Critical relaxation in AdS/CFT

Mario Flory

JAGIELLONIAN UNIVERSITY
IN KRAKOW

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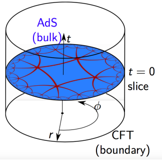
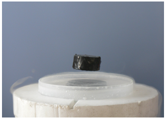
Overview

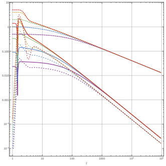
- What is AdS/CFT?



- Critical relaxation in Holographic Superconductors



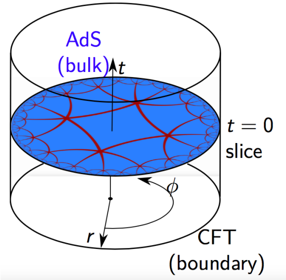
- Boundary model



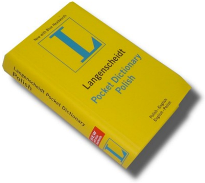
What is AdS/CFT?

The AdS/CFT correspondence:

The physics of certain conformal field theories (CFT, "boundary") can be equivalently encoded in the gravitational physics of a higher dimensional Anti-de Sitter space (AdS, "bulk") [Maldacena 1999].



Via the *holographic dictionary*, mathematical problems of the boundary theory may be translated into a problem in the bulk theory (or vice versa), where a solution may be easier, or where a new perspective on the problem may emerge.



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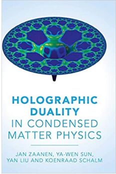

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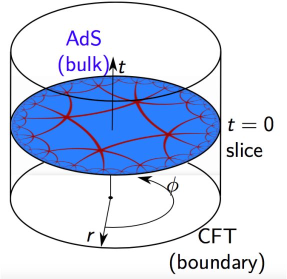
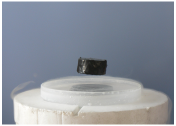


Example for the *holographic dictionary*:

Bulk geometry $\leftrightarrow$ Field theory state

Black Hole, temperature T $\leftrightarrow$ Finite temperature T

Scalar hair $\varphi \neq 0$ $\leftrightarrow$ Superconducting state, oder parameter $\Psi \neq 0$

 $\leftrightarrow$ 

Bulk theory of a *Holographic Superconductor* [Hartnoll et al. 2008b] [Hartnoll et al. 2008a]:

$$S = \int d^4x \sqrt{-g} \left[R - 2\Lambda + \frac{1}{2\kappa^2} \left(-\frac{1}{4\,q^2} F_{\mu\nu} F^{\mu\nu} - |\mathcal{D}\varphi|^2 - m^2 |\varphi|^2 \right) \right]$$

- Cosmological constant $\Lambda < 0$; $\rightarrow$ asympt. AdS solutions:

$$ds^2 = \frac{1}{u^2} \left[-f(u)\, dt^2 - 2\, dt du + dx^2 + dy^2 \right].$$

  Asymptotic boundary at $u = 0$.

- Complex bulk scalar φ. Near boundary: $\varphi = \Psi u^2 + ...$
- Bulk $U(1)$-gauge field A_μ. Near boundary: $A_t = \mathcal{A}_t + \rho u + \ldots$

Phenomenology:

- For $\rho < \rho_c$, AdS-RN black hole is stable ($\varphi = 0$).
- Second order phase transition at $\rho = \rho_c$.
- Scalar hair forms for $\rho > \rho_c$, $\varphi \neq 0$.

Critical relaxation

in

Holographic Superconductors

Quench: Start with equilibrium state $\rho > \rho_c$ ($\Rightarrow \varphi \neq 0$), then artificially switch it to $\rho = \rho_c$ suddenly.

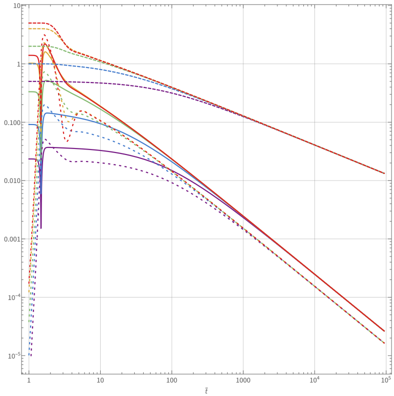
Observation: Universally,

$$\phi(t) \approx \frac{4.07}{\sqrt{t+\delta t}}$$

$$\dot{\psi}(t) - (\mathcal{A}_t(t) - \rho_c) \approx \frac{0.93}{t+\delta t}$$

for $t \gg 1$ with $\Psi = \phi e^{i\psi}$.

