UNIWERSYTET WARSZAWSKI

WYDZIAŁ FIZYKI UNIWERSYTET WARSZAWSKI

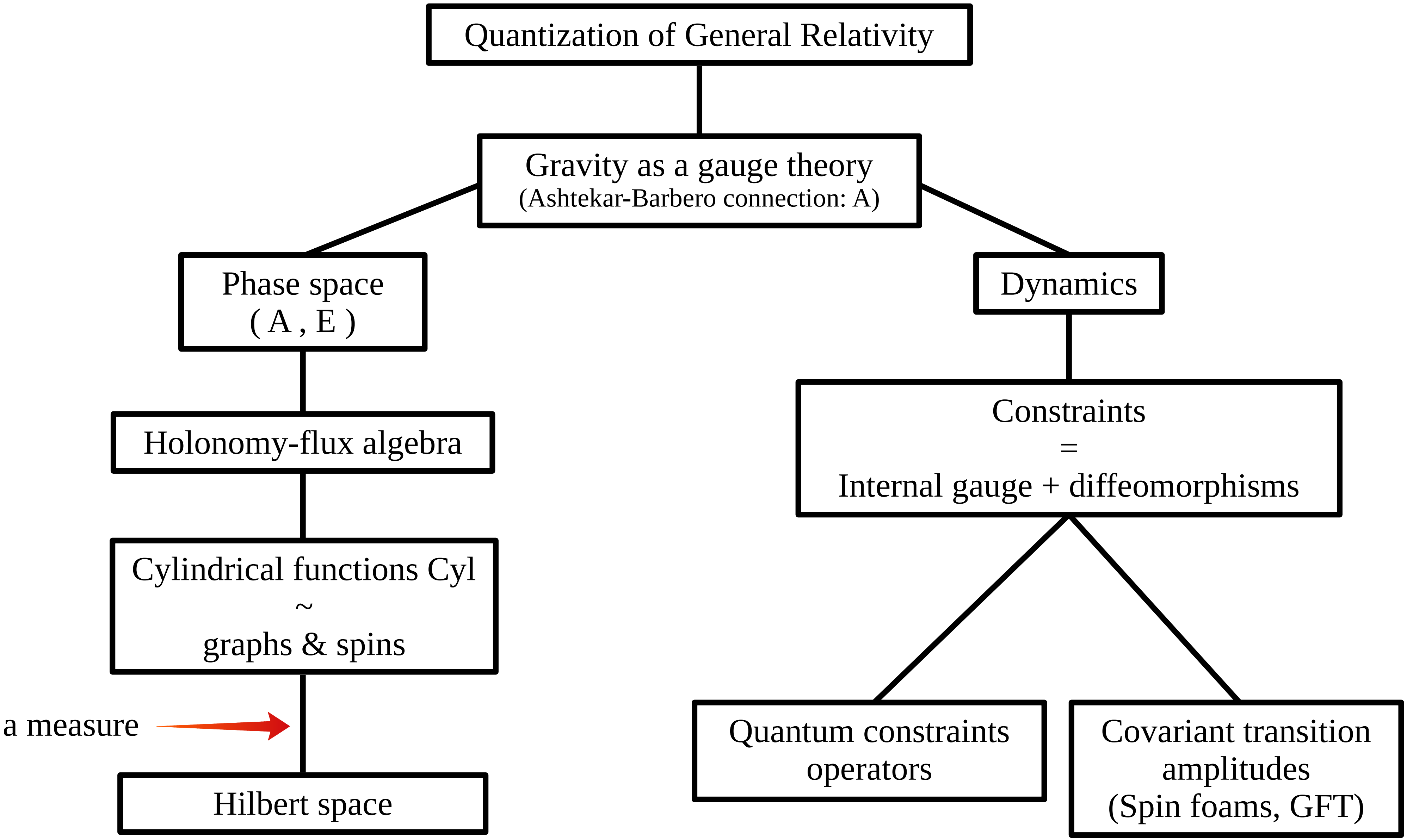
Matter fields in loop quantum gravity

Mehdi Assanioussi

Faculty of Physics, University of Warsaw

8th Conference of the Polish Society on Relativity
Warsaw, Poland

Loop Quantum Gravity



Loop Quantum Gravity + Matter

Scalar field	**Gauge field**	**Fermions**
Point holonomies	holonomies	half-densities
Cylindrical functions		
Hilbert space + quantum dynamics		

Canonical dynamics = Quantum constraint operators

What are the relevant physical & semi-classical states?

