Early evolution of fully convective stars in scalar-tensor gravity

In collaboration with Aneta Wojnar (arXiv:2206.04464)

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The 8th Conference of the Polish Society on Relativity

Why Modify Gravity

General Relativity (GR) is not complete:

- It does not explain the late accelerated expansion of the Universe (dark energy) and the discrepancy between the observed and the predicted galaxy rotation curves (dark matter);
- It lacks a quantum counterpart;
- It may not be correct at all scales.

Gravity needs to be modified:

- Consider higher curvature term (R^2, $R_{\mu\nu}R^{\mu\nu}$);
- Include of extra fields (scalar, vector, etc);
- Consider higher dimensions;

Tests of Modified Gravity theories

- Hydrogen Burning [1];
- Cooling process of brown dwarfs [2]
- Helioseismology[3];
- Seismic data [4];
- ...

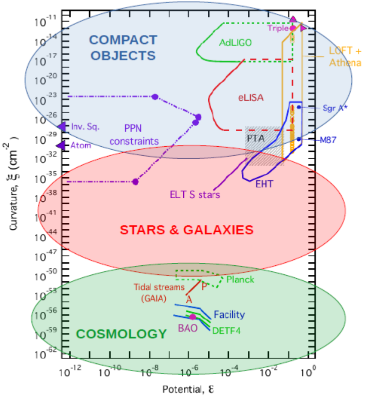


Figure: Baker, Psaltis & Skordis (2015)

[1] J. Sakstein, Phys. Rev. D 92 (2015) 124045; J. Sakstein, Phys. Rev. Lett. 115 (2015) 201101

[2] M. Benito and A. Wojnar, Phys. Rev. D 103 (2021) 064032.

[3] I. D. Saltas and I. Lopes, Phys. Rev. Lett. 123 (2019) 091103.

[4] A. Kozak and A. Wojnar, Phys. Rev. D 104 (2021) 084097.

Low-mass stars ($M \lesssim 0.5M_\odot$) can be treated as an object with uniform composition and therefore are perfect objects to study gravitational physics. Some features of pre-main sequence low-mass stars are dependent on the gravitational model:

- Minimum main sequence mass (MMSM) [5];
- Hayashi tracks and upper mass limit of fully convective stars on the Main Sequence [6];
- Light elements' abundances (hydrogen, **lithium** [7]);
- ...

[5] J. Sakstein, Phys. Rev. Lett. 115 (2015) 201101

[6] A. Wojnar, Phys. Rev. D 102, 124045 (2020)

[7] A. Wojnar, Phys. Rev. D 103, 044037 (2021)

Lane-Emden equation

The most general scalar-tensor theory having second order field equations is known as **Horndeski Gravity**.
The hydrostatic equilibrium equation in Horndeski gravity is modified as [8] ($-10^{-3} < \Upsilon < 5 \times 10^{-4}$ [9])

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{G_{\mathrm{N}} M(r)\rho(r)}{r^2} - \frac{\Upsilon}{4} G_{\mathrm{N}} \rho(r) M''(r). \qquad (1)$$

Low-mass stars can be modeled by a polytropic equation $P = K\rho^{\gamma}$, from which we can obtain the Modified Lane-Endem equation:

$$\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left[\left(1 + \frac{n}{4}\Upsilon\xi^2\theta^{n-1}\right)\xi^2\frac{\mathrm{d}\theta}{\mathrm{d}\xi} + \frac{\Upsilon}{2}\xi^3\theta^n\right] = -\theta^n. \qquad (2)$$

[8] K. Koyama and J. Sakstein, Phys.Rev. D91 (2015) 124066.

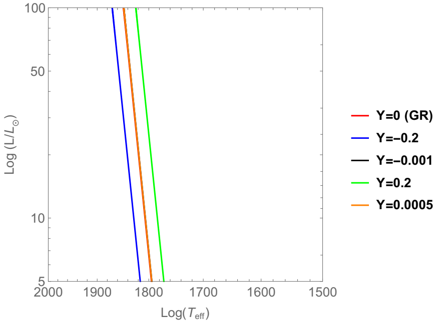
[9] I. D. Saltas, J. Christensen-Dalsgaard, arXiv:2205.14134 (2022)

Hayashi tracks

The Hayashi tracks are shifted[10]

$$T_{\text{ph}} = 2487.77\mu^{\frac{13}{51}} \left(\frac{L}{L_\odot}\right)^{\frac{1}{102}} \left(\frac{M}{M_\odot}\right)^{\frac{7}{51}} \left(\frac{\left(\frac{\left(1+\frac{\Upsilon}{2}\right)}{Z}\right)^{\frac{4}{3}}}{\xi_R^5\sqrt{-\theta'}}\right)^{\frac{1}{17}} \text{K}, \tag{3}$$

where $L_\odot$ and $M_\odot$ are the solar luminosity and mass, respectively.



[10] D.A. Gomes, A. Wojnar, arXiv:2206.04464 (2022).

Lithium abundance in Horndeski Gravity

Lithium depletion method:

- It is one of the most reliable method for age determination of young globular clusters;
- It is used to distinguish brown dwarfs from true stars;
- It is used to calibrate other age determination techniques;
- It depends on the gravitational model [11];
- It may contribute to the solution of the cosmological lithium problem [12].

