Scalarization in Einsteinian Cubic Gravity with General Relativity as a cosmic attractor

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Introduction

- General relativity (GR) is well tested in the weak-field regime, whereas the strong-field regime still remains essentially unexplored and unconstrained.

- The attempts to construct a unified theory of all the interactions naturally lead to scalar-tensor-type generalizations of GR viewed as an effective field theory (EFT) operating in the strong-gravity regime.

- The effects of higher-order curvature operators become more significant exciting the scalar degree of freedom in the strong-field regime. When the effective mass is chosen to be tachyonic, GR solutions may become unstable, while stable black hole or star solutions acquire scalar hair absent in GR.

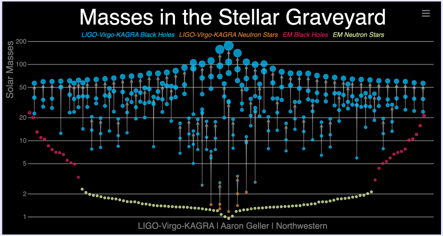
- "Spontaneous scalarization" is a distinctive manifestation of gravitational interactions in the strong-field regime. Einstein Scalar-Gauss-Bonnet (ESGB) gravity is the higher-order theory up to quadratic order in the curvature invariants:
  - Neutron star scalarization [Damour & Esposito-Farese '93].
  - Black hole scalarization [Silva et al '18][Doneva & Yazadjiev '18]

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - 2\nabla_\mu \varphi \nabla^\mu \varphi + \lambda^2 f(\varphi) \mathcal{R}^2_{\rm GB} \right]$$

- Generalizations of the spontaneous scalarization framework: Coupled to R [Brihaye et al '19], coupled to Chern-Simons invariant [Brihaye et al '19], flat asymptotics [Doneva et al '20], AdS/dS asymptotics [Bakopoulos et al '19 & '20][Brihaye et al '20], holographic applications in AdS [Kiorpelidi et al '20][Brihaye et al '20], f(R) [E. Papantonopoulos '21], spin induced black hole spontaneous scalarization [D. Doneva, L. Collodel '20]

ESGB can not provide black hole scalarization without an unstable cosmological background.

- **Is it possible to construct a theory beyond ESGB gravity as an EFT?**
- **Is there a theory able to reconcile black hole scalarization with a stable cosmology?**
- **Is scalarization of astrophysical black holes possible?**



Along this talk we want:

- To address a scalar-tensor EFT that exhibits curvature-induced scalarization for black hole solutions, triggered by a set of suitable invariants made up of Riemann tensor, up to cubic order
- To investigate within this framework, how the new operators modify a catastrophic instability triggered by quantum fluctuations during the inflationary stage in ESGB theory
- To explore the Big Bang Cosmology (BBC) of the model, and check that GR is indeed a late-time cosmological attractor as experiments seem to demand

Outline

The model and black hole scalarization

- We want to consider a theory that exhibits scalarization triggered by higher-order curvature operators other than the Gauss-Bonnet invariant.
- For definiteness, we will consider the so-called Einsteinian Cubic Gravity (ECG) theory [Bueno, Cano '16], which possesses some basic "good" properties such as:
    - Having a spectrum identical to that of Einstein gravity, i.e., the metric perturbation (on a maximally symmetric background) propagates only a transverse massless graviton.
    - It is neither topological nor trivial in four dimensions.
    - It is defined such that it is independent of the number of dimensions.

Cubic operators

- We start by recalling the cubic operator $\mathcal{P}$ in ECG theory, which reads

$$\mathcal{P} = 12 R_{\mu\ \nu}^{\ \rho\ \sigma} R_{\rho\ \sigma}^{\ \gamma\ \delta} R_{\gamma\ \delta}^{\ \mu\ \nu} + R_{\mu\nu}^{\ \ \rho\sigma} R_{\rho\sigma}^{\ \ \gamma\delta} R_{\gamma\delta}^{\ \ \mu\nu} - 12 R_{\mu\nu\rho\sigma} R^{\mu\rho} R^{\nu\sigma} + 8 R^{\nu}_{\ \mu} R^{\mu}_{\ \rho} R^{\rho}_{\ \nu} \,,$$

  while the operator $\mathcal{C}$, found in GQTG [Hennigar et al '17], is given by the combination

$$\mathcal{C} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho}{}_{\delta} R^{\sigma\delta} - \frac{1}{4} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} R - 2 R_{\mu\nu\rho\sigma} R^{\mu\rho} R^{\nu\sigma} + \frac{1}{2} R_{\mu\nu} R^{\mu\nu} R.$$

- It has been proven that it is the exact combination $\mathcal{P} - 8\mathcal{C}$, the one that leads to cosmologies with a well-posed initial value problem [Arcienaga et al '20].