From Q.G.+M. to q. fields on a fixed background

How to connect the low energy physics to the loop quantum gravity framework?

How to relate the Fock quantization of matter fields and the loop quantization?

From Q.G.+M. to q. fields on a fixed background

How to connect the low energy physics to the loop quantum gravity framework?

How to relate the Fock quantization of matter fields and the loop quantization?

R-Fock representations (measures)

M. Varadarajan [PRD ‘00, ‘01, ‘02] – ***$U(1)^n$*** gauge theory
A. Ashtekar, J. Lewandowski, H. Sahlmann [CQG ‘03] – ***Scalar field***
M. A., J. Lewandowski [PRD ‘22] – ***SU(N)*** gauge theory

- Original motivation:
  understanding the relation between quant. states of linearized gravity and states in LQG.

- What is it:
  - Roughly: r-Fock reps. are reps. for matter fields propagating on Minkowski spacetime, which connect the standard Fock reps. to the background independent loop reps.
  - measurements at a given “scale”: $\exists$ r-Fock rep. $\equiv$ Fock rep.

- Role:
  - Provide new measures for the loop states.
  - Provide mappings of Fock states to the Loop space.

Abelian gauge theory

$$\{A_a(x), E^b(y)\} = \tfrac{1}{q}\delta^b_a \delta^{(3)}(x,y)$$

Fock rep. = rep.

Smearing function
(Schwartz)

$$\lim_{r\to 0} f_r(x-y) = \delta^{(3)}(x,y)$$

HA: (h, E_r)

$$h_\gamma(A) := \exp\left[i\int_\gamma ds\ \dot\gamma^a(s)A_a(\gamma(s))\right]$$

$$E^a_r(x) := \int_{R^3} d^3y f_r(x-y)E^a(\vec{y})$$

Isomorphic

HA$_r$: (h^r, E)

$$A^r_a(x) := \int_{R^3} d^3y f_r(x-y)A_a(y)$$

$$h^r_\gamma(A) := \exp\left[i\int_\gamma ds\ \dot\gamma^a(s)A^r_a(\gamma(s))\right]$$

"smeared holonomy"

The Fock rep. of ***HA$_r$*** induces the r-Fock rep. of ***HA***

~~Abelian~~ Non-Abelian gauge theory

$$\{A_a(x), E^b(y)\} = \frac{1}{q}\delta_a^b \delta^{(3)}(x,y)$$

Smearing function (Schwartz)

$$\lim_{r\to 0} f_r(x-y) = \delta^{(3)}(x,y)$$

Fock rep. = rep.

HA: (h, E_r)

$$h_\gamma(A) := \exp\left[i\int_\gamma ds\ \dot{\gamma}^a(s) A_a(\gamma(s))\right]$$

$$E_r^a(x) := \int_{R^3} d^3y f_r(x-y) E^a(\vec{y})$$

~~Isomorphic~~

HA$_r$: (h^r, E)

$$A_a^r(x) := \int_{R^3} d^3y f_r(x-y) A_a(y)$$

$$h_\gamma^r(A) := \exp\left[i\int_\gamma ds\ \dot{\gamma}^a(s) A_a^r(\gamma(s))\right]$$

"smeared holonomy"

