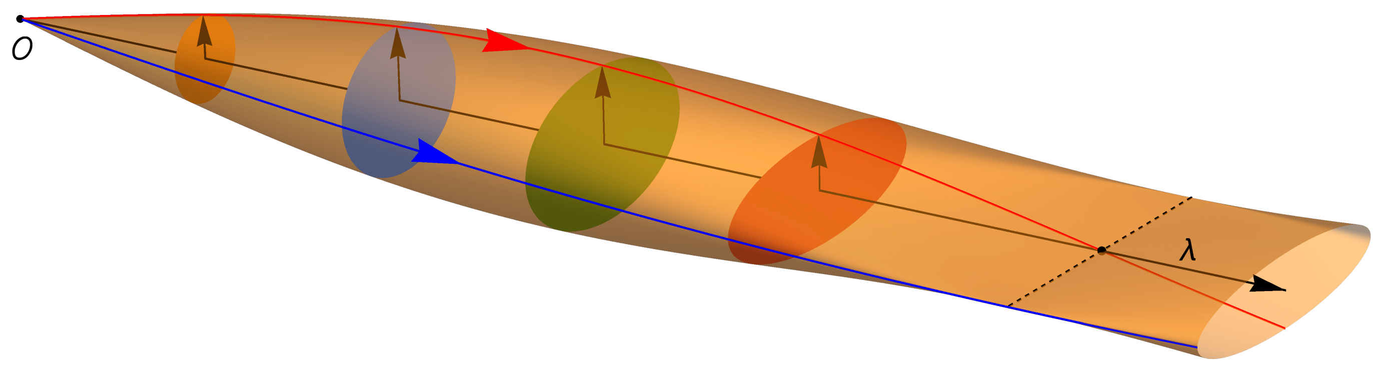
Optical distance measures in general relativity



Mikołaj Korzyński, Julius Serbenta

Centre for Theoretical Physics,
Polish Academy of Sciences
Warsaw

8th POTOR Meeting, IMPAN Warsaw, September 2022

Motivation

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Geometric optics in GR beyond the Sachs formalism, beyond a single emission and observation point

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Beyond the standard lensing/weak lensing formalism - exact formulas

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Based on papers:

M. Grasso, MK, J. Serbenta, *Geometric optics in general relativity using bilocal operators,* Phys. Rev. D **99**, 064038 (2019)

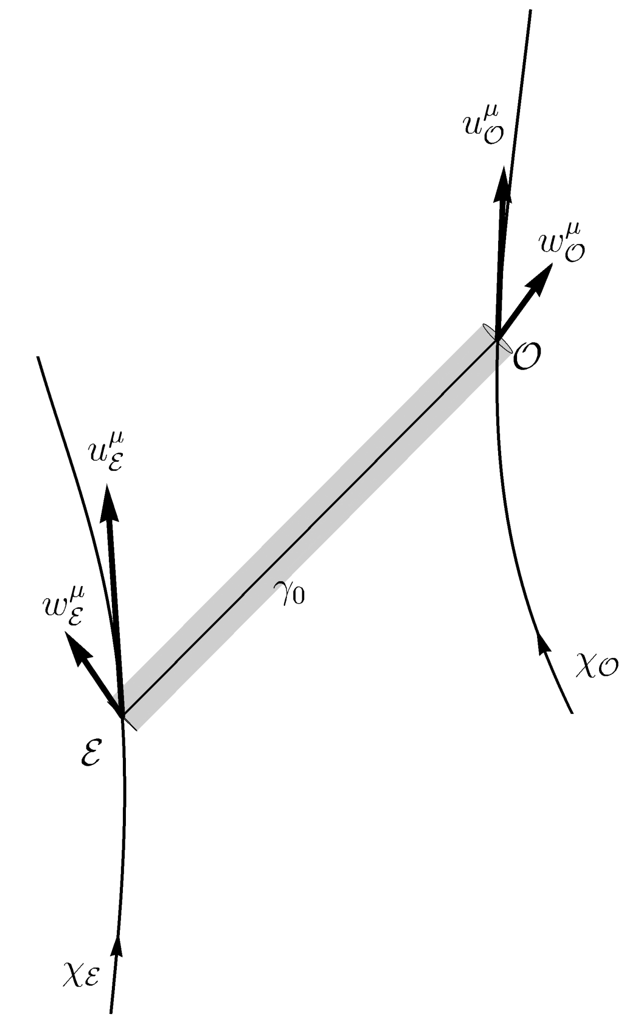
MK, E. Villa, *Geometric optics in relativistic cosmology: New formulation and a new observable,* Phys. Rev. D **101**, 063506 (2020)

MK, J. Serbenta, *Testing the null energy condition with precise distant measurements,* Phys. Rev. D **105**, 084017 (2022)

Distance measures in GR

Distance measure along a null geodesic

$$D \equiv D(\mathscr{E}, \mathscr{O}, \gamma_0, u_{\mathscr{O}}, u_{\mathscr{E}})$$



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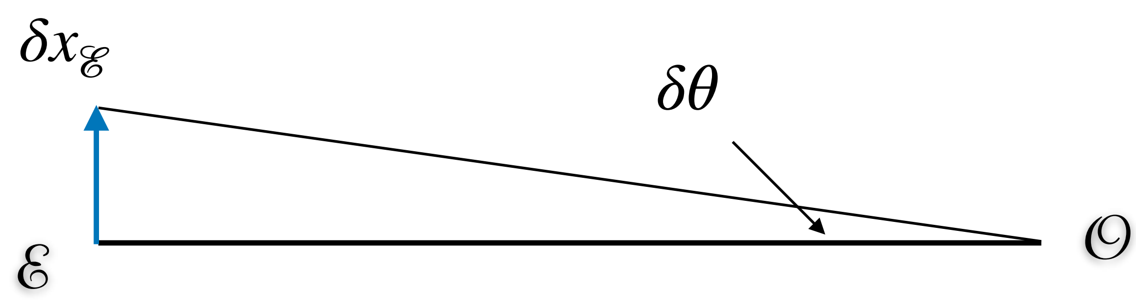
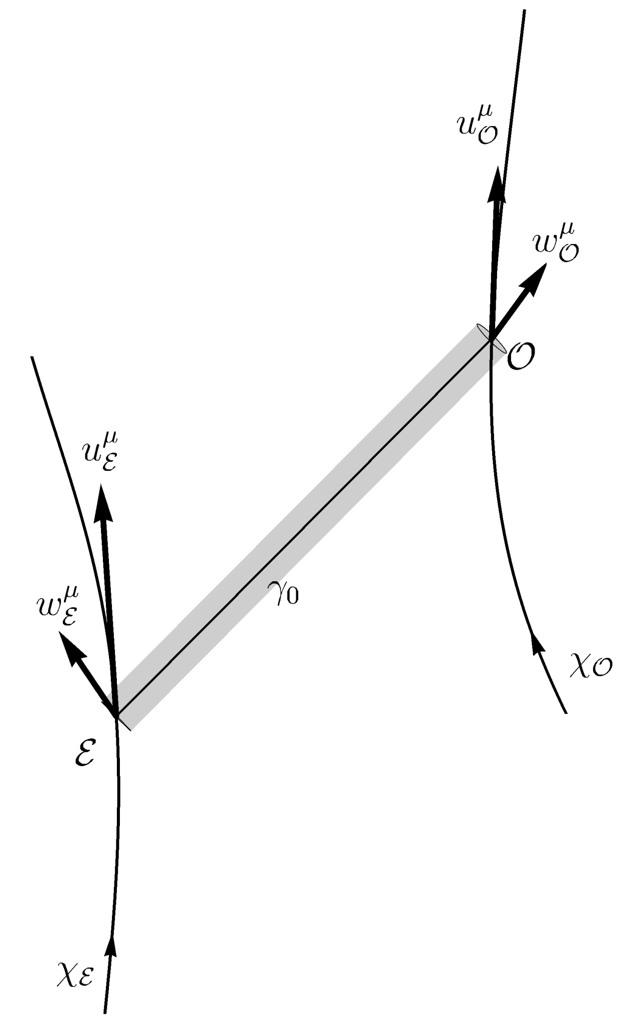
Angular diameter distance

flat spacetime

$$\delta\theta = \frac{\delta x_{\mathscr{E}}}{D}$$





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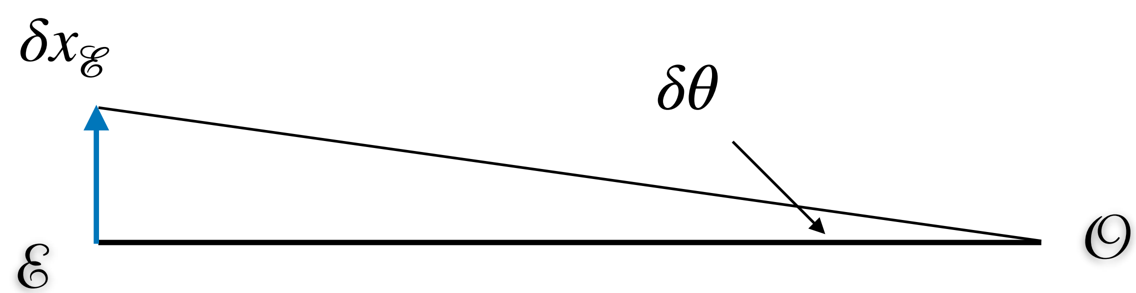
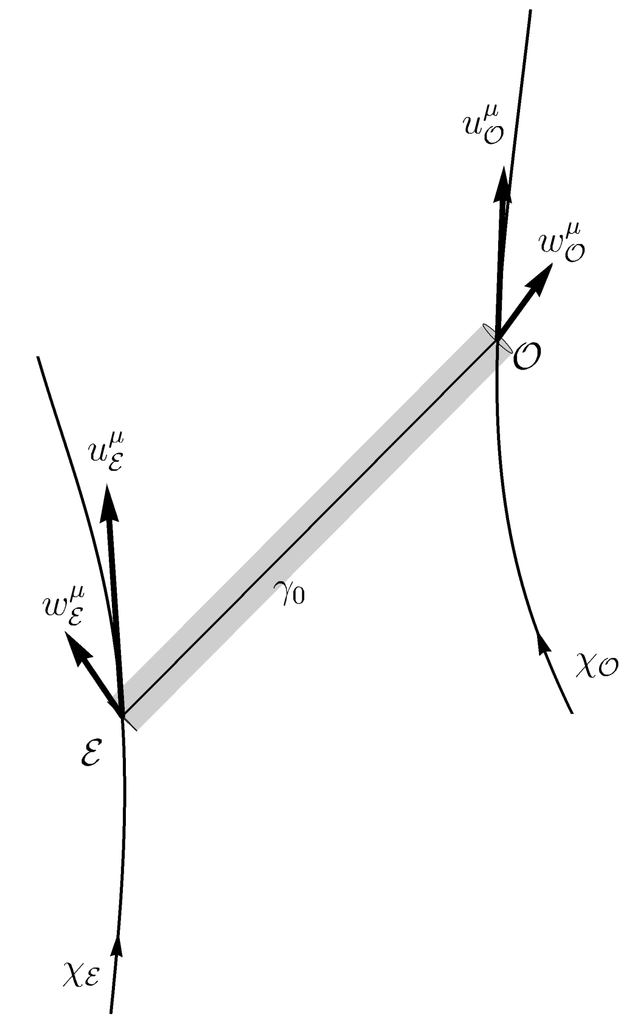




general spacetime

$$\delta\theta^A = M^A{}_B\, \delta x^B_{\mathscr{E}}$$

$$D_{ang} = \left|\det M^A{}_B\right|^{-1/2} = \left|\frac{A_{\mathscr{E}}}{\Omega_{\mathscr{O}}}\right|^{1/2}$$

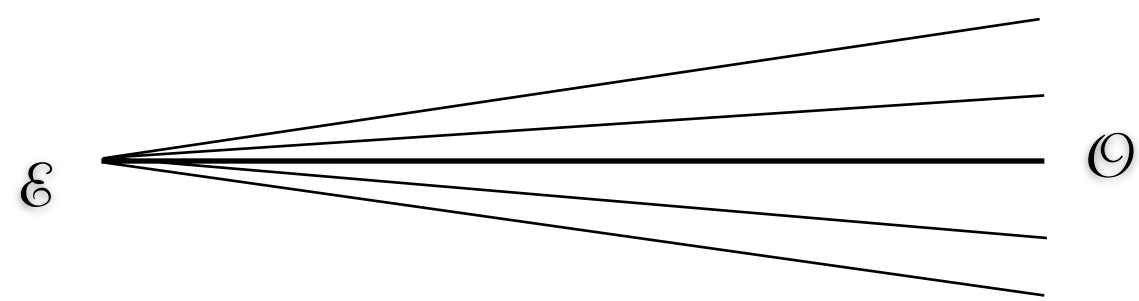
$$D_{ang} \equiv D_{ang}(\mathscr{E}, \mathscr{O}, \gamma_0, u_{\mathscr{O}})$$

Distance measures in GR

Luminosity distance

flat spacetime, no relative motion

$$F = \frac{I}{4\pi D^2}$$



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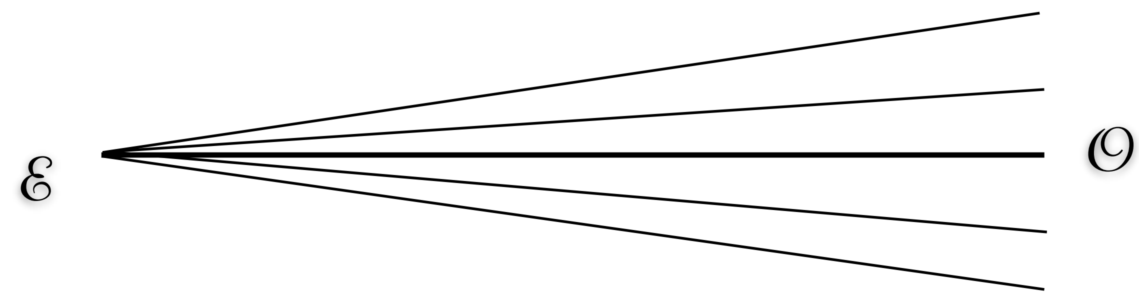

general spacetime

$$D_{lum} = \sqrt{\frac{I}{4\pi F}} \qquad D_{lum} \equiv D_{lum}(\mathscr{E}, \mathscr{O}, \gamma_0, u_{\mathscr{O}}, u_{\mathscr{E}})$$