Power law instead of QNM-like exponential falloff! $\dot{\psi} \sim 1/t$ means $\psi(t) \sim \log(t)$, i.e. log-periodic oscillations of complex phase similar to [Hirschmann and Eardley 1995].



$|\mathcal{A}_t(t) - \rho_c|$ (solid lines), $|\Psi| \equiv \phi(t)$ (dashed lines), and $|\dot{\psi}(t)|$ (dotted lines) for multiple exactly critical quenches.

Boundary model

Via AdS/CFT, this corresponds to the physics of a superconductor (or superfluid), where the dynamics of Ψ should be described by something like a Ginzburg-Landau (or Gross–Pitaevskii) equation [Tsuneto et al. 1998].

We propose the phenomenological equation

$$\left[\partial_t - iC_1\left(\mathcal{A}_t(t) - \rho + C_5|\Psi(t)|^2\right)\right]\Psi(t) \equiv -(C_2 + iC_3)\left[|\Psi(t)|^2 - C_4(\rho - \rho_c)\right]\Psi(t)$$

with parameters $C_1 = 1$ and

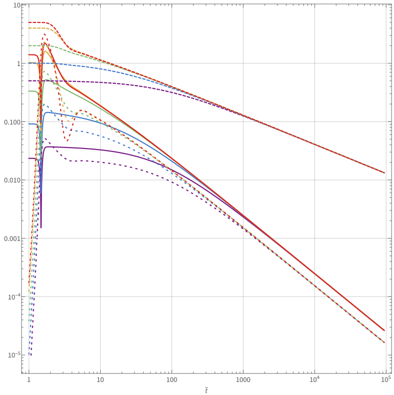
$$\begin{aligned} C_2 &\approx 0.03018 & C_3 &\approx 0.09308 \\ C_4 &\approx 4.09192 & C_5 &\approx 0.14967 \end{aligned}$$

determined by fitting to static behaviour and exponential falloff (at $t \gg 1$) after near-critical quenches.

Exactly critical solutions:

$$\phi(t) = \frac{1}{\sqrt{2C_2 t + \frac{1}{\phi_0^2}}} \approx \frac{4.07}{t^{1/2}} + \ldots$$

$$\dot{\psi} - C_1(\mathcal{A}_t - \rho_c)$$
$$= \frac{C_1 C_5 + C_3}{2C_2 t + \frac{1}{\phi_0^2}} \approx \frac{0.94}{t} + \ldots .$$



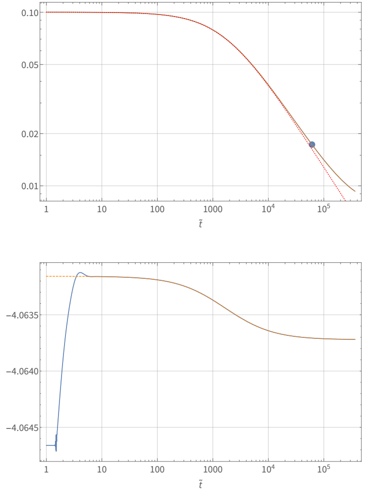
$|\mathcal{A}_t(t) - \rho_c|$ (solid lines), $|\Psi| \equiv \phi(t)$ (dashed lines), and $|\dot{\psi}(t)|$ (dotted lines) for multiple exactly critical quenches.

Near-critical solutions

$$\phi(t) = \sqrt{\frac{C_4(\rho-\rho_c)}{1-\left(1-\frac{C_4(\rho-\rho_c)}{\phi_0^2}\right)e^{-2C_2C_4t(\rho-\rho_c)}}}$$

describe the system not just at late, but already at early and intermediate times.

For early times $t < \frac{1}{\rho-\rho_c}$, this exact solution is well approximated by the critical solution ($\sim$ power law falloff).



Numerical (blue) and analytical (orange) results for a near-critical quench. Top: $\phi(t)$, bottom: $\dot{\psi}(t) - C_1\mathcal{A}_t(t)$.

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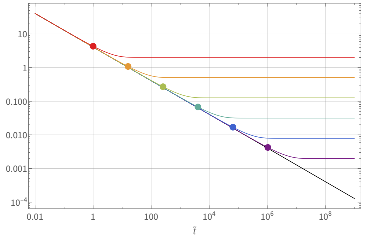
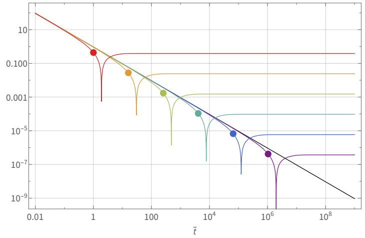




Top: $\phi(t)$, bottom: $|\dot{\psi} - C_1(\mathcal{A}_t - \rho_c)|$ for varying values of ρ_{final}.

