Halilsoy spacetime

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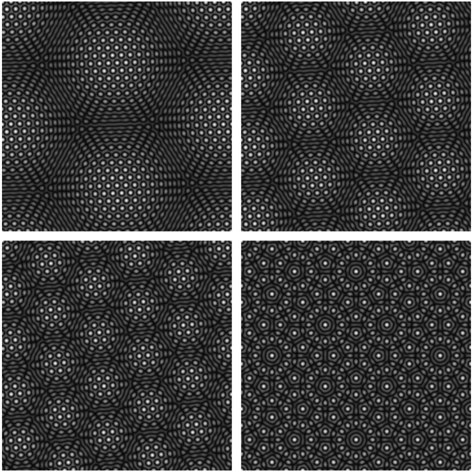
a joint project with Syed Naqvi

motivation

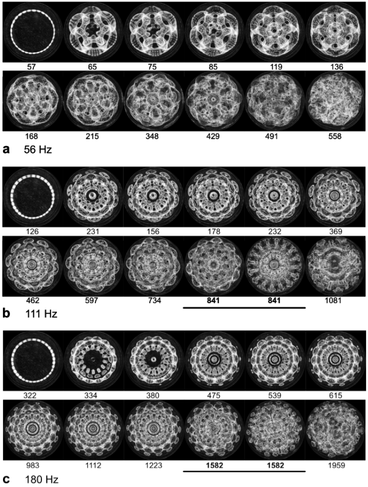
- Jarek Duda — analog systems
- nonlinearities $\implies$ new phenomena
- examples
  - Faraday waves – the Navier Stokes equation, 1831
  - Couder and Fort, 2005, a hydrodynamic pilot-wave system, walking droplets

Faraday waves



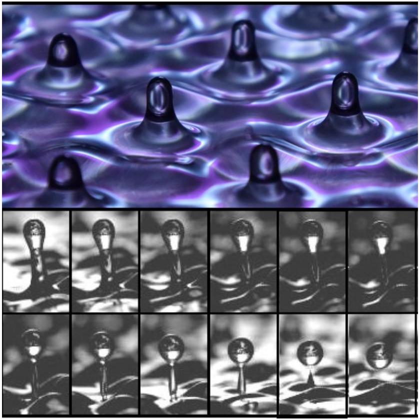
credits: Stéphan Fauve and Gérard Ioos, Quasipatterns versus superlattices resulting from the superposition of two hexagonal patterns, Comptes Rendus Mécanique Volume 347, Issue 4, April 2019, Pages 294-304

Faraday waves



credits: Merlin Sheldrake, Rupert Sheldrake, Determinants of Faraday Wave-Patterns in Water Samples Oscillated Vertically at a Range of Frequencies from 50-200 Hz, Water 9, 1–27, 2017

Faraday waves—droplets



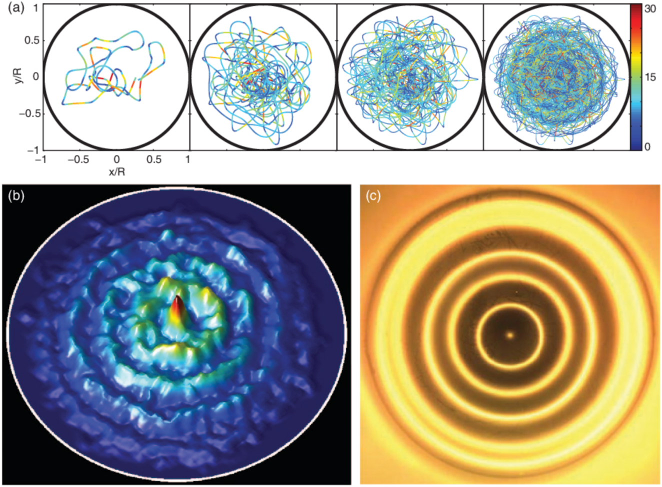
credits: Naresh Sampara, Tristan Gilet, Two-frequency forcing of droplet rebounds on a liquid bath,

Phys. Rev. E 94, 053112 (2016)

walking droplets

credits: Aleks Labuda (winner of the Royal Society Publishing Photography Competition)

walking droplets



credits: D. Harris, J. M. Bush, The pilot-wave dynamics of walking droplets, : Phys. Fluids 25, 091112 (2013)