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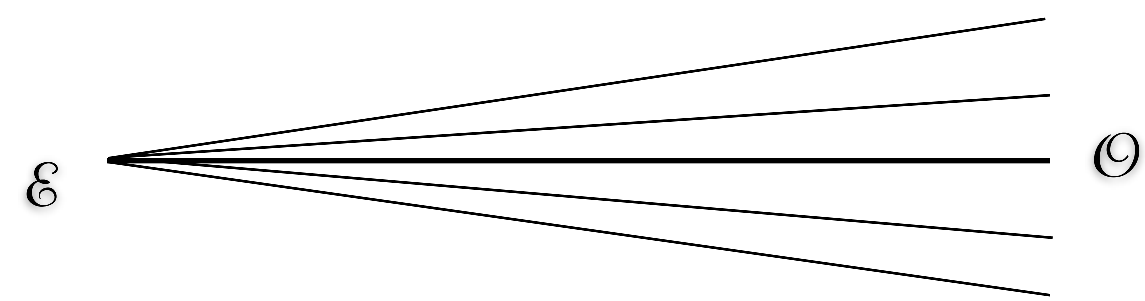

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Related to the angular diameter distance via the Etherington's reciprocity relation

$$D_{lum} = (1+z)^2 \, D_{ang}$$

[Etherington 1933, Penrose 1966, … Uzun 2019]

Angular diameter distance

Expressing the distance measures using curvature

Main tool: geodesic deviation equation around a null geodesic

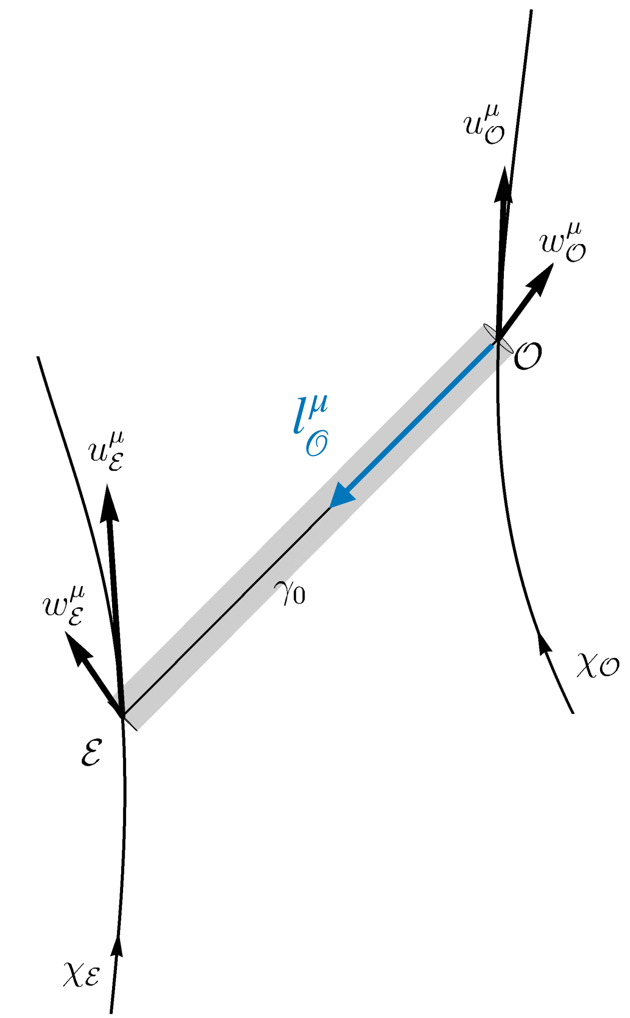
$$M^A{}_B = (l_{\mathcal{O}\,\mu}\, u^\mu_{\mathcal{O}})\,\mathcal{D}^{-1\,A}{}_B$$

Jacobi matrix

$$\ddot{\mathcal{D}}^A{}_B - R^A{}_{llC}\,\mathcal{D}^C{}_B = 0$$

$$\mathcal{D}^A{}_B(\lambda_{\mathcal{O}}) = 0$$

$$\dot{\mathcal{D}}^A{}_B(\mathcal{O}) = \delta^A{}_B$$



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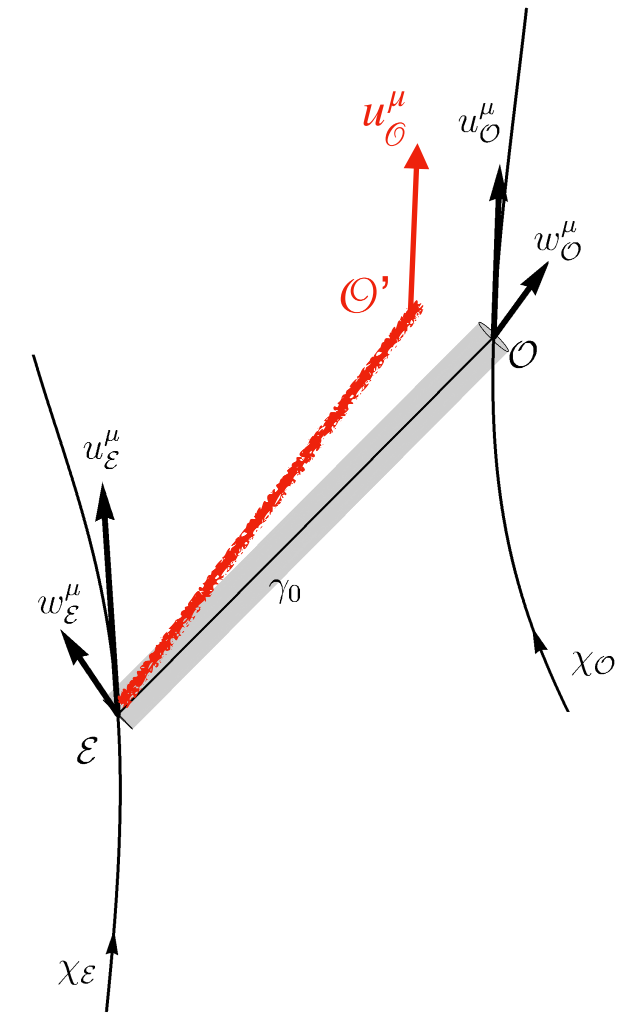
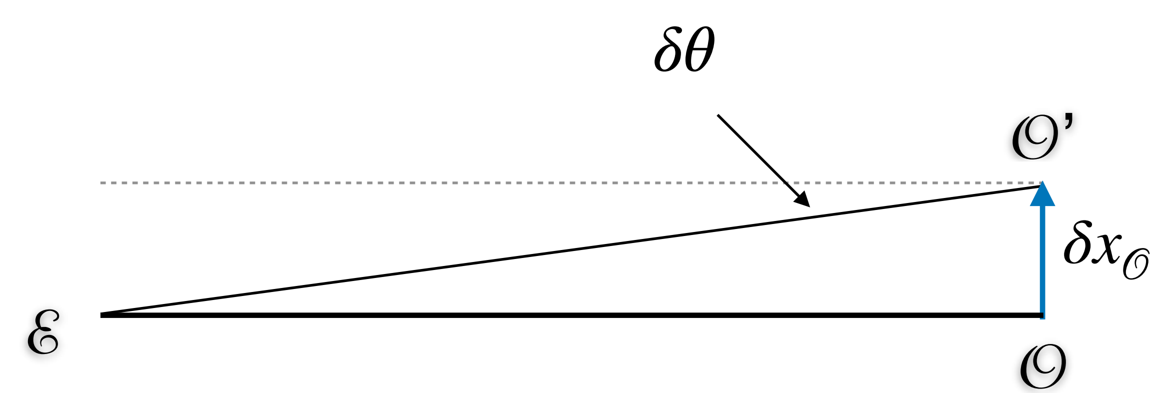
$$D_{ang} = (l_{\mathcal{O}\,\mu}\, u^\mu_{\mathcal{O}})^{-1}\left|\det \mathcal{D}^A{}_B\right|^{1/2}$$

Parallax in GR

Parallax effect - difference in apparent position of a light source between two nearby observers [Grasso, MK, Serbenta 2019]





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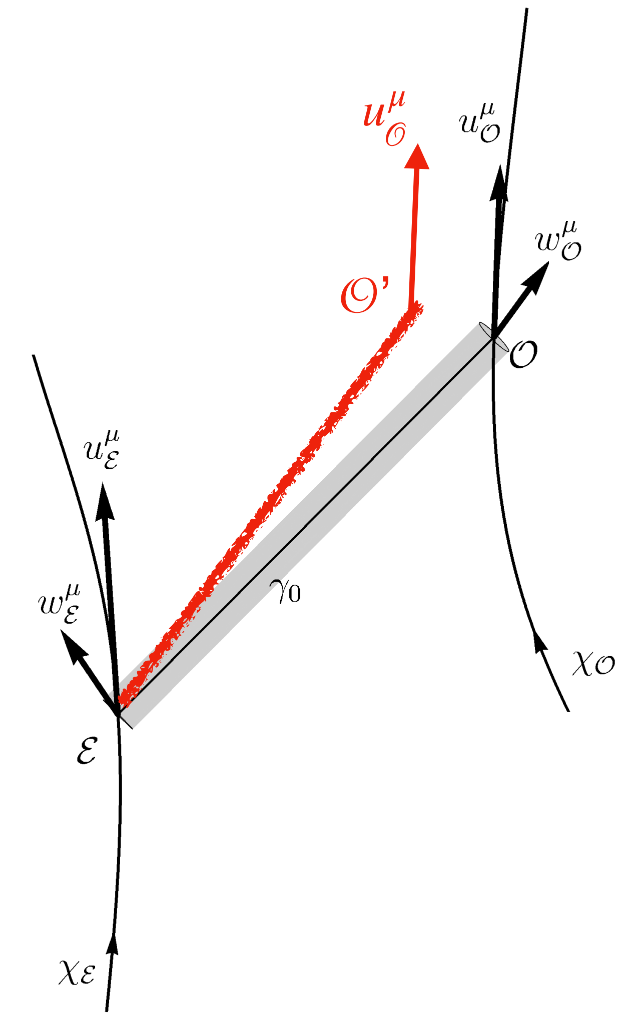

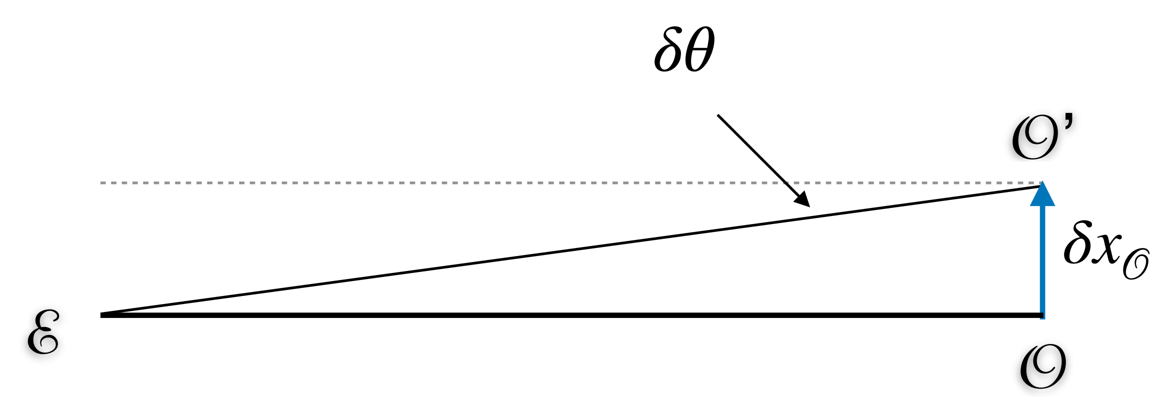

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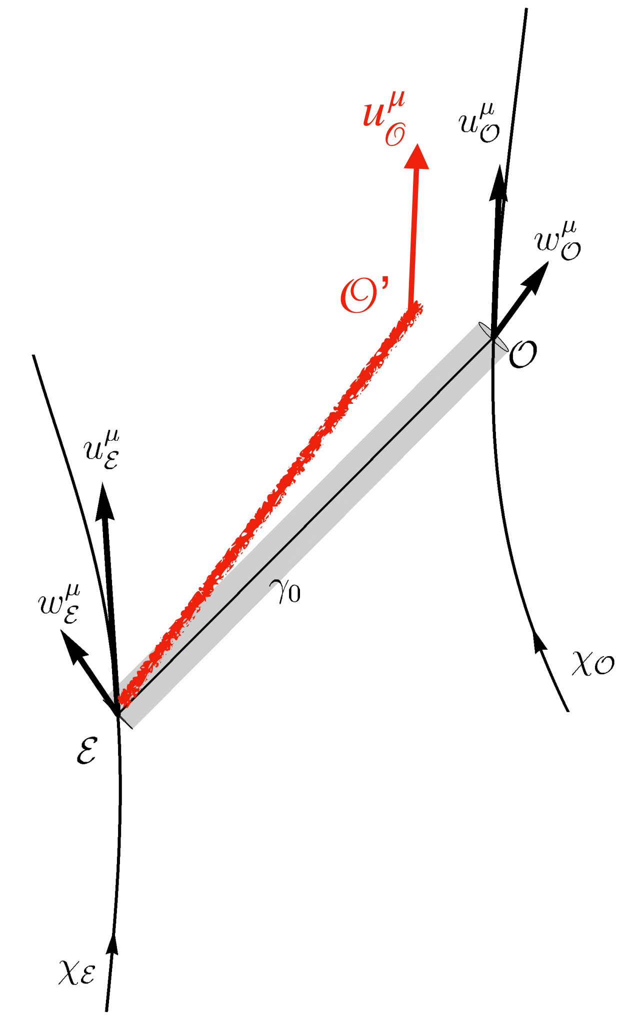