Summary

- AdS/CFT allows us to apply methods of gravitational physics to problems in QFT and condensed matter theory.
- Hairy black holes in AdS correspond to (holographic) superconductors.
- Exactly at the critical point where scalar hair forms (phase transition), we find the system relaxes after perturbations in a power-law manner.
- This behaviour can be reproduced by a phenomenological model with high precision.
- Even near-critical quenches show approximate power-law behaviour at intermediate time scales.

New SONATA BIS grant at Jagiellonian U. in Kraków

Timeframe: September 2022 - August 2027

Topics:

- Applications of AdS/CFT to QFT and condensed matter theory
- The nature of AdS/CFT and the holographic principle
- Connections between AdS/CFT and Quantum Information Theory

Personnel:

- *1 Postdoc to be hired soon*
- *1 PhD to be hired soon*
- 1 PhD to be hired next year

Contact:

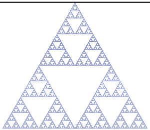
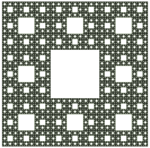
mflory@th.if.uj.edu.pl

Thank you very much for your attention

Back-up slides...

Discrete scale invariance (DSI)

Discrete scale invariance (DSI) is the symmetry underlying the self-similarity of fractal structures w.r.t scalings by a preferred factor.

Fractal	Name	Hausdorff dimension, Preferred scaling factor
	Sierpinski triangle	$\frac{\log(3)}{\log(2)}$, 2
	Sierpinski carpet	$\frac{\log(8)}{\log(3)}$, 3

Discrete scale invariance (DSI)

Scale invariance (SI):

$$\mathcal{O}_{SI}(x) = \mu(\lambda)\mathcal{O}_{SI}(\lambda x)$$

for *any* $\lambda \in \mathbb{R}^+$ and some $\mu(\lambda) = \lambda^\alpha$.

Discrete scale invariance (DSI):

$$\mathcal{O}_{DSI}(x) = \mu(\lambda_0)\mathcal{O}_{DSI}(\lambda_0 x)$$

only for a specific scale $\lambda_0 \in \mathbb{R}^+$ and the related scales $\lambda_0^m,\ m \in \mathbb{Z}$. E.g.:

$$\mathcal{O}_{DSI}(x) \propto x^\alpha, \quad \alpha = -\frac{\log \mu}{\log \lambda_0} + i\frac{2\pi n}{\log \lambda_0}, \quad n \in \mathbb{Z}.$$

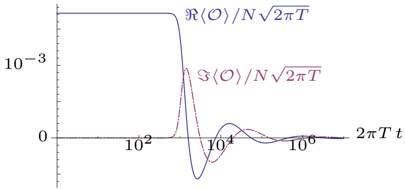
For $n \neq 0$, we hence find *complex critical exponents* and, because of

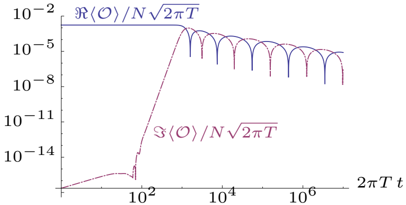
$$\mathcal{O}_{DSI}(x) \propto x^\alpha = x^{\Re(\alpha)}\left(\cos\left[\Im(\alpha)\log(x)\right] + i\sin\left[\Im(\alpha)\log(x)\right]\right),$$

log-periodic oscillations.

Discrete scale invariance (DSI), Example

Oscillations (after quench) in holographic Kondo model [Erdmenger et al. 2017]:





A log-linear and log-log plot of the boundary quantity $\langle\mathcal{O}\rangle(t)$. Bulk fields show log-periodicity in time:

$$\phi(t,z) \approx t^{\upsilon_I} \cos(\upsilon_R \log(2\pi T t))\tilde{\phi}(z), \text{ with } \upsilon_I \approx -0.5,\ \upsilon_R \approx 1.5.$$

Discrete scale invariance (DSI)

DSI plays a role in:

- Fractals, stock markets and earthquakes [Sornette 1998]
- Black hole formation [Choptuik 1993], [Hirschmann and Eardley 1995]
- The Efimov effect [Hammer and Platter 2011]
- Quantum Gravity [Calcagni 2017]
- AdS/CMT models: [Liu et al.; Faulkner et al. 2011; 2011], [Hartnoll et al. 2016], [Erdmenger et al. 2017], [Brattan et al. 2017], [Ammon et al. 2018]
- Cyclic RG flows, see e.g. [Wilson 1971] [Bulycheva and Gorsky 2014]

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