- In order to explore the phenomenon of scalarization we must include a scalar field φ, while keeping the healthy features of ECG.

The action of Scalar Einsteinian Cubic Gravity (SECG)

- For simplicity, we impose a $\varphi \to -\varphi$ (discrete) symmetry. The action of the theory is then given by

$$S[g_{\mu\nu}, \varphi] = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + \frac{\alpha}{M_{\rm Pl}^2}(\mathcal{P} - 8\,\mathcal{C}) - \frac{1}{2}g^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi + f\,(\varphi/M_{\rm Pl})\,\mathcal{I} + \cdots\right.$$

["Scalar-Einsteinian Cubic Gravity" (SECG)]

- Here, $f(\varphi/M_{\rm Pl})$ is a dimensionless "coupling function" between the canonically normalized scalar field φ and a set of curvature invariants given by

$$\mathcal{I} = -\beta\,M_{\rm Pl}^2 R + \gamma\,\mathcal{G} - \frac{\lambda}{M_{\rm Pl}^2}\,(\mathcal{P} - 8\mathcal{C})\,,$$

  where $\mathcal{G}$ is the well-known Gauss-Bonnet operator $\mathcal{G} \equiv R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$.

- The dimensionless coupling constants α, β, γ, and λ are expected to be $\mathcal{O}(1)$ numbers ("naturalness" argument).

Field equations

- We will *not* set the (reduced) Planck scale $M_{\rm Pl} = 2.4 \times 10^{18}$ GeV to unity as it is usually done in the literature since we want to keep track of it to easily emphasize its role of being the ultimate EFT cut-off of any gravitational system.
- The EOM that stem from extremizing the action $S[g_{\mu\nu}, \varphi] = \int d^4x \sqrt{-g}\, \mathcal{L}$ read

$$R^{\alpha\beta\rho}{}_{\mu} P_{\nu\rho\alpha\beta} + 2\nabla^{\alpha}\nabla^{\beta} P_{\alpha\mu\nu\beta} + \frac{1}{2}\nabla_{\mu}\varphi\nabla_{\nu}\varphi + \frac{1}{2} g_{\mu\nu}\mathcal{L} = 0 \; ,$$
$$\Box\varphi + f_{,\varphi}(\varphi/M_{\rm Pl})\, \mathcal{I} = 0 \; ,$$

where $P_{\alpha\beta\mu\nu} \equiv \dfrac{\partial \mathcal{L}}{\partial R^{\alpha\beta\mu\nu}}$.

- The EOM for the scalar field fluctuation $\delta\varphi \equiv \varphi - \varphi_0$ is given by

$$\Big[\Box + f_{,\varphi\varphi}(\varphi_0/M_{\rm Pl})\, \mathcal{I}\Big]\delta\varphi = 0,$$

where φ_0 is the scalar field background, while the d'Alembertian operator and $\mathcal{I}$ are computed in a fixed background.

- We need to demand that $f_{,\varphi}(0) = 0$ so that GR vacuum solutions together with $\varphi_0 = 0$ are admissible solutions of the field equations.

- Moreover, $f_{,\varphi\varphi}(0) > 0$ is necessary for the emergence of a tachyonic instability, which triggers the spontaneous scalarization.

- Without loss of generality we then take $f(x) = \frac{1}{2}x^2 + \ldots$, implying a scalar field fluctuation effective mass squared given by

$$m_{\text{eff}}^2 = -f_{,\varphi\varphi}\left(\varphi_0/M_{\text{Pl}}\right)\mathcal{I} = \beta R - \frac{\gamma}{M_{\text{Pl}}^2}\mathcal{G} + \frac{\lambda}{M_{\text{Pl}}^4}\left(\mathcal{P} - 8\mathcal{C}\right).$$

- We are interested in models that exhibit spontaneous scalarization around compact objects such as Schwarzschild black holes for which $R = 0$ and $\mathcal{G} > 0$. As the cubic operator is further suppressed by the cut-off for natural values of λ, this implies the condition $\gamma > 0$. Hereafter, we will take $\boldsymbol{\gamma > 0}$.