The Fock rep. of ***HA$_r$*** ~~induces the r-Fock rep. of *HA*~~

induces an r-Fock measure on ***HA***

Provide mappings of Fock states to the Loop space

r-Fock measures for SU(N) gauge theories

- Defining the measure in the case of U(1):

$$\int d\mu^r_{U(1)}\ h_\gamma(A) := \langle 0|\hat{h}^r_\gamma|0\rangle \qquad \Rightarrow \qquad \int_{\bar{\mathcal{A}}/\bar{\mathcal{G}}} d\mu^r_{U(1)}\ \Psi(A) := \langle 0|\psi(\hat{h}^r_{\gamma_1}, \ldots, \hat{h}^r_{\gamma_K})|0\rangle$$

consequence of Mandelstam identities for U(1): every U(1) cylindrical function can be expressed as a linear combination of ***Wilson loops***.

r-Fock measures for SU(N) gauge theories

- Defining the measure in the case of U(1):

$$\int d\mu^r_{U(1)}\ h_\gamma(A) := \langle 0|\hat{h}^r_\gamma|0\rangle \qquad \Rightarrow \qquad \int_{\bar{\mathcal{A}}/\bar{\mathcal{G}}} d\mu^r_{U(1)}\ \Psi(A) := \langle 0|\psi(\hat{h}^r_{\gamma_1},\ldots,\hat{h}^r_{\gamma_K})|0\rangle$$

consequence of Mandelstam identities for U(1): every U(1) cylindrical function can be expressed as a linear combination of ***Wilson loops***.

- Defining the measure in the case of SU(N):
  - We "cannot" use smeared holonomies: gauge transformations are peculiar.
  - ***Mandelstam identities*** for SU(N) imply that the natural generalization is in terms of Wilson loops:

$$\int d\mu^r_{SU(N)}\ W^J_{\gamma_1}(A)\ldots W^J_{\gamma_{N-1}}(A) := \underline{\langle 0|\hat{W}^{r,J}_{\gamma_1}\ldots\hat{W}^{r,J}_{\gamma_{N-1}}|0\rangle}$$

To calculate
Not easy!

$$\hat{W}^{r,J}_\gamma := \mathrm{Tr}\left[\hat{h}^{r,J}_\gamma\right] \qquad \text{"r-Wilson loop operator"}$$

$$\hat{W}^{r,J}_\gamma = \sum_{n=0}^{\infty}\mathrm{Tr}\left[\prod_{m=1}^{n}\tau^J_{i_m}\right]\mathcal{P}\!\!\oint_\gamma ds_1\ldots ds_n\prod_{m=1}^{n}\int\frac{d^3k_m}{q\sqrt{2|k_m|}}\tilde{X}^{a_m}_{\gamma,r}(s_m,k_m)\left(c^{i_m\,\dagger}_{a_m}(k_m)+c^{i_m}_{a_m}(-k_m)\right)$$

r-Fock measures for SU(N) gauge theories

- Linear functional:

j_o – fundamental rep.

$$\Phi_F^r : \mathcal{HA} \longrightarrow \mathbb{C}$$

$$\Phi_F^r \left[\sum_{i=1}^{M} a_i W_{\gamma_1^i}^{j_o} \ldots W_{\gamma_{N-1}^i}^{j_o} \right] := \sum_{i=1}^{M} a_i \langle 0 | \hat{W}_{\gamma_1^i}^{r,j_o} \ldots \hat{W}_{\gamma_{N-1}^i}^{r,j_o} | 0 \rangle$$

- Definiteness : ***boundedness of the coefficients*** implies the convergence of the expansion of the expectation value;

- Positivity : ***Mandelstam identities*** for the smeared Wilson loop operators;

- **Existence of an induced measure on $\overline{A}/\overline{G}$:** continuity w.r.t. the C*-norm on $\overline{A}/\overline{G}$

$$\left| \Phi_F \left[\sum_{i=1}^{M} a_i W_{\gamma_i}^{r,j_o} \right] \right| \leq \sup_{A \in \bar{\mathcal{A}}/\bar{\mathcal{G}}} \left| \sum_{i=1}^{M} a_i W_{\gamma_i}^{j_o}(A) \right|$$

the smearing : $A \in \mathcal{S}^* \longrightarrow A^r \in \mathcal{S}^* \cap \bar{\mathcal{A}}$

