SPHERICAL AND NON-SPHERICAL COLLAPSE OF A FINITE TWO-LAYER BODY

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Background

- Schwarzschild interior for incompressible fluid (1916).
- Oppenheimer-Snyder collapse of homogeneous body (1939)
- Lemaitre-Tolman-Bondi (LTB) solutions
- Slow-collapse approximation (Bondi, Misner +Sharp)
- Penrose Conjecture
- Joshi
- Lynden-Bell, Bicak (2017).
- Price-Cunningham-Moncrief (1980). Abbreviated P-C-M.
- Darmois Matching Conditions – ‘metric and derivatives’

Aims of this lecture

- Review of the spherical Oppenheimer-Snyder and Incompressible interiors.
- Convert Oppenheimer-Snyder interior into Gaussian Coordinates, and use this for both outer and inner layer.
- Attempt to perform Darmois matching between the two layers, but encounter a limitation.
- Instead, investigate the somewhat simpler problem of a static two-layer incompressible body, and describe what a slow-collapse through equilibrium states could look like.
- Outline how this could be extended to non-spherical cases.

Static Incompressible Interior

- The simplest description of a static interior with pressure.
- Incompressibility (μ=constant) is a good approximation for liquid-like behavior, e.g. Neutron Stars (but not ‘gas-like’).
- \+ (Ω = solid angle)
- Three field equations essentially amount to i) Hydrostatic Equilibrium, and ii) Conditions on dt and dr coefficients.
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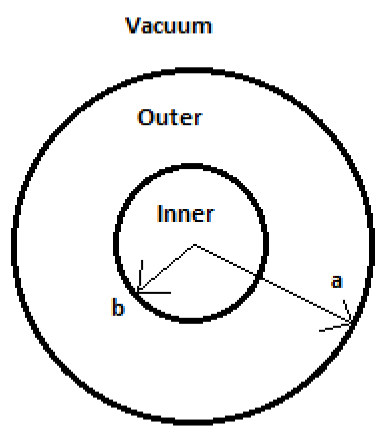
- Once solved, the Darmois matching conditions must be applied. They reduce to matching of pressure (which = 0 for vacuum), as well as metric coefficients and .

Oppenheimer-Snyder collapse

- The simplest interior solution that describes spherical collapse. Again μ=constant, but pressure is zero.
- A strong requirement is that different layers of the star *must* stay ‘in line’ with each other and do not overtake.
- Metric expressed in ‘synchronous’ comoving coordinates
- + (Ω=solid angle)
- Apply Darmois matching conditions to match with Schwarzschild exterior. Geodesic equation for boundary.
- Smooth onset of Horizon and Trapped Surface for O-S. But not so in LTB solutions!

Two-layer setup

- a (on diagram) is denoted in our equations. ‘Boundary’
- b (on diagram) is denoted in our equations. ‘Interface’



Two-layer OS collapse

- It is more useful to use Gaussian (boundary) Coordinates for matching between two dust layers of different densities.
- Create alternative radial coordinate: Let
- O-S metric becomes +
- The coordinates can now be ‘continued’ across layers.
- Outer metric +
- Inner metric +
- and bare their respective scale factors.

Two-layer OS collapse

- In the Gaussian Coordinate system (only), the Darmois Matching Conditions reduce to the matching of the metric components and their normal derivatives.
- Actually, theand coefficients are already matched, so only the angular metric coefficients and their R derivatives need to be matched at the interface .
- Matching of outer layer to the vacuum proceeds in the usual manner.
- Matching both angular metric coefficients at gives
- = b

Two-layer OS collapse

- While matching the normal derivatives results in
- 2b = 2a
- The above expression can be simplified using earlier result
- = b
- to produce b
- So the two layers must collapse at the same rate? The combined entity then behaves like a single entity? Original integration of the O-S equation may need to be revisited!
- Instead, we now focus on a two-layer incompressible fluid.

Two-layer Incompressible

- Pursue a similar solution procedure to Stephani book (2004):
- Inner layer metric will be the same as the original metric:
- +
- While the metric for the outer layer will have an extra piece for both λ and ν (and the pressure):
- Field Equation for λ (denotes r-derivative):
- , (μ = mass density), let
- Integrate, , g is a constant. Typically, g would represent a diverging term at r=0, but we shall keep it.