Halilsoy spacetime

- an exact solution to the vacuum Einstein equations
  M. Halilsoy, Cross-polarized cylindrical gravitational waves of Einstein and Rosen. Nuovo Cimento B Series 102, 563-571, 1988
- regular
- cross-polarized
- cylindrical symmetry
- not asymptotically flat (even in the radial direction)
- a standing gravitational wave
- a generalization of Einstein-Rosen waves (for $\alpha = 0$ we recover Einstein-Rosen waves)
- a localized gravitational energy — a partial similarity to the John Wheeler geons
- we study test particles — no backreaction!

Halilsoy spacetime

- the cylindrical Jordan–Ehlers–Kundt–Kompaneetz line element

$$g = e^{2(\gamma-\psi)}\left(-dt^2 + d\rho^2\right) + \rho^2 e^{-2\psi} d\varphi^2 + e^{2\psi}(dz + \omega d\varphi)^2$$

- the metric functions (parameters: A, λ, α)

$$\begin{aligned} e^{-2\psi(t,\rho)} =& e^{AJ_0(\rho/\lambda)\cos t/\lambda}\sinh^2\frac{\alpha}{2} + e^{-AJ_0(\rho/\lambda)\cos t/\lambda}\cosh^2\frac{\alpha}{2}\ , \\ \omega(t,\rho) =& -(A\sinh\alpha)\rho J_1(\rho/\lambda)\sin\lambda t\ , \\ \gamma(t,\rho) =& \frac{1}{8}A^2\left(\lambda^2\rho^2(J_0^2(\rho/\lambda) + J_1^2(\rho/\lambda))\right. \\ & \left. -2\lambda\rho J_0(\rho/\lambda)J_1(\rho/\lambda)\cos^2 t/\lambda\right) \end{aligned}$$

- the Ernst equation $\Longrightarrow$ other solutions of this type exist, but γ and ω, in general, cannot be found in an explicit form

cylindrical gravitational waves

- Einstein–Rosen waves: A. Einstein, N. Rosen. "On Gravitational waves". Journal of the Franklin Institute. 223: 43–54 (1937)
- Tsvi Piran, Pedro N. Safier, "A gravitational analogue of Faraday rotation", Nature, 318, 21 (1985)
  - null coordinates $u = \frac{1}{2}(t-\rho)\ , \quad v = \frac{1}{2}(t+\rho)$
  - the Einstein equations

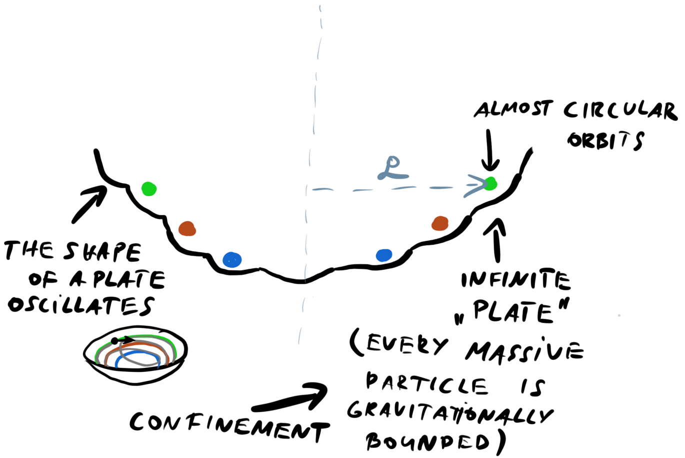
$$
\begin{aligned}
O_+ =& 2(\psi_{,t} + \psi_{,\rho})\ , \quad I_+ = 2(\psi_{,t} + \psi_{,\rho})] \\
O_x =& \frac{e^2\psi}{\rho}(\omega_{,t} - \omega_{,\rho})\ , \quad I_x = \frac{e^2\psi}{\rho}(\omega_{,t} + \omega_{,\rho})
\end{aligned}
$$