- Mapping between measures:

$$d\mu^r_{SU(N)} = \left(\sum_{\Gamma} \sum_{\{j,\iota\}_\Gamma} \Phi^r_F \left[\sum_{(\{\gamma^i_k\},a_i)\in \mathcal{I}_\Gamma\left(\Psi_{\Gamma,\{j,\iota\}}\right)} a_i W^{j_o}_{\gamma^i_1} \ldots W^{j_o}_{\gamma^i_{N-1}} \right] \overline{\Psi_{\Gamma,\{j,\iota\}}} \right) d\mu^o_{SU(N)}$$

lift to a gauge invariant measure on $\overline{A}$ via group averaging.

r-Fock measures for SU(N) gauge theories

- Mapping between measures:

$$d\mu^r_{SU(N)} = \left(\sum_\Gamma \sum_{\{j,\iota\}_\Gamma} \Phi^r_F \left[\sum_{(\{\gamma^i_k\},a_i)\in\mathcal{I}_\Gamma\left(\Psi_{\Gamma,\{j,\iota\}}\right)} a_i W^{j_o}_{\gamma^i_1} \ldots W^{j_o}_{\gamma^i_{N-1}} \right] \overline{\Psi_{\Gamma,\{j,\iota\}}} \right) d\mu^o_{SU(N)}$$

lift to a gauge invariant measure on $\overline{A}$ via group averaging.

- U(1) example:
  - r-Fock measure & AL-measure:

n_I – rep. label; G^r_{IJ} – smearing factor

$$d\mu^r_{U(1)} = \left(\sum_{\Gamma,\vec{n}} \exp\left[-\frac{1}{4q^2} \sum_{I,J} n_I n_J G^r_{IJ} \right] \overline{\mathcal{N}_{\Gamma,\vec{n}}} \right) d\mu^o_{U(1)}$$

  - Fock states mapped to Cyl*, in particular the vacuum st. & canonical coherent states:

$$\mathcal{Z}^r_F = \sum_{\Gamma,\vec{n}} \exp\left[-\frac{1}{q^2} \sum_I n_I Z^r_I \right] \exp\left[-\frac{1}{q^2} \sum_{I,J} n_I n_J G^r_{IJ} \right] \langle \mathcal{N}_{\Gamma,\vec{n}}|$$

  - Shadow states = projections of Fock states on separable sub-Hilbert spaces:
    exp. : fixed graph, dynamical super-selected sector, ...

Comments & outlook

✔ Results:

- Construction of r-Fock measures to the general SU(N) case:
  Use of Wilson loops and their properties provides a systematic procedure;
- Construction of an r-Fock measure for the fermions sector (to appear soon);

✔ Implications:

- Fock states as shadow states :
  - Matter states encoding Minkowski geometry;
  - Non-local coefficients;
  - Graphs superposition could be restricted by the dynamics;

🔍 To explore:

- Non-locality (entanglement) & semi-classical prop. of shadow states;
- Effective dynamics for the shadow states as approximate physical states;
- Role in the construction of a continuum limit for LQG;

Comments & outlook

✔ Results:

- Construction of r-Fock measures to the general SU(N) case:
  Use of Wilson loops and their properties provides a systematic procedure;
- Construction of an r-Fock measure for the fermions sector (to appear soon);

✔ Implications:

- Fock states as shadow states :
  - Matter states encoding Minkowski geometry;
  - Non-local coefficients;
  - Graphs superposition could be restricted by the dynamics;

To explore:

- Non-locality (entanglement) & semi-classical prop. of shadow states;
- Effective dynamics for the shadow states as approximate physical states;
- Role in the construction of a continuum limit for LQG;

THANK YOU!