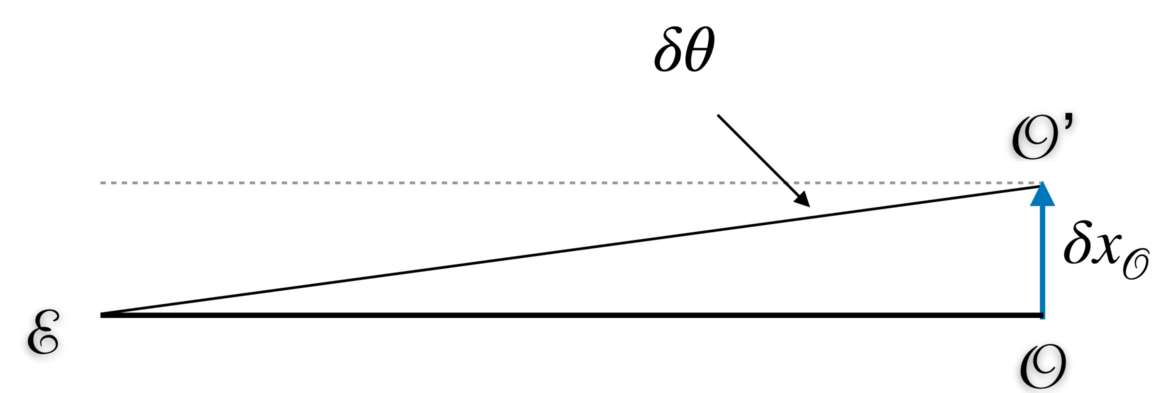

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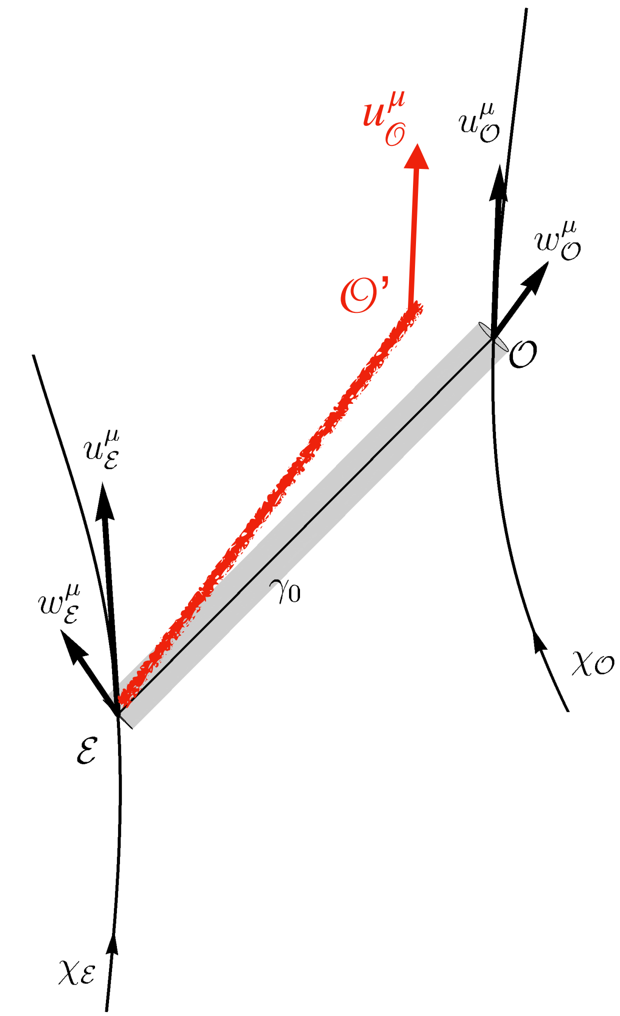

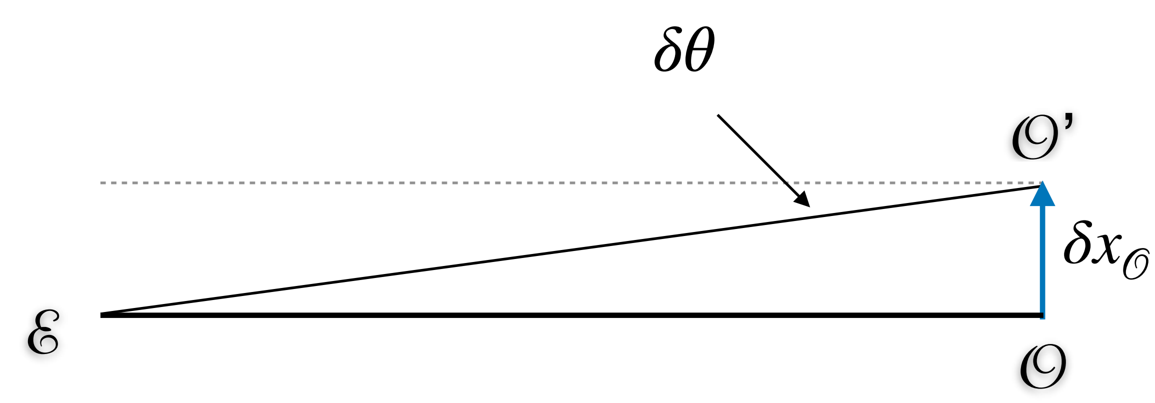

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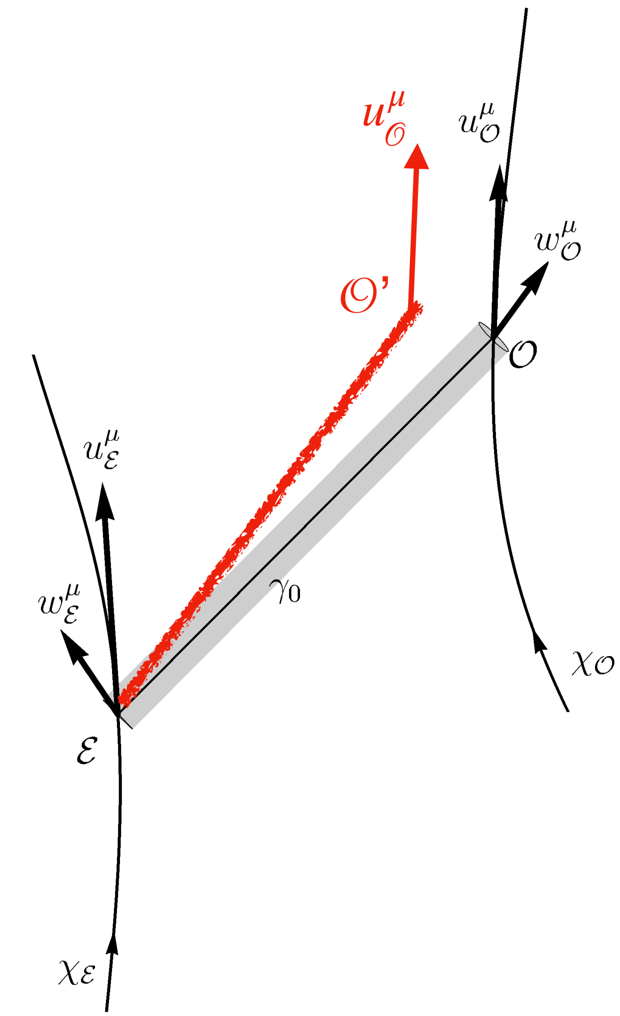

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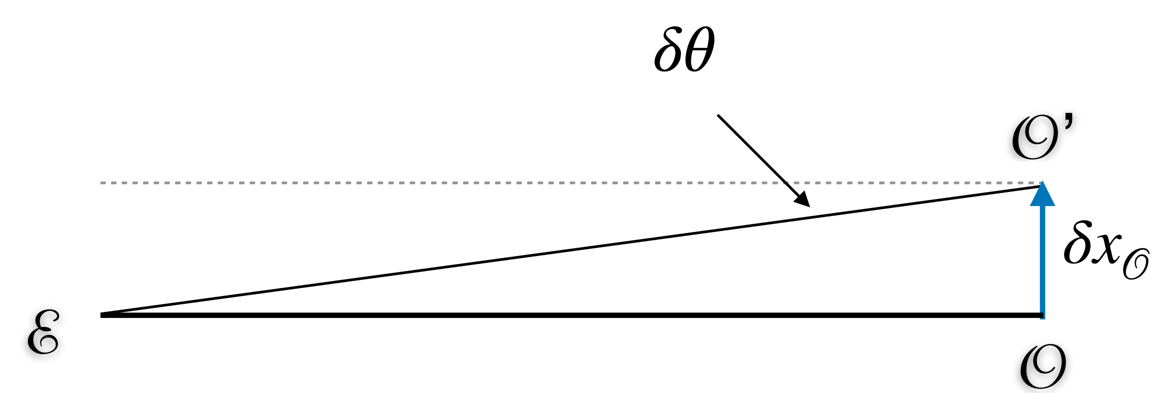

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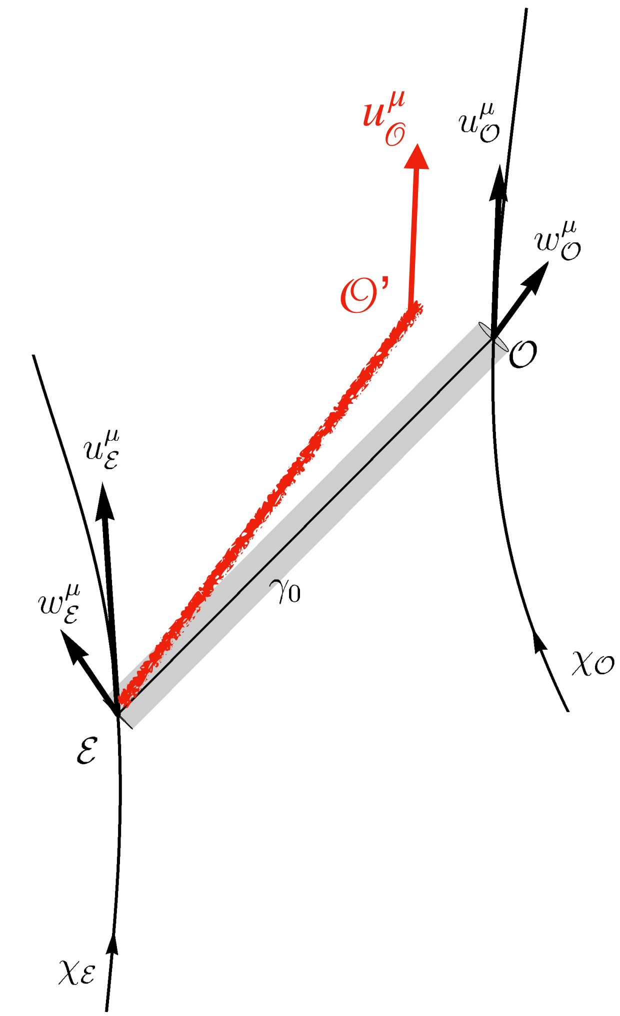

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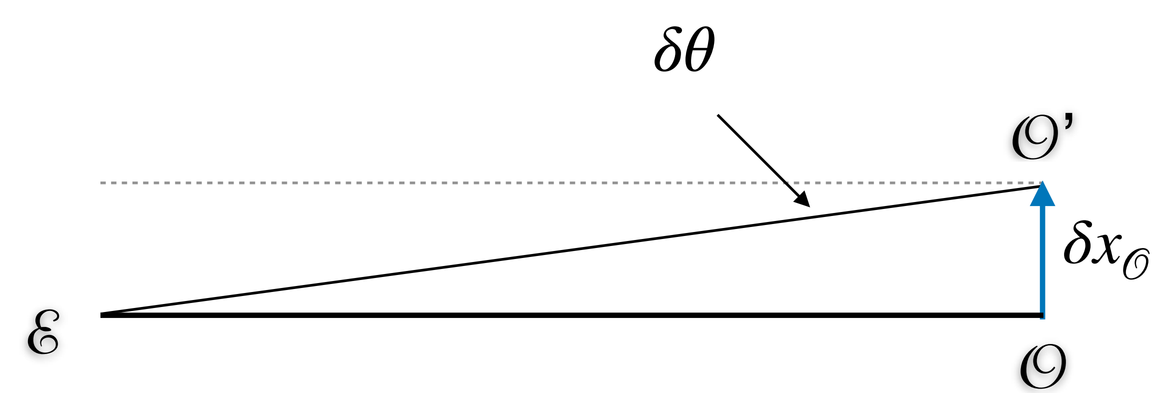
$$\Pi_{AB} = \Pi_{BA}$$