$$
\begin{aligned}
I_{+,u} =& \frac{I_+ - O_+}{2\rho} + I_x O_x\ , \quad O_{+,v} = \frac{I_+ - O_+}{2\rho} + I_x O_x \\
I_{x,u} =& \frac{I_x + O_x}{2\rho} + I_+ O_x\ , \quad O_{x,v} = -\frac{I_x + O_x}{2\rho} - I_x O_+
\end{aligned}
$$

cylindrical gravitational waves

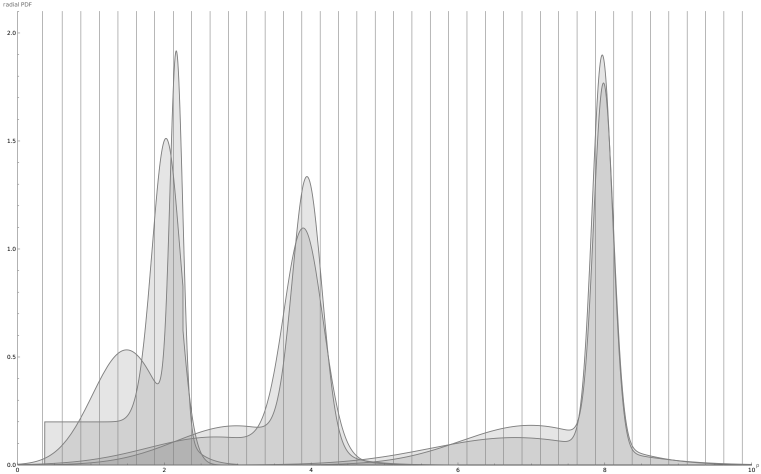
- S. Chandrasekhar, Proc. R. Soc. London, Ser. A, 408, 209 (1986)
- geodesics for Einstein-Rosen waves:
  Adam Cieślik, the bachelor thesis, 2018
- we study timelike geodesics in the Halilsoy spacetime (the geodesic equation is too large to be usefully presented here)

geodesics

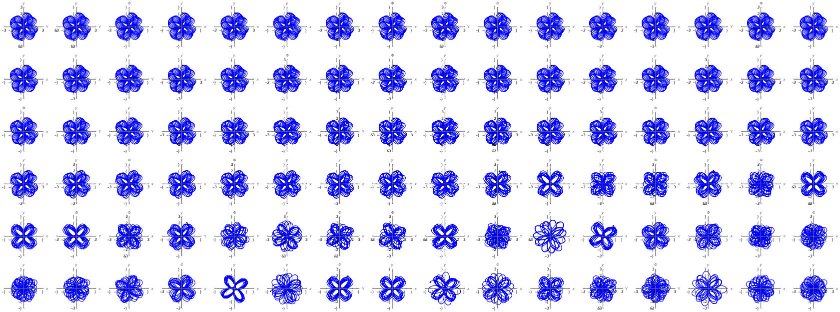


The "mechanical model."

