]

Thank you for your attention!

Jaime de Cabo Martín
(PhD student at NCBJ, Warsaw)
Mixmaster universe: semiclassical dynamics and inflation from bouncing
Ongoing work in collaboration with:
Hervé Bergeron, Jean-Pierre Gazeau, and Przemysław Małkiewicz