[11] A. Wojnar, Phys. Rev. D 103 (2021) 044037; D. Gomes and A. Wojnar, Early evolution of fully convective stars in scalar-tensor gravity (in preparation)

[12] A. Kozak, M. Saal, A. Wojnar (in preparation)

For a star with mass M and hydrogen fraction X we can write the depletion rate as

$$M\frac{\mathrm{d}f}{\mathrm{d}t} = -\frac{Xf}{m_H}\int_0^M \rho\langle\sigma v\rangle dM, \tag{4}$$

where the non-resonant reaction rate for the temperature range $T < 6\times10^6\mathrm{K}$ is given by

$$N_A\langle\sigma v\rangle = Sf_{\mathrm{scr}}T_6^{-2/3}\exp\left[-aT_6^{-\frac{1}{3}}\right]\frac{\mathrm{cm}^3}{\mathrm{s\ g}}, \tag{5}$$

where $T_6 \equiv T/10^6\mathrm{K}$ and f_{scr} is the screening correction factor. $S = 7.2\times10^{10}$ and $a = 84.72$ for the reaction ${}^7\mathrm{Li}(p,\alpha)\,{}^4\mathrm{He}$.

For Horndeski gravity, the lithium depletion rate is[13]

$$\mathcal{F} = \ln\frac{f_0}{f} = 5.6\times 10^{14}\,T_{eff}^{-4}\left(\frac{X}{0.7}\right)\left(\frac{0.1M_\odot}{M}\right)^3\left(\frac{0.6}{\mu_{eff}}\right)^6 S f_{scr}\,a^{16} \times g(u)\left(1+\frac{3\Upsilon}{2}\right)^{-3/2}\frac{\xi_R^7(-\theta'(\xi_R))^2}{\delta^2}, \quad (6)$$

where $u \equiv aT_{c6}^{-1/3}$ and $g(u) = u^{-37/2}e^{-u} - 29\Gamma(-37/2, u)$. For different values of the parameter Υ, we have

Υ	$T_c/10^6K$	$t(Myr)$	$R/R_\odot$	$L/L_\odot$
-2×10^{-1}	3.00650	17.6690	1.40157	0.264446
-10^{-3}	3.05366	13.1133	1.39486	0.261920
-10^{-4}	3.05385	13.0972	1.39483	0.261909
0 (GR)	3.05388	13.0951	1.39482	0.261907
10^{-4}	3.05390	13.0935	1.39482	0.261906
5×10^{-4}	3.05398	13.0863	1.39481	0.261901
2×10^{-1}	3.09503	10.1109	1.38787	0.259303

[13] D.A. Gomes, A. Wojnar, arXiv:2206.04464 (2022).

Schwarzschild criterion

The heat transport with respect to radiative/conductive process is given by

$$\frac{dp}{dr} = -\frac{G_N M(r)}{r^2}\rho(r)\left(1 + \frac{\Upsilon}{2}\right). \tag{7}$$

Using the modified hydrostatic equilibrium equation, together with Kramer's law and Boltzmann law, we can show that

$$\begin{aligned} \nabla_{rad} &= 2.79 \times 10^{-148} \frac{k_B^{8.5} N_A^{8.5} \xi_R^{10.83} (-\theta'(\xi_R))^{2.167} \kappa_o L^{1.25}}{bcG^{7.5} \delta_{3/2}^{2.33} \mu^{8.5} M_{-1}^{5.5} T_{\text{eff}}} \\ &\times \left(1 + \frac{\Upsilon}{2}\right)^{-1}, \end{aligned} \tag{8}$$

where κ_0 denotes the total opacity, which can be bound-free and free-free.

The Schwarzschild criterion is given by:

$$\nabla_{\text{rad}} \leq \nabla_{\text{ad}} \quad \text{pure diffusive radiative or conductive transport}$$
$$\nabla_{\text{rad}} > \nabla_{\text{ad}} \quad \text{adiabatic convection is present locally.}$$

In the case of an ideal gas model, the adiabatic gradient is a constant, that is, $\nabla_{\text{ad}} = 0.4$. The luminosity of hydrogen burning in Horndeski gravity is[14]

$$\frac{L_H}{5.2 \times 10^6 L_\odot} = \frac{\delta_{3/2}^{5.487} M_{-1}^{11.973} \eta^{10.15}}{\omega_{3/2} \gamma_{3/2}^{16.46} \left(1 + \frac{3\Upsilon}{2}\right)^{3/2} (\eta + \alpha)^{16.46}}. \qquad (9)$$

[14] J. Sakstein, Phys. Rev. D 92 (2015) 124045; J. Sakstein, Phys. Rev. Lett. 115 (2015) 201101

We can relate the modified mass M^{mod} with the GR mass M^{GR} as

$$
\begin{aligned}
M^{\text{mod}} &= \left(\frac{\gamma_{3/2}}{\gamma_{3/2}^{\text{GR}}}\right)^{2.17} \left(\frac{\delta_{3/2}^{\text{GR}}}{\delta_{3/2}}\right)^{0.48} \left(\frac{\theta'^{\text{GR}}}{\theta'}\right)^{0.23} \left(\frac{\xi_R^{\text{GR}}}{\xi_R}\right)^{1.14} \\
&\times \left(\frac{\omega}{\omega^{\text{GR}}}\right)^{0.13} \left(1+\frac{\Upsilon}{2}\right)^{0.11} \left(1+\frac{3\Upsilon}{2}\right)^{0.2} M^{\text{GR}}
\end{aligned}
\tag{10}
$$

Conclusions

- The Hayashi track method can be used to bound the values of the parameter Υ;
- As an age dependent quantity, lithium abundance allows the age determination of young clusters and it is employed in the calibration of other age determination techniques;
- It is a gravitational model dependent quantity;
- It may explain the existence of "too old" stars;
- The Schwarzschid criterion is also affected by the ST modifications.

Thank you!