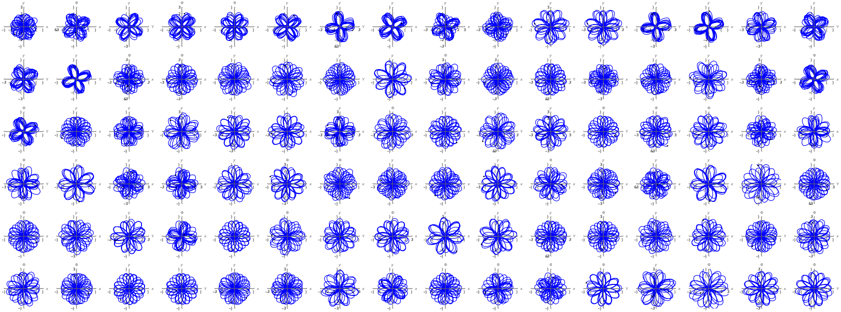
probability density function



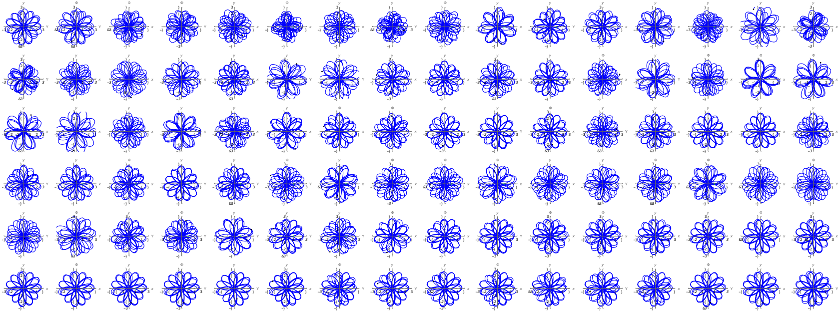
sensitivity to initial conditions



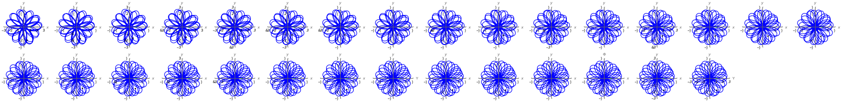
sensitivity to initial conditions



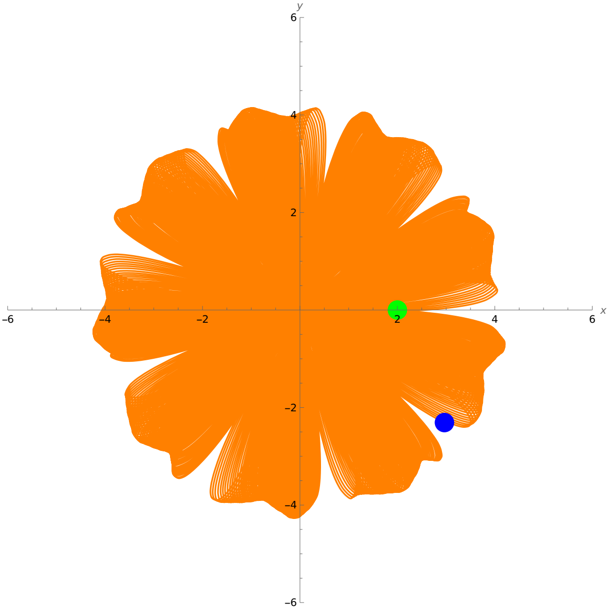
sensitivity to initial conditions



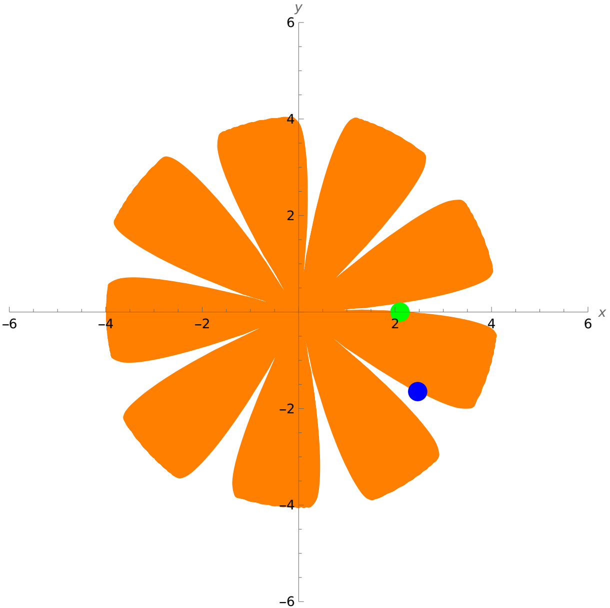
sensitivity to initial conditions



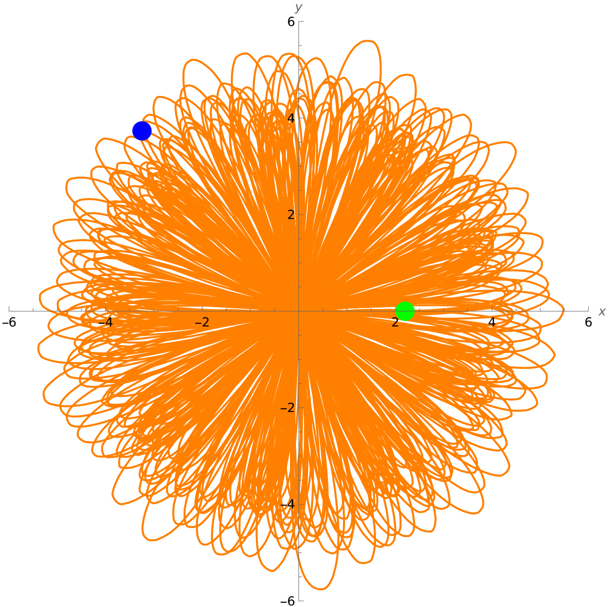
profile 1a: $\alpha = 0,\ A = 1,\ \lambda = (2\pi)^{-1},\ \rho_0 = 2,\ \dot{\rho} = -1,\ I = 1/10$