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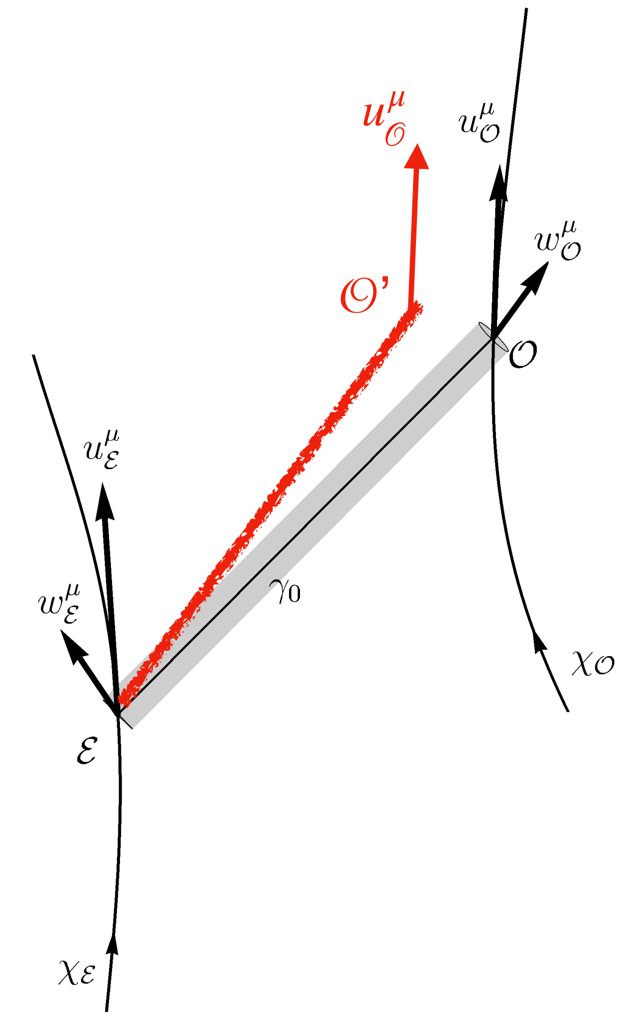

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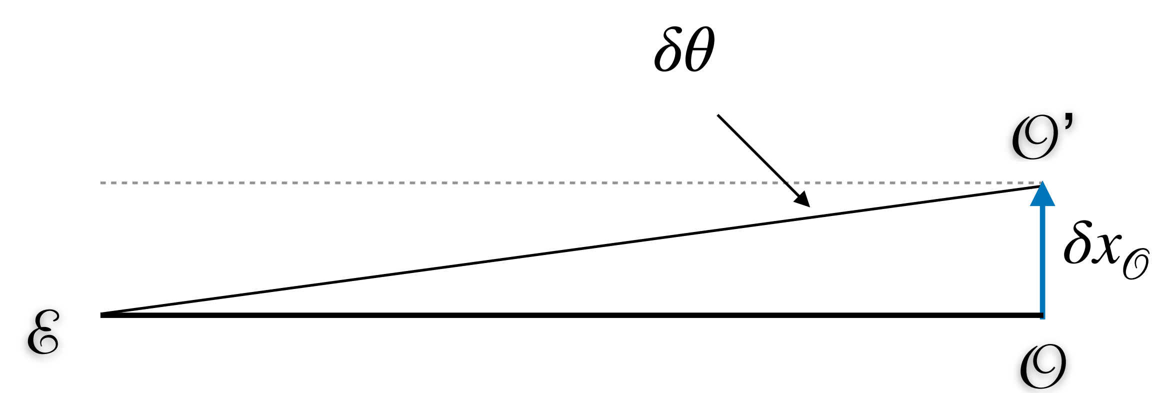

$$D_{par} = \left|\det \Pi^A{}_B\right|^{-1/2} \qquad D_{par} \equiv D_{par}(\mathcal{E}, \mathcal{O}, \gamma_0, u_{\mathcal{O}})$$

Parallax distance

Expressing the distance measures using curvature

$$\Pi^A{}_B = (l_{\mathcal{O}\,\mu}\, u^\mu_{\mathcal{O}})\, \overbrace{\mathcal{D}^{-1\,A}{}_C\, \left(\delta^C{}_B + m^C{}_B\right)}^{W_{XX}{}^A{}_B}$$

$$\dot{m}^A{}_B - R^A{}_{llC}\, m^C{}_B = R^A{}_{llB}$$

$$m^A{}_B(\mathcal{O}) = 0$$

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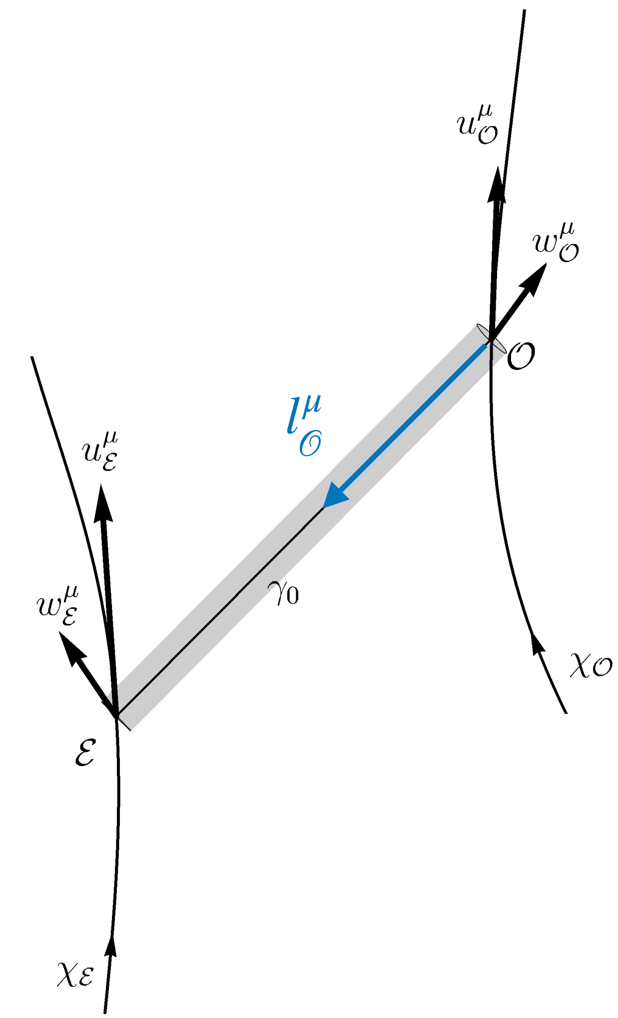

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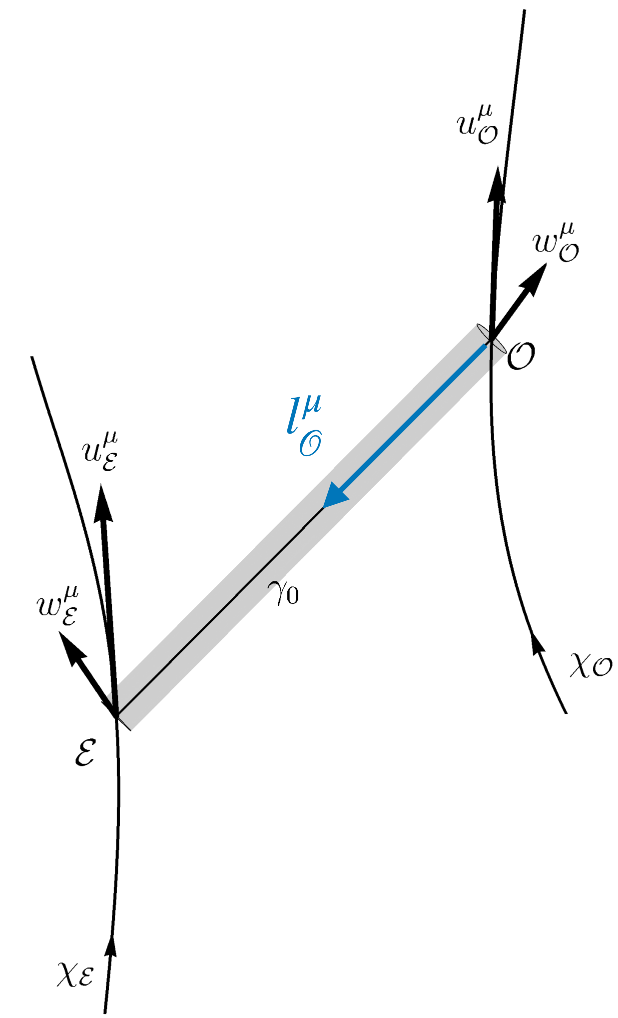

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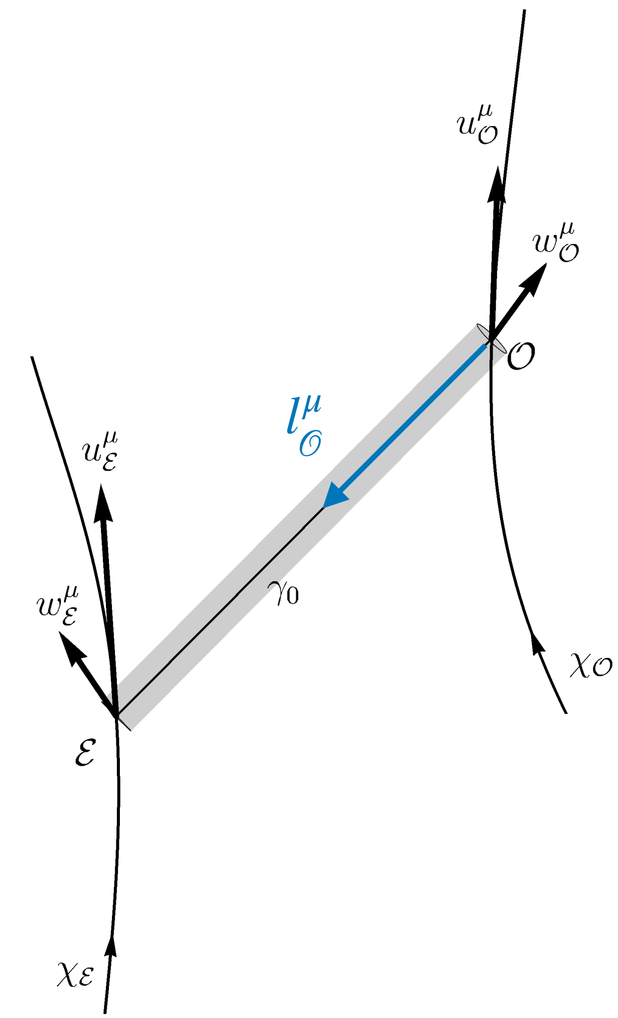

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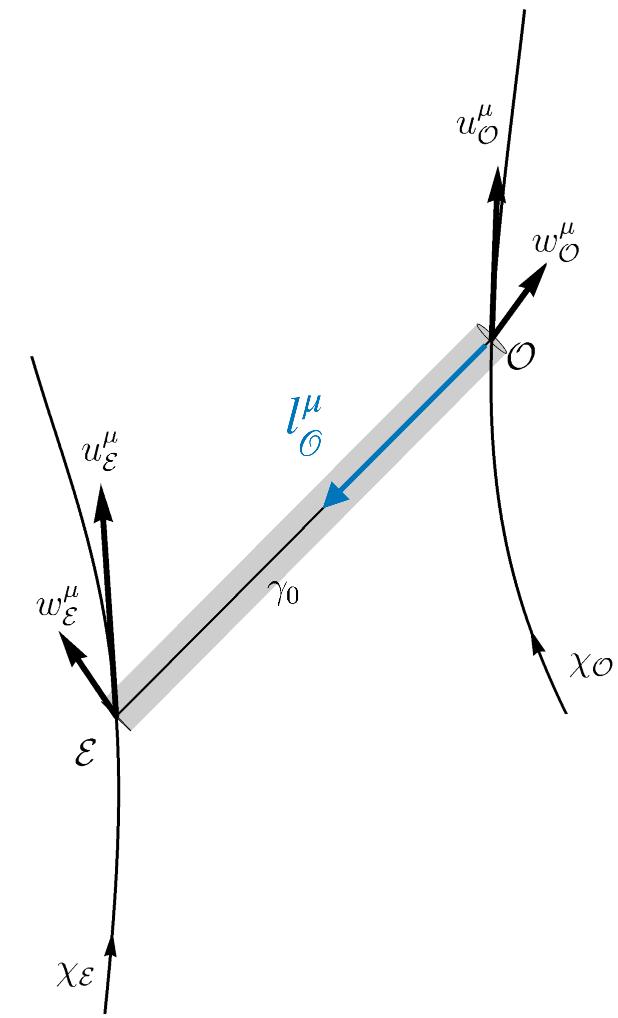

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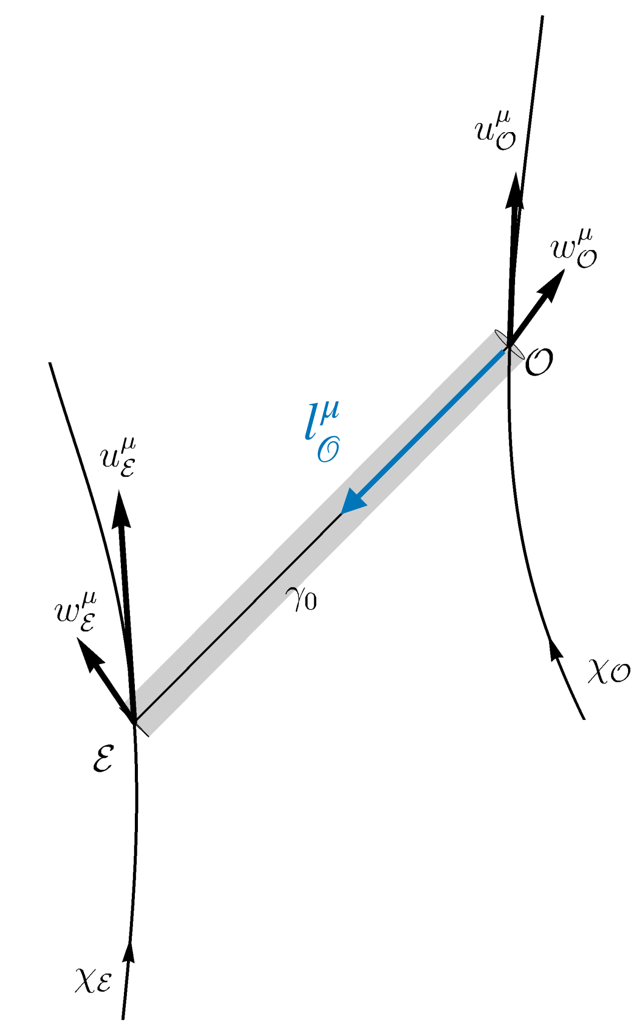
Distance slip