- Scalar field fluctuations evolve according to

$$[\Box + f_{,\varphi\varphi}\,(\varphi_0/M_{\rm Pl})\,\mathcal{I}]\,\delta\varphi = 0$$

- Perturbations on a fixed Schwarzschild background,

$$\delta\varphi = \frac{u(r)}{r}\, e^{-i\omega t}\, Y_{lm}\,(\theta, \phi).$$

- In tortoise coordinates r_*, the Klein-Gordon equation becomes "Schrödinger-like", meaning

$$\frac{d^2 u}{dr_*^2} + \omega^2 u = V_{\rm eff}(r) u,$$

where the effective potential $V_{\rm eff}$ is,

$$V_{\rm eff}(r) = \left(1 - \frac{r_g}{r}\right)\left(\frac{l(l+1)}{r^2} + \frac{r_g}{r^3} - \frac{\gamma}{M_{\rm Pl}^2}\frac{12\, r_g^2}{r^6} + \frac{\lambda}{M_{\rm Pl}^4}\frac{84\, r_g^3}{r^9}\right).$$

- It can be shown that there exists a sufficient condition for the existence of an unstable mode given by [Buell and Shadwick '95]

$$\int_{r_g}^{\infty} dr \, \frac{V_{\text{eff}}(r)}{\left(1 - \frac{r_g}{r}\right)} < 0,$$

where $r_g \equiv \mathcal{M}/4\pi M_{\text{Pl}}^2$, and $\mathcal{M}$ stands for the black hole mass.

- The above condition implies that a Schwarzschild background is unstable for a specific range of masses.

- The maximum mass $\mathcal{M}$ of Schwarzschild black holes that may be scalarized is

$$\mathcal{M}_{MAX} \sim 10^{-37} M_\odot \ ,$$

precluding any possibility of such a version of SECG theory to be compared with observations. We will have more to say about this soon enough.

- In a FLRW background with metric

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu = -dt^2 + a(t)^2 \delta_{ij}\, dx^i dx^j, \quad \text{and} \quad H \equiv \frac{\dot{a}}{a},$$

it so happens that the EOM for the fluctuation reads

$$\delta\ddot{\varphi} + 3H\delta\dot{\varphi} - \frac{\nabla^2 \delta\varphi}{a^2} + m_{\rm eff}^2\, \delta\varphi = 0\ .$$

- For a FLRW spacetime

$$R = 6\left(2H^2 + \dot{H}\right), \quad \mathcal{G} = 24H^2\left(H^2 + \dot{H}\right), \quad \mathcal{P} - 8\mathcal{C} = -48H^4\left(2H^2 + 3\dot{H}\right)\ ,$$

so that the effective mass squared becomes

$$m_{\rm eff}^2 = 6\beta\left(2H^2 + \dot{H}\right) - \frac{24\,\gamma}{M_{\rm Pl}^2} H^2\left(H^2 + \dot{H}\right) - \frac{48\,\lambda}{M_{\rm Pl}^4} H^4\left(2H^2 + 3\dot{H}\right).$$

Einstein Scalar Gauss-Bonnet gravity revisited ($\beta = \lambda = 0$)

- In the ESGB theory the effective squared mass (recall that $\gamma > 0$)

$$m_{\rm eff}^2 = -\frac{\gamma}{M_{\rm Pl}^2}\mathcal{G} \qquad \text{(Schwarzschild background)}$$

$$m_{\rm eff}^2 = -\frac{24\,\gamma}{M_{\rm Pl}^2}H^2\left(H^2 + \dot{H}\right) = -\frac{24\,\gamma}{M_{\rm Pl}^2}H^2\frac{\ddot{a}}{a} \qquad \text{(FLRW background)}$$

Problems in ESGB:

- $m_{\rm eff}^2 < 0 \Longleftrightarrow \ddot{a} > 0$ implies the existence of a **tachyonic instability during any (quasi)-de Sitter (dS) phase of our universe.**
- The possibility of scalarizing astrophysical compact objects, e.g., neutron stars, introduces a **hierarchy problem** since it requires $\boldsymbol{\gamma \sim 10^{74}}$.
- $\frac{t_{\rm inst}}{t_{\rm inf}} \sim 10^{-34}$ implies that **inflation is not compatible with the phenomenon of black hole scalarization**

- From an EFT point of view a linear term on Ricci scalar must nonminimally coupled to the scalar field i.e. $\beta \neq 0$

Scalar Einsteinian Cubic Gravity

In order to cure the the highly unnatural value for γ we need to introduce a new scale M within the coupling sector operators.

$$S[g_{\mu\nu},\varphi] = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + \frac{\alpha}{M_{\rm Pl}^2}(\mathcal{P} - 8\,\mathcal{C}) - \frac{1}{2}g^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi + f\left(\varphi/M\right)\mathcal{I}\right]$$

[**"Scalar-Einsteinian Cubic Gravity" (SECG)**] with

$$\mathcal{I} = -\beta\, M^2 R + \gamma\,\mathcal{G} - \frac{\lambda}{M^2}\left(\mathcal{P} - 8\mathcal{C}\right).$$

- Now the effective squared mass for Schwarzschild and FLRW background is, repectively,

$$m_{\rm eff}^2 = -\frac{\gamma}{M^2}\mathcal{G} + \frac{\lambda}{M^4}\left(\mathcal{P} - 8\mathcal{C}\right)$$

$$m_{\rm eff}^2 = 12\left[\beta\left(1 - \frac{\epsilon}{2}\right) - 2\,\gamma\,(1-\epsilon)\,\chi - 8\,\lambda\left(1 - \frac{3}{2}\,\epsilon\right)\chi^2\right]H^2, \quad \text{with} \quad \chi \equiv \left(\frac{H}{M}\right)^2.$$

Some good properties of SECG

- **Tachyonic instability is removed** during any (quasi)-de Sitter (dS) phase of our universe provided $\boldsymbol{\beta > 0}$
- Scalarization of astrophysical black holes with characteristic length scale $L \equiv M^{-1} \sim 10$ km requires $\boldsymbol{\gamma \sim 10^{-2}}$ in **agreement with naturalness argument**.
- Maximum mass allowed for scalarized black holes grows up to $\boldsymbol{\mathcal{M}_{MAX} \sim 180 M_\odot}$.

- Unlike ESGB, scalarization in SECG scenario **is restricted by an upper bound of the curvature** at the event horizon,

$$\frac{\mathcal{K}_H}{M_{\rm Pl}^4} \in \left[\frac{3072\,\pi^4}{q_+^2}, \frac{3072\,\pi^4}{q_-^2}\right] \qquad \text{if} \quad 0 < \lambda \leq \frac{48}{175}\,\gamma^2,$$

$$\frac{\mathcal{K}_H}{M_{\rm Pl}^4} \in \left[0, \frac{3072\,\pi^4}{q_-^2}\right] \qquad \text{if} \quad \lambda \leq 0.$$

General Relativity as a Cosmic Attractor

- The scalar EOM in a FLRW background is given by

$$\ddot{\varphi} + 3H\dot{\varphi} + m_{\text{eff}}^2\,\varphi = 0 \quad \text{where} \quad m_{\text{eff}}^2 = \beta R - \frac{\gamma}{M^2}\mathcal{G} + \frac{\lambda}{M^4}\left(\mathcal{P} - 8\mathcal{C}\right)$$

while the "t-t" Einstein equation reads

$$M_{\text{Pl}}^2 G_{tt} = \rho_{\text{eff}} + \rho_a ,$$

with

$$\rho_{\text{eff}} \equiv \rho_{\mathcal{PC}} + \rho_{\varphi}, \quad \rho_{\mathcal{PC}} = -\frac{48\,\alpha}{M_{\text{Pl}}^2} H^6,$$

$$\rho_{\varphi} = \frac{1}{2}\dot{\varphi}^2 + 6\left(\beta - 4\,\gamma\,\chi - 24\,\lambda\,\chi^2\right) H\varphi\dot{\varphi} + 3\left(\beta + 8\,\lambda\,\chi^2\right) H^2\varphi^2, \text{ and } \chi \equiv (H/M)^2.$$

- We shall demand usual cosmic evolution, i.e.

$$\rho_a \approx 3M_{\text{Pl}}^2 H^2,$$

with $\rho_a = \{\rho_{\text{r}}, \rho_{\text{m}}, \rho_{\text{de}}\}$

- Since we do not want φ to play any role in late-time cosmology, we shall assume that

$$\rho_{\varphi} \ll \rho_a .$$

It is mandatory to check if such an assumption is dynamically consistent.