Two-layer Incompressible

- To deal with the Field Equation for ν,
- ,   where
- Let . End up with following ode (RHS extra σ!):

- This o.d.e. can be solved iff g=0

- But cannot solve the g ≠ 0 case in closed form!

Two-layer Incompressible

- One possible way forward is to assume that the quantity g is small compared to the other coefficients in the ode.
- By comparing with the ‘mass function’ used in other treatments of the zero-order case, our new approximation would correspond to an inner boundary that is close to r=0.
- Indeed, the quantity in looks suspiciously like a ‘mass’ term, possibly the total mass of the inner layer.
- Therefore, let , and
- and B now correspond to the well-known zero-order case describing a single layer.

Two-layer Incompressible

- It has now been established that under our new approximation, the inner layer must have a small (coordinate) radius compared to the outer layer.
- Inner layer is effectively a core, or ‘dweller in the depths’.
- Solve the first order equation for the outer layer to obtain:
- ,

- k, A, B, D are zero-order constants. E, b, g are first order.

Two-layer Incompressible

- On the other hand, the inner core solution is basically of the same form as the single-layer case.
- ,
-
- ρ is the (rest) mass density of the inner core, while β and δ are integration constants.
- Apply Darmois conditions, and match both metric components and the pressure at the interface ().

Two-layer Incompressible

- Matching must take place in two separate stages:
- i) The zero-order outer layer, with constants B and D, must match as usual to the vacuum Schwarzschild, with constant m. At this stage the inner core is not considered.
- ii) Once the above has taken place, the inner core with constants β and δ must match to the outer layer with constant g. Then, the outer layer with remaining constants E and b must match to the ‘perturbed’ Schwarzschild exterior, the latter possessing a constant Δm.
- Obtain the inner interface and the external boundary .

Two-layer Incompressible

- Our result must be converted so that it can be used as a ‘initial value matching’ for the Oppenheimer-Snyder collapse. Once again use the hybrid Synchronous-Gaussian form.
- Create alternative radial coordinate: Let and substitute . Similarly, .
- dt conversion is simple, but find that the dr integral cannot be evaluated in closed form; for small g the integral can be Taylor-expanded about the single-layer case, and then inverted for r.
- where

Implications

- Having obtained our static incompressible metrics for both the outer and inner layers, we can ask the following question: For what value of does the central pressure diverge?
- Even if is still outside r=2M (and also the Buchdahl limit), would it allow the density of the inner core to be *above* the critical mass density, provided that the average mass density over both layers remains below the critical density?
- Then consider a slow collapse of the combined body, such that it passes through a sequence of static states. But likely to cease being valid when the central pressure gets too high!
- Could the analysis for the static case shed new clues on the Oppenheimer-Snyder collapse?

Non-spherical Collapse

- Price-Cunningham-Moncrief (1980) perturbed the O-S solution to second-order in the rotation speed. Darmois Matching Conditions proved difficult in closed form.
- Provided that the earlier issue for spherical collapse can be resolved, we could imagine perturbing the two dust layers with rotation, and Darmois matching them using the earlier (perturbed) Gaussian-Synchronous coordinates instead?
- If the inner layer does produce a naked singularity, then the emitted Gravitational Waves will carry this information.
- Convert vacuum metric to Bondi-Sachs coordinates, which have invariant meaning (c.f. stationary Weyl coordinates).

Tasks still remaining

- Investigate slow collapse of (quasi) incompressible body.
- Resolve the issue in the Oppenheimer-Snyder collapse, and apply the two-layer setup to P-C-M non-Spherical collapse. Express vacuum in Bondi-Sachs form.
- On a somewhat different note, for a **single** body investigate the slow collapse of the quasi-incompressible interior, then match it at late times to constant pressure O-S collapse. Compare the result against the O-S dust collapse matched at late times to O-S pressure collapse. Then…
- …for late-time non-Spherical P-C-M, match dust to pressure.
- Papers to be published!