Define a scalar quantity $$\mu = 1 - \frac{\det \Pi^A{}_B}{\det M^A{}_B}$$



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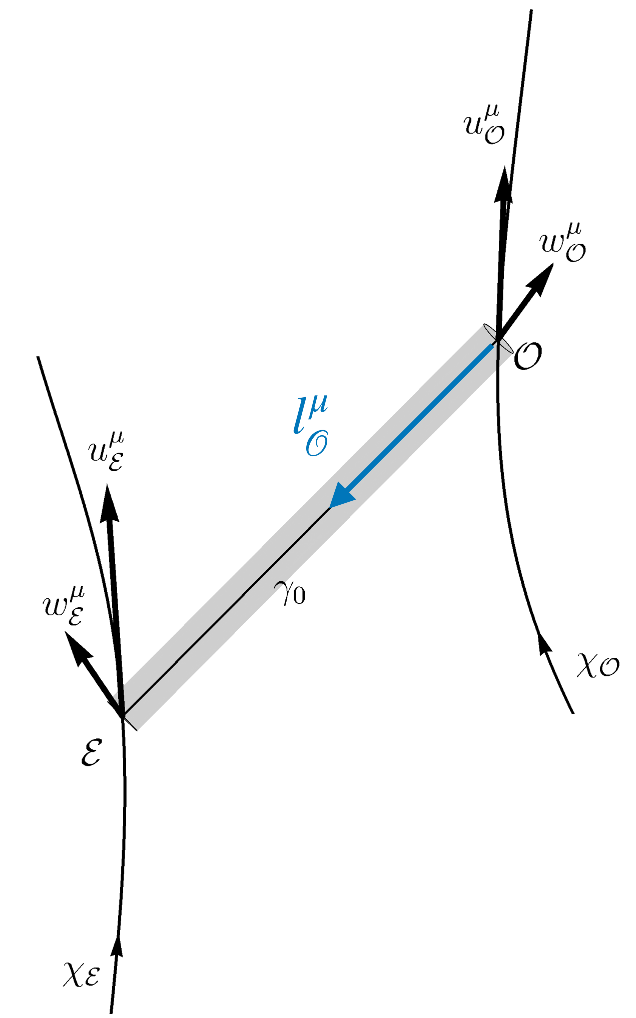

Expressed via distance measures $$\mu = 1 - \sigma \frac{D^2_{ang}}{D^2_{par}}$$

±1, but usually 1

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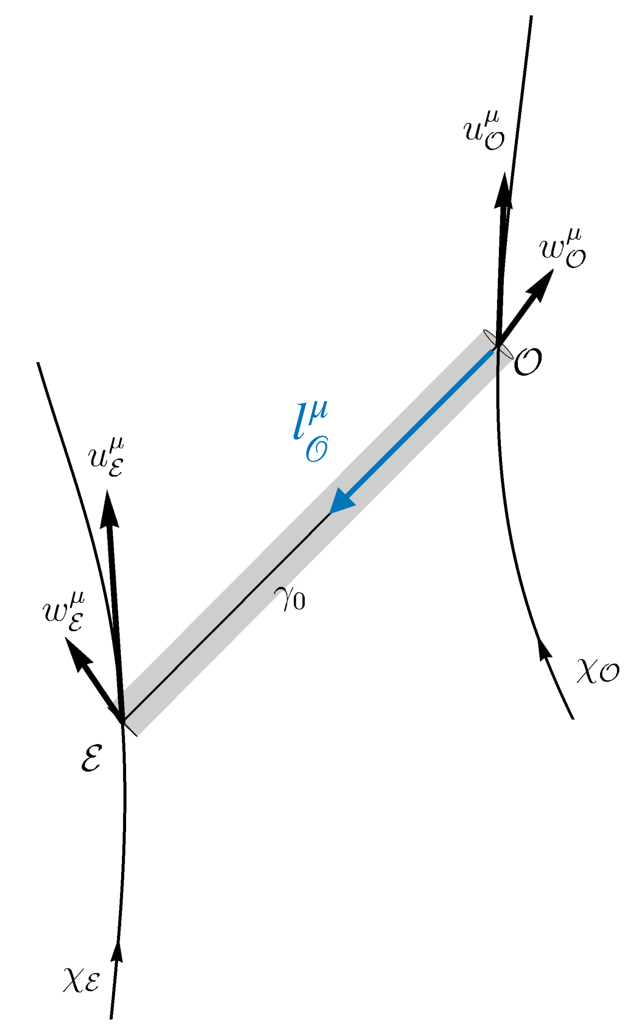

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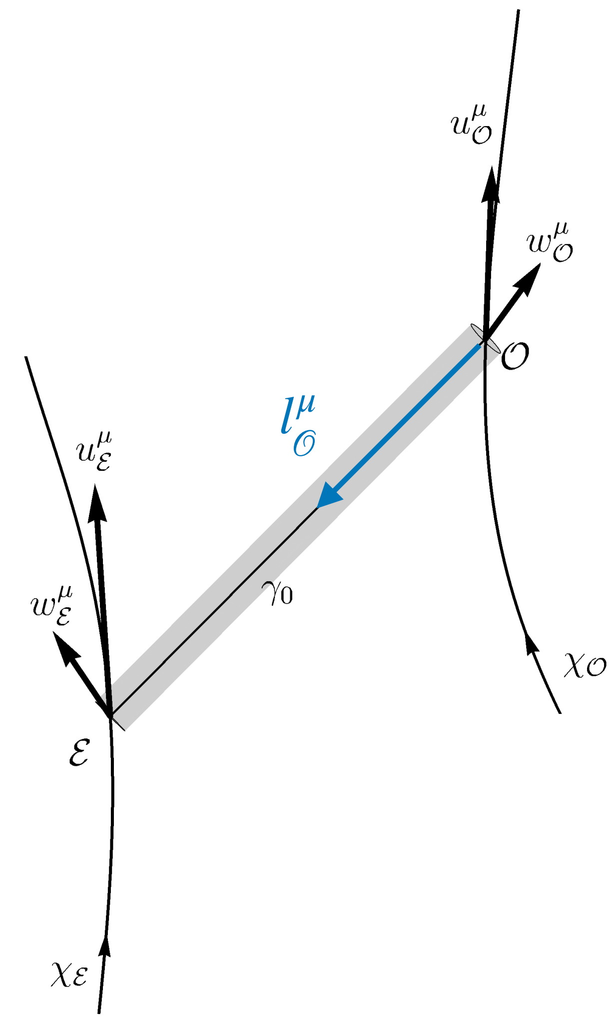

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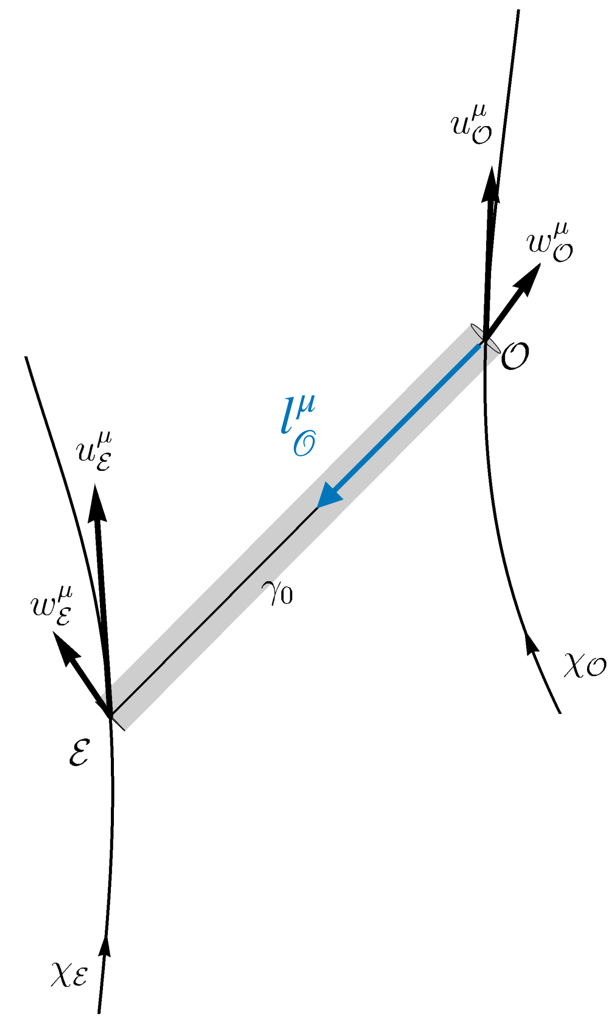

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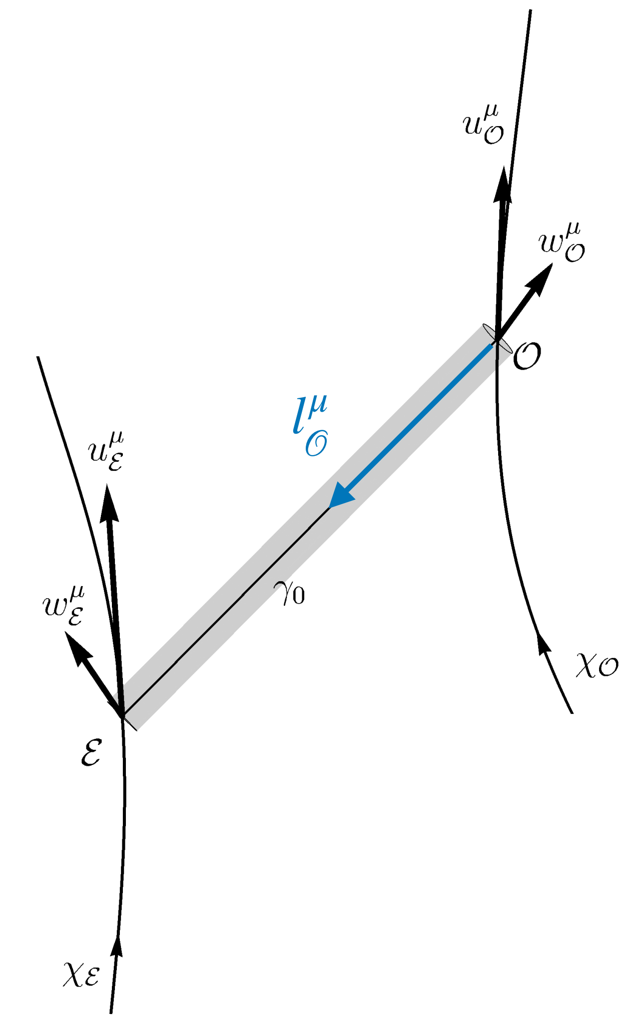

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no $C^\mu{}_{\nu\alpha\beta}$ or Λ

Short distance approximation: $$\mu = \frac{8\pi G}{c^4} \int_{\mathscr{O}}^{\mathscr{E}} T_{ll}(\lambda)\,\left(\lambda_{\mathscr{E}} - \lambda\right)\, d\lambda + O(\mathrm{R}^2)$$

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Magnitude of the effect locally:

negligible pressure (dust) $T^{\mu\nu} = \rho\, U^{\mu} U^{\nu}$

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Galactic scales

mass density of the thin disc of the Milky Way $\rho \approx 1\,M_\odot\,\mathrm{pc}^{-1}$

most distant trigonometric parallax measured $r \approx 20\,\mathrm{kpc}$

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$\mu \approx 2 \cdot 10^{-4}$

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…but we may use the motion of the Solar System wrt CMB frame in the future [Kardashev 1986, Rosquist 1988, Kasai 1988, Räsänen 2014, Quercellini *et al* 2012, Marcori *et al* 2018], 78AU/year

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Need sources for which two methods of distance determination are possible (+ big sample)

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Assume this measurement is possible. Signal? What can we learn?

μ in FLRW spacetime

μ in FLRW spacetime

$$ds^2 = -\,dt^2 + a(t)^2\left(d\chi^2 + S_k(\chi)^2\,d\Omega^2\right)$$

$$S_k(\chi) = \begin{cases} \frac{1}{\sqrt{k}}\sin(\sqrt{k}\chi) & \text{if } k > 0 \\ \chi & \text{if } k = 0 \\ \frac{1}{\sqrt{|k|}}\sinh(\sqrt{|k|}\chi) & \text{if } k < 0, \end{cases}$$

μ in FLRW spacetime

$$ds^2 = -\,dt^2 + a(t)^2\left(d\chi^2 + S_k(\chi)^2\, d\Omega^2\right)$$

$$\mu = 1 - \left(\frac{1}{1+z}(C_k(\chi) + H_0\, S_k(\chi))\right)^2$$

$$\chi(z) = \int_0^z \frac{dz'}{H(z')}$$

$$S_k(\chi) = \begin{cases} \frac{1}{\sqrt{k}}\sin(\sqrt{k}\chi) & \text{if } k > 0 \\ \chi & \text{if } k = 0 \\ \frac{1}{\sqrt{|k|}}\sinh(\sqrt{|k|}\chi) & \text{if } k < 0, \end{cases}$$

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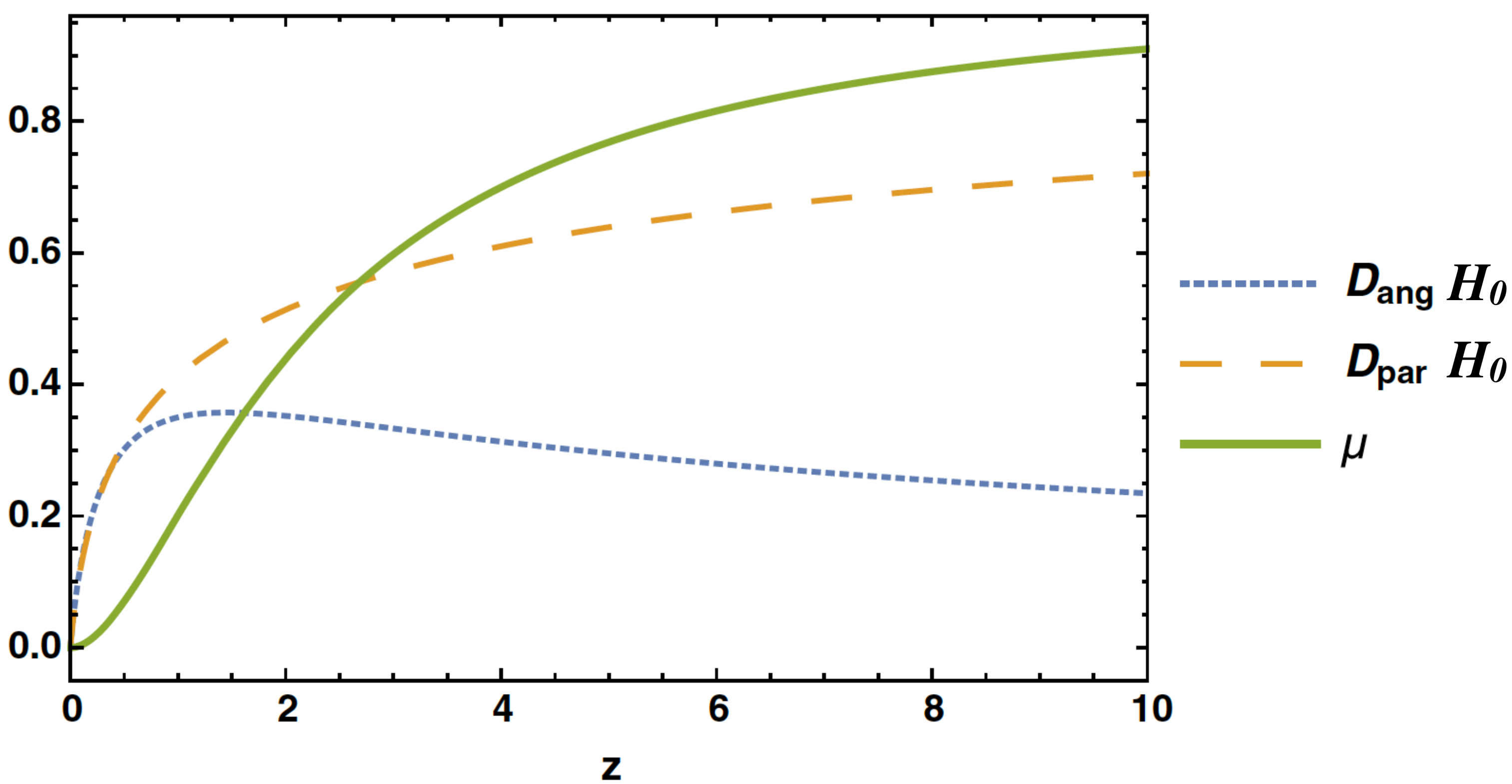

ΛCDM, Planck values

$H_0 = 67.4$

$\Omega_{m0} = 0.266018$

$\Omega_{k0} = 0$

$\Omega_{\Lambda 0} = 0.732982$

μ in cosmology

Low redshift expansion

$$\mu(z) = \frac{3}{2}\Omega_{m0}\, z^2 + \left(-\frac{1}{2}\Omega_{m0} - \frac{3}{2}\Omega_{m0}\,\Omega_{k0} - \frac{9}{4}\Omega_{m0}^2\right) z^3 + O(z^4)$$

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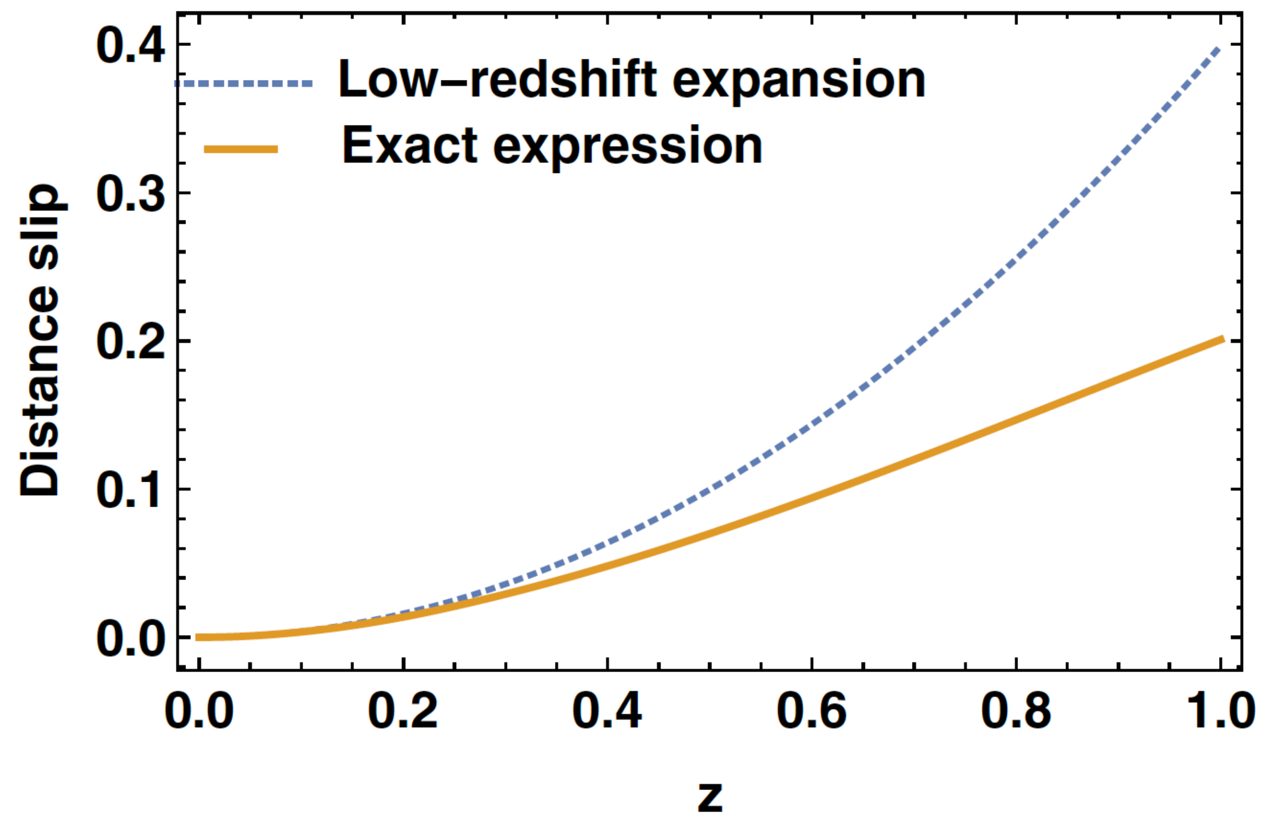

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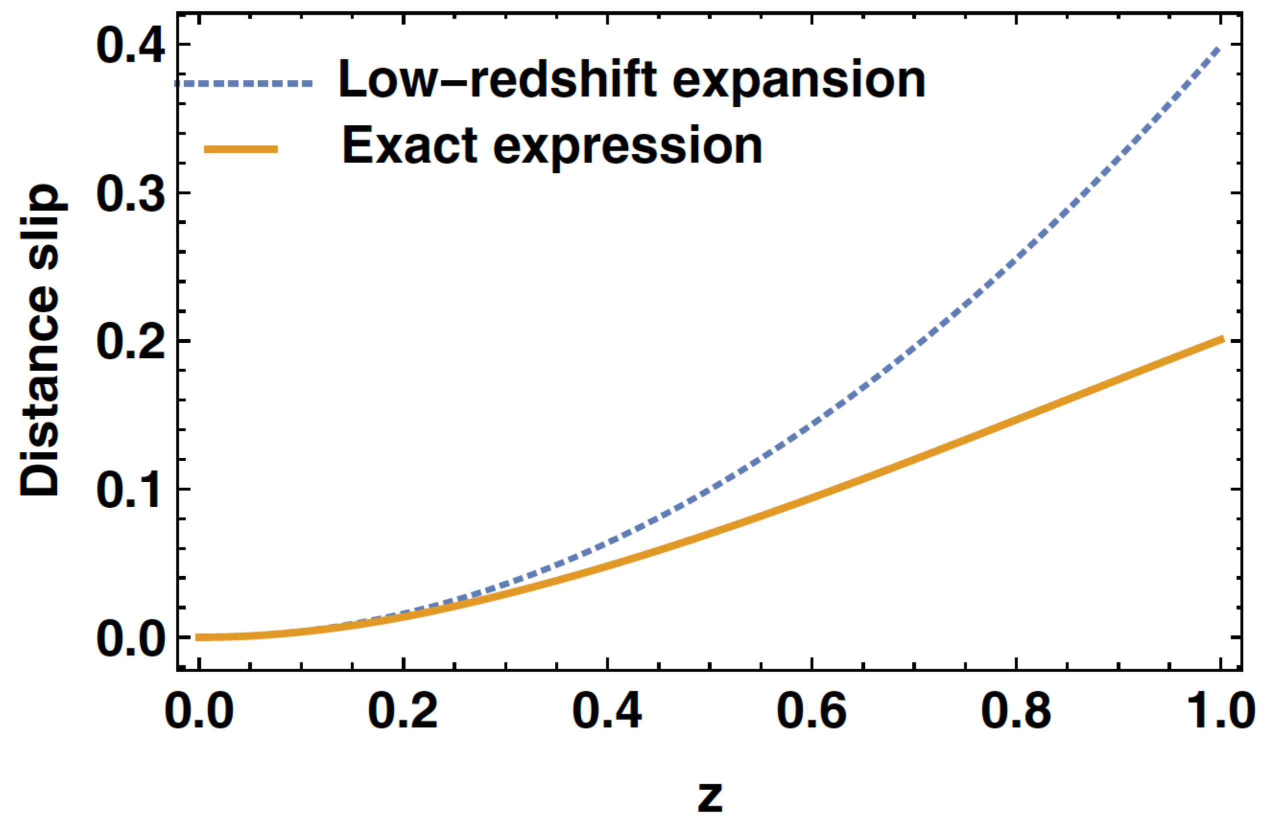
dimensionless μ vs dimensionless $z \implies$ no H_0



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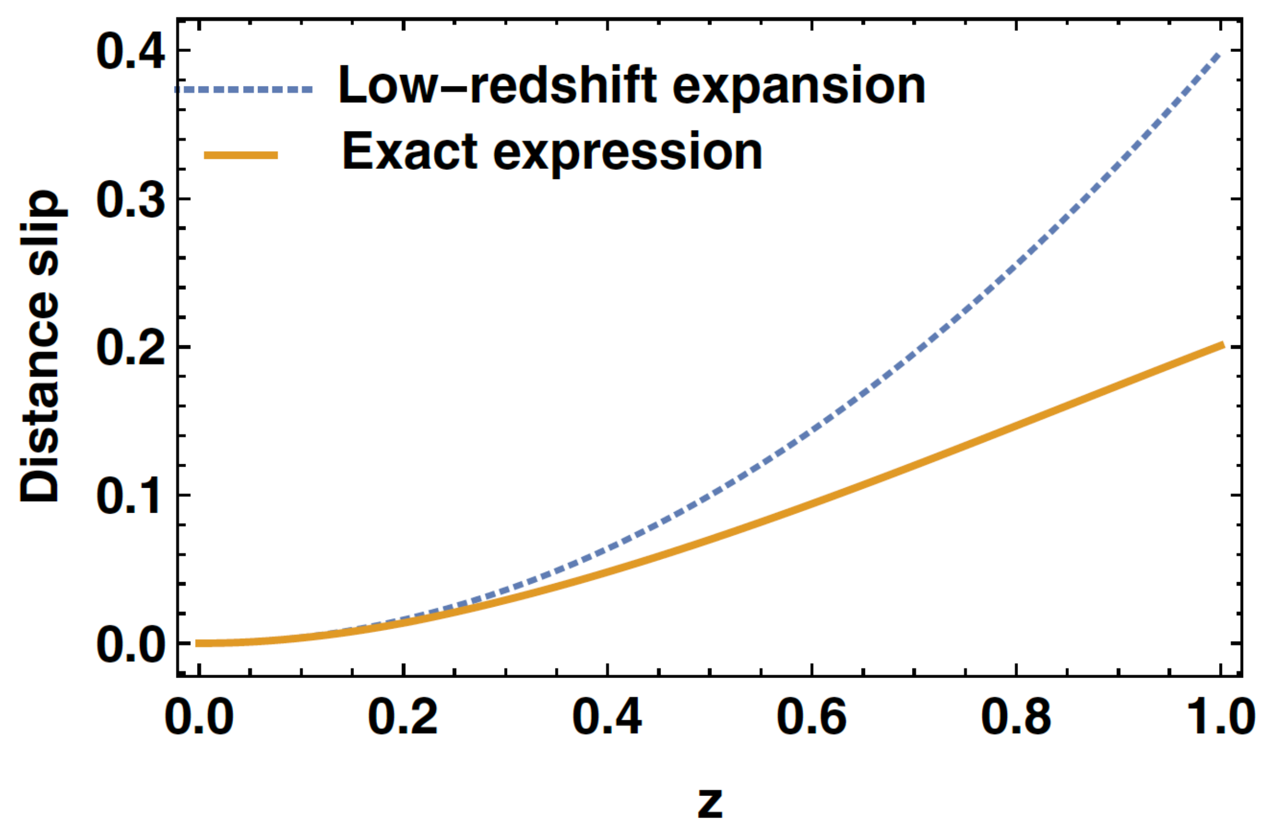
leading order term gives a measurement of Ω_{m0}



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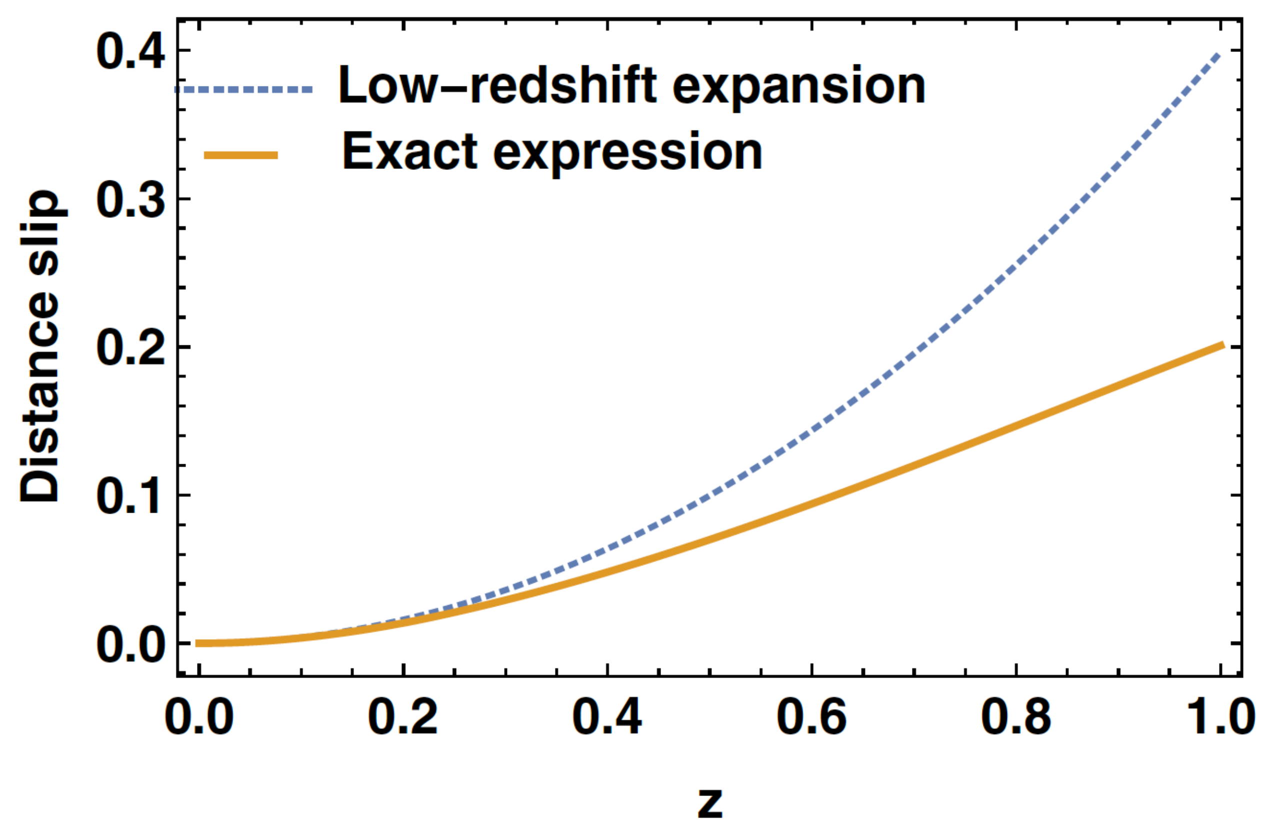
leading order term gives a measurement of Ω_{m0}

independent from any other measurements



μ in cosmology

$\boldsymbol{\mu}$ vs D_{ang} diagram

$$\mu(D_{ang}) = \frac{3}{2}\Omega_{m0}\, H_0^2\, D_{ang}^2 + \frac{5}{2}\Omega_{m0}\, H_0^3\, D_{ang}^3 + O(D_{ang}^4)$$

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bypassing z as observable

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both quantities independent of $u_{\mathcal{E}}$!

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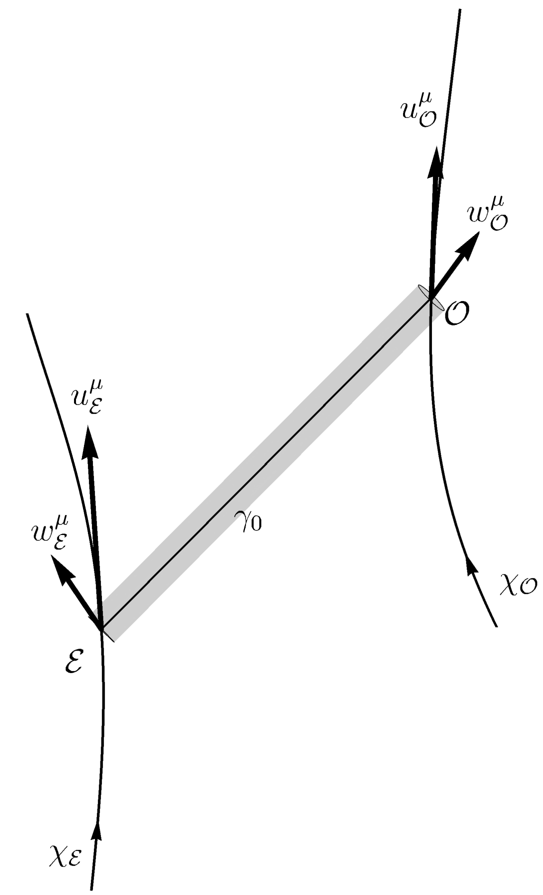
diagram insensitive to the peculiar motions of the sources! No redshift space distortions.

potentially very robust measurement, independent from others

Distance inequality

Recall that:

$$\mu = 1 - \frac{D_{ang}^2}{D_{par}^2} = \frac{8\pi G}{c^4}\int_{\lambda_{\mathscr{O}}}^{\lambda_{\mathscr{E}}} T_{\mu\nu}\, l^\mu\, l^\nu\, (\lambda_{\mathscr{E}} - \lambda)\, d\lambda + O(\textbf{Riemann}^2)$$



If $T_{\mu\nu}\, l^\mu\, l^\nu \geq 0$, we have initially $\mu \geq 0$

$$\Longrightarrow \quad D_{par} \geq D_{ang}$$

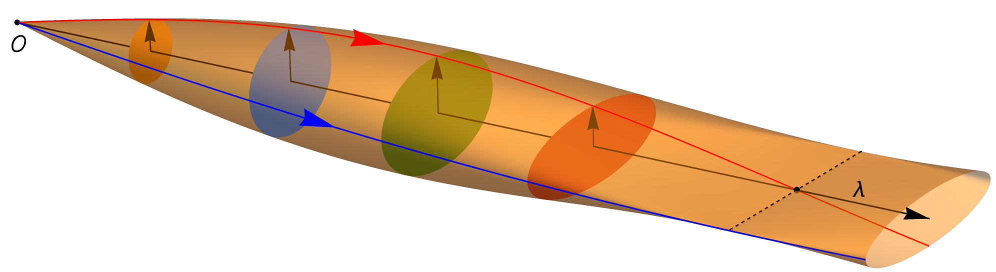
This can be extended to a general, non-perturbative result

M. K., J. Serbenta, *„Testing the null energy condition with precise distance measurements”*, Phys. Rev. D **105**, 084017 (2022)

Distance inequality

Infinitesimal bundles of null rays

$$\xi^A(\lambda) = W_{XL}{}^A{}_B(\lambda)\, \nabla_l \xi^B(\lambda_{\mathcal{O}})$$



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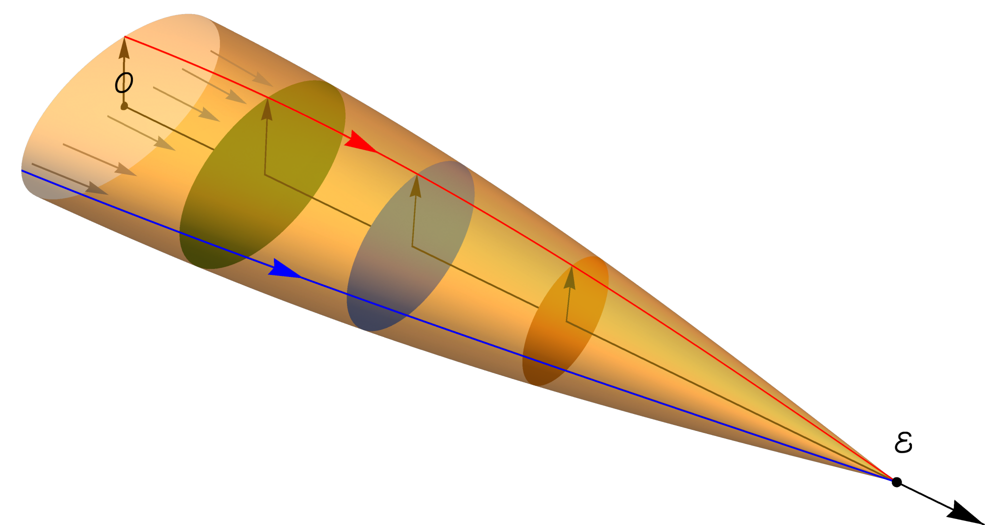

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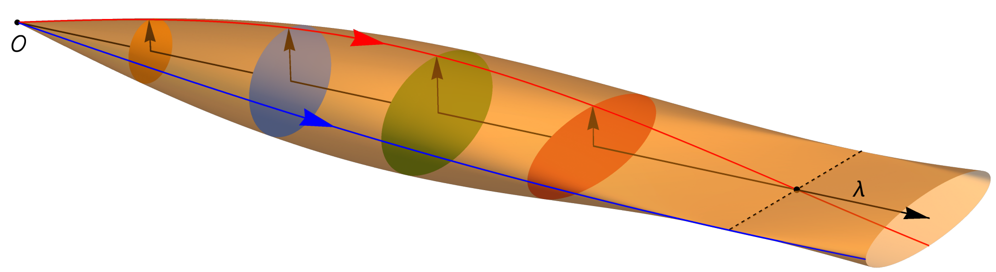

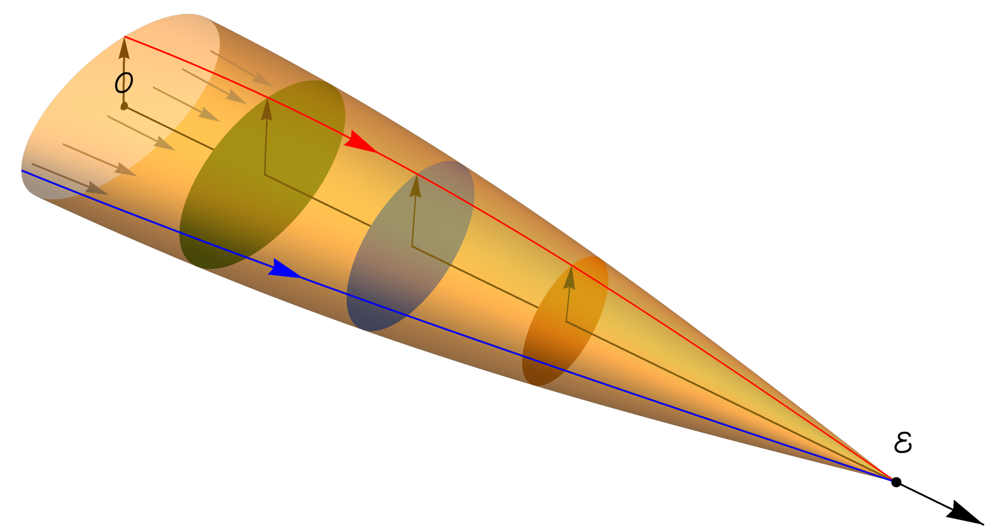

Alternative description

$$\nabla_l \xi^A(\lambda) = B^A{}_C(\lambda)\, \xi^C(\lambda) \qquad\qquad B_{AB} = \frac{1}{2}\theta\, \delta_{AB} + \sigma_{(AB)} + \omega_{[AB]}$$

$= 0$

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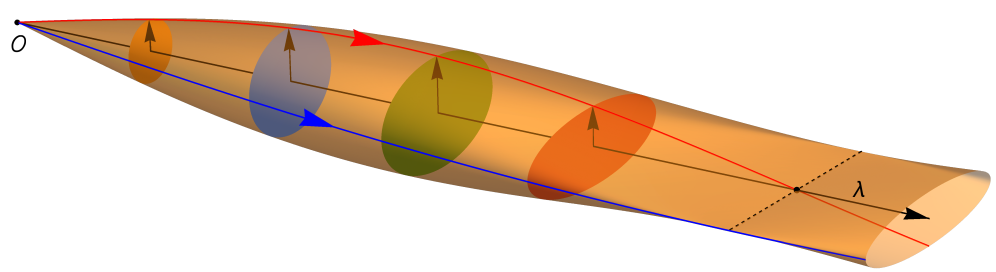

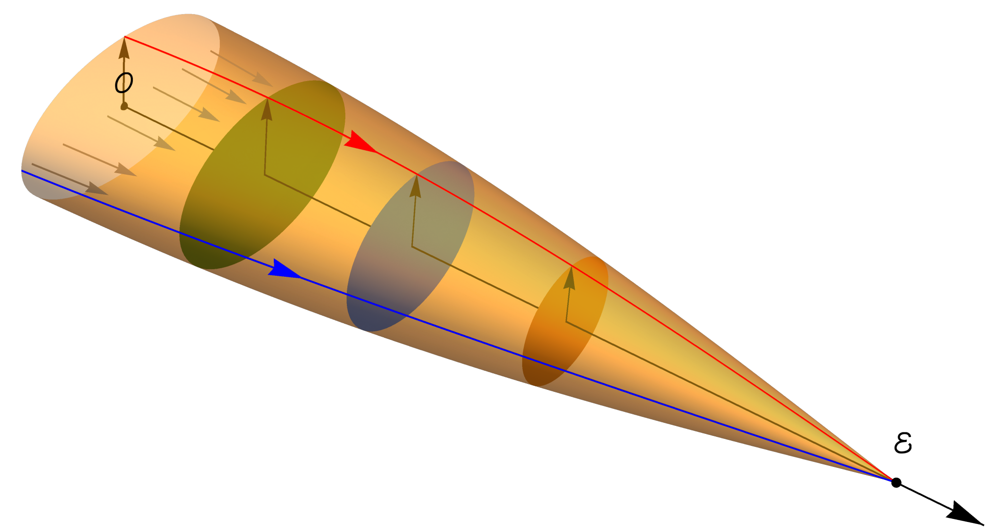

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$= 0$

Singular points of bundles

$$\det W^A{}_B = 0$$

$$\theta, \sigma_{AB} \to \infty$$

Focal point:

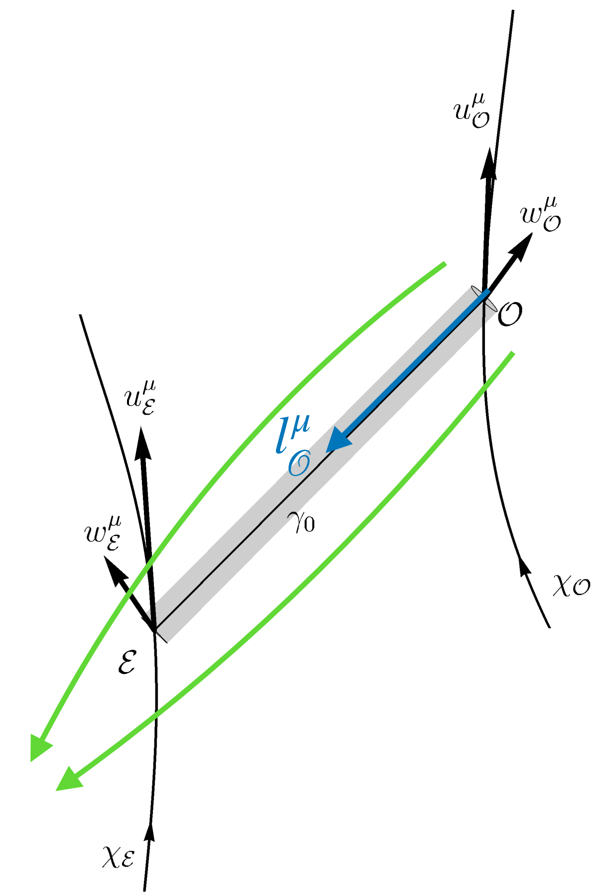
$$\det W_{XX}{}^A{}_B = 0$$

$$D_{par} \to \infty$$

Distance inequality

Theorem:

- NEC holds, i.e. $R_{\mu\nu}\, l^\mu\, l^\nu \geq 0$
- The bundle of rays parallel at $\mathcal{O}$ has no singular points between $\mathcal{E}$ and $\mathcal{O}$



then

- $D_{par} \geq D_{ang}$
- Moreover, $D_{par} = D_{ang}$ iff no Ricci focusing or Weyl focusing along the way, i.e.

$$R_{\mu\nu}\, l^\mu\, l^\nu = 0 \qquad C^A{}_{\mu\nu B}\, l^\nu\, l^\nu = 0$$

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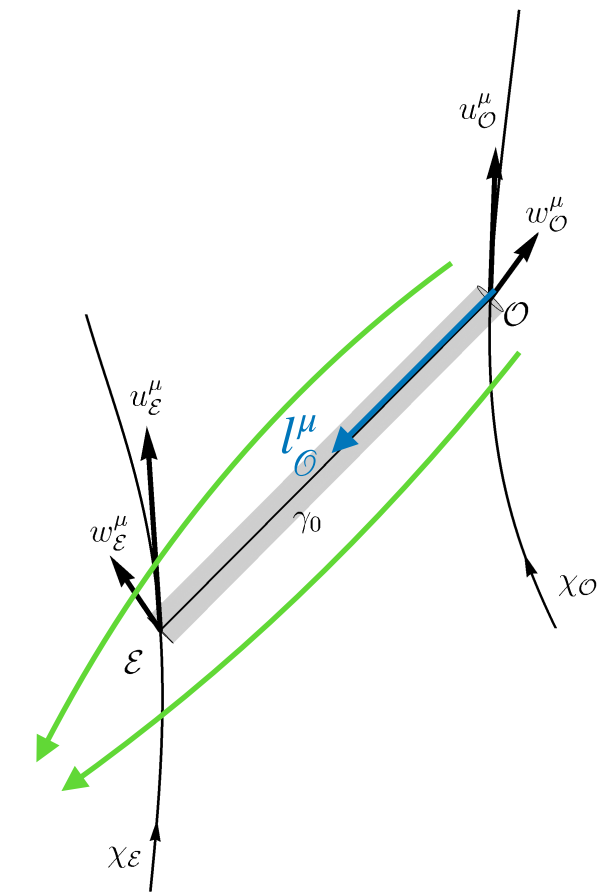
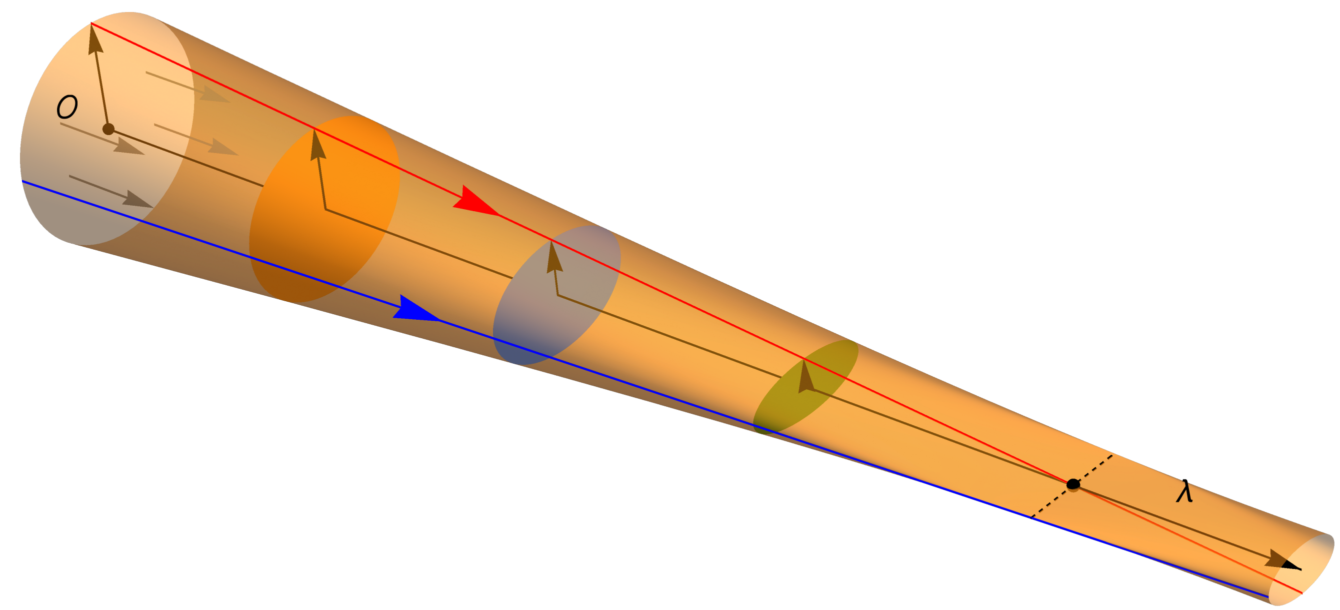


If NEC holds, then both Ricci focusing and Weyl shear cause $D_{par} > D_{ang}$ at least up to the first focal point counting from $\mathcal{O}$

Distance inequality

Idea of the proof: relate μ to the gravitational focusing of rays

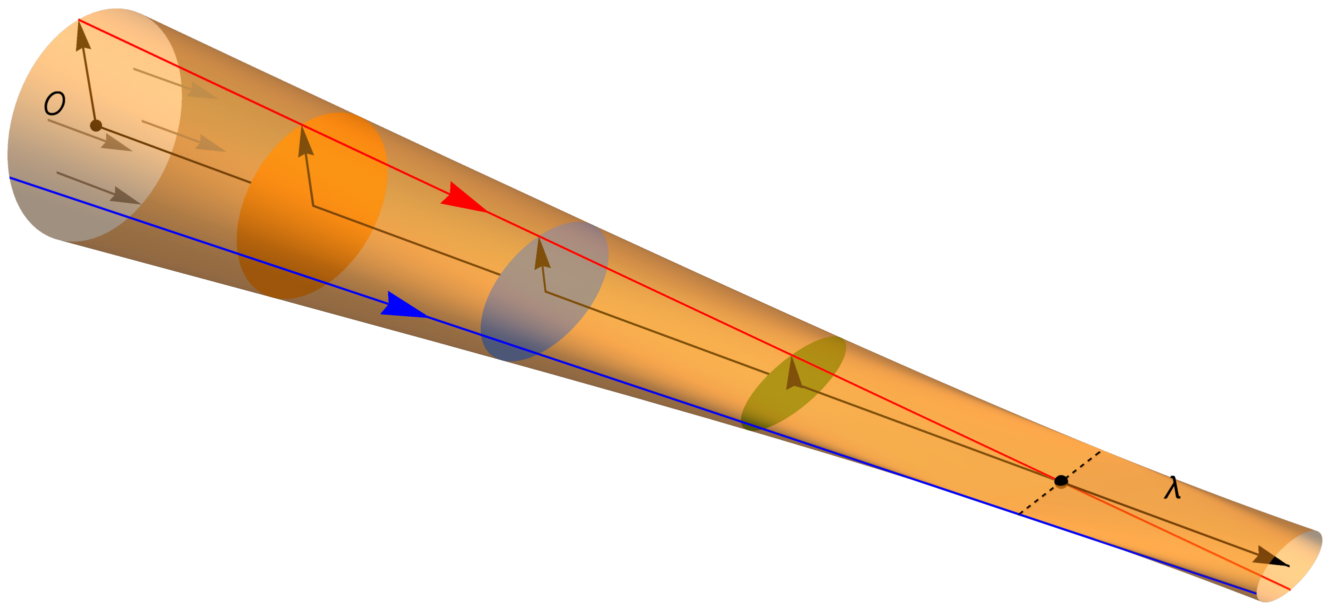


Distance inequality

Idea of the proof: relate μ to the gravitational focusing of rays

$$\mu = 1 - \det\left(W_{XX}\right)^{A}{}_{B} = 1 - \frac{D_{ang}^{2}}{D_{par}^{2}}$$

up to the first focal point



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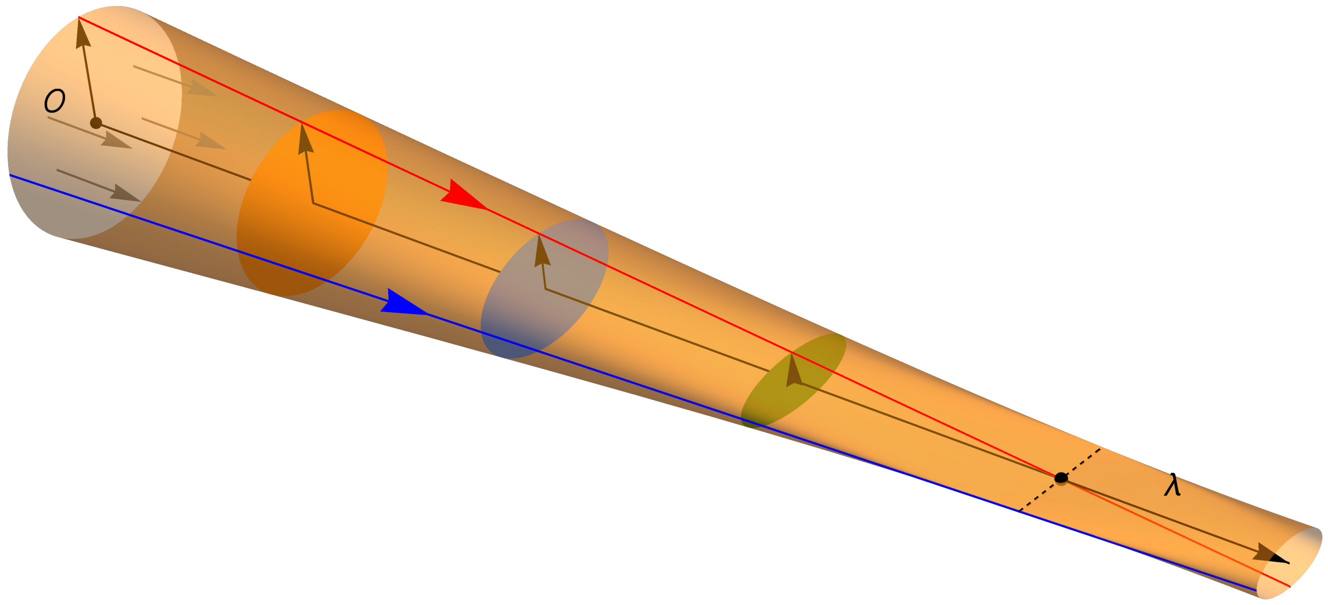
For the bundle parallel at $\mathscr{O}$ we have:



$$A(\lambda) = \det\left(W_{XX}\right)^{A}{}_{B}(\lambda) \cdot A(\lambda_{\mathscr{O}})$$

$$\mu = 1 - \frac{A(\lambda)}{A(\lambda_{\mathscr{O}})}$$

Distance inequality

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