Numerical integration from cosmological redshift $z = 0$ to $z = 10^{12}$

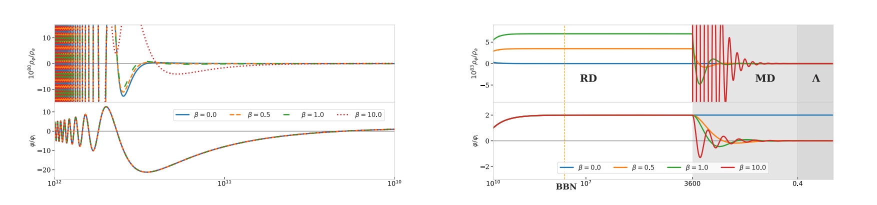


Figure 1: *Top panel*: Effective energy density ρ_φ relative to the energy density of the cosmic fluid ρ_a. *Bottom panel*: Scalar field value relative to its initial value fixed at $z_i = 10^{10}$. The values of the coupling constants are taken to be $\gamma = 1$ and $\lambda = 48/175$

The solution is consistent with assumption $\rho_\varphi(z) \lll \rho_a(z)$ for the whole range of numerical integration.

Evolution of scalar field

- During early times, or high redshift, m^2_{eff} dominates over Hubble friction within the scalar field equation. However, as we "move" forward in time, m^2_{eff} decays much faster than the Hubble friction which rapidly takes over, so it is expected that the scalar field freezes to a constant way before entering MD era.

- For even higher redshift values the relative scalar field and the relative energy density oscillate with ever increasing frequency as can be seen in figure **??**.

- During radiation domination (RD) the scalar field is completely insensitive to the value of β as the Ricci scalar identically vanishes, while the relative energy density does marginally depend on such a constant even though all the curves, for high enough z, eventually converge.

- By the time the MD era begins, the Ricci scalar stops being trivial, and in fact it entirely determines the relative scalar and energy density evolution because the higher-order operators become irrelevant considering that $\chi \sim 10^{-36} \lll 1$ when $z = 3600$.

As it was expected, the scalar field profile in SECG exhibits a manifest deviation from its quadratic counterpart for very high cosmological redshifts, as can be appreciated in the two following figures.

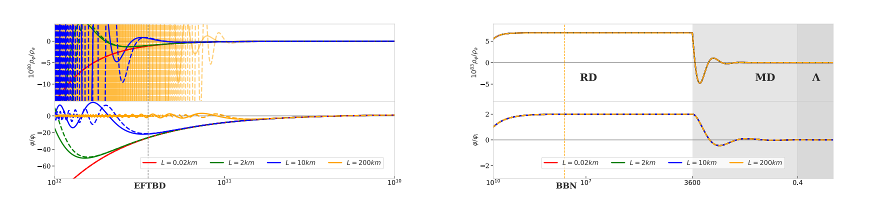


Figure 2: The continuous and dashed curves represent the profile stemming from ESGB and SECG, respectively.

Ending remarks

- We propose scalar-Einstenian Cubic Gravity as a scalar tensor theory constructed as as an effective field theory where **black hole scalarization is compatible with a stable cosmological evolution**.
- The introduction of a second scale M, which is associated with the characteristic energy scale of a compact object to be scalarized, is necessary to have "natural" coupling constants.
- After the introduction of such a scale, the scalarization bound was increased from $10^{-37}M_{\odot}$ to $180M_{\odot}$.
- Unlike ESGB, scalarization in SECG scenario is restricted by an upper bound of the curvature at the event horizon.
- We integrated the scalar field equation to find that **SECG admits GR as a cosmological attractor**, under very sensible assumptions for the initial conditions of the system.

Thank you!

Scalarization in Einstein Scalar-Gauss-Bonnet