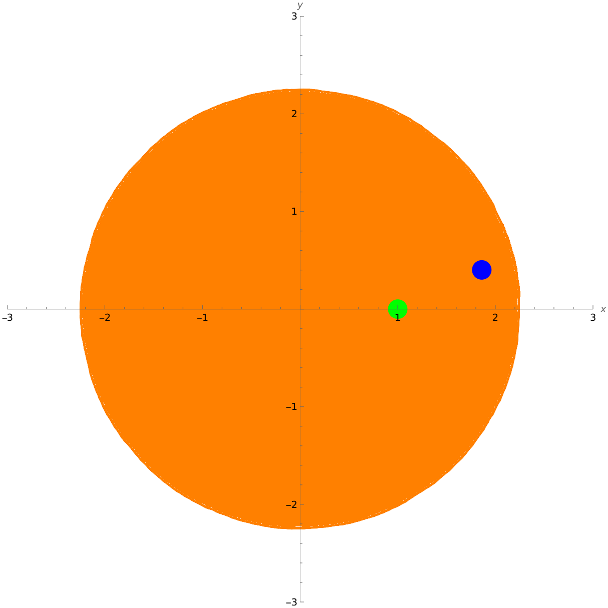
profile 1b: $\alpha = 0,\ A = 1,\ \lambda = (2\pi)^{-1},\ \rho_0 = 2.1,\ \dot{\rho} = -1,\ l = 1/10$



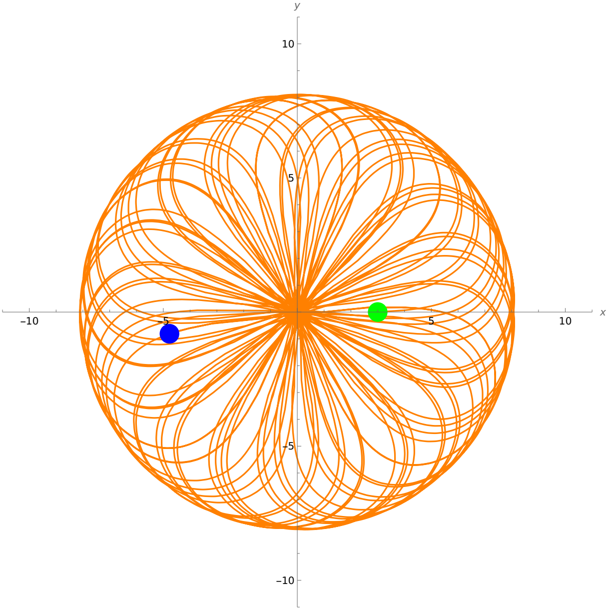
profile 1c: $\alpha = 0$, $A = 1$, $\lambda = (2\pi)^{-1}$, $\rho_0 = 2.2$, $\dot{\rho} = -1$, $I = 1/10$



profile 2: $\alpha = 0,\ A = 1,\ \lambda = (2\pi)^{-1},\ \rho_0 = 1,\ \dot{\rho} = 1,\ l = 1$



profile 3: $\alpha = 0$, $A = 1$, $\lambda = (2\pi)^{-1}$, $\rho_0 = 3$, $\dot{\rho} = 3$, $l = 1/3$



summary

- a "classical" behavior of test particles on scales $\gg \lambda$
- hypothesis: complex, possibly discrete in a probabilistic sense behavior on scales $\sim \lambda$
- relativistic precession of orbits
- antinodes attract test particles via Coulomb terms in the geodesic deviation equation (longitudinal higher order effect of nonlinear gravitational waves)
- sensitivity to initial conditions $\rightarrow$ chaos
- the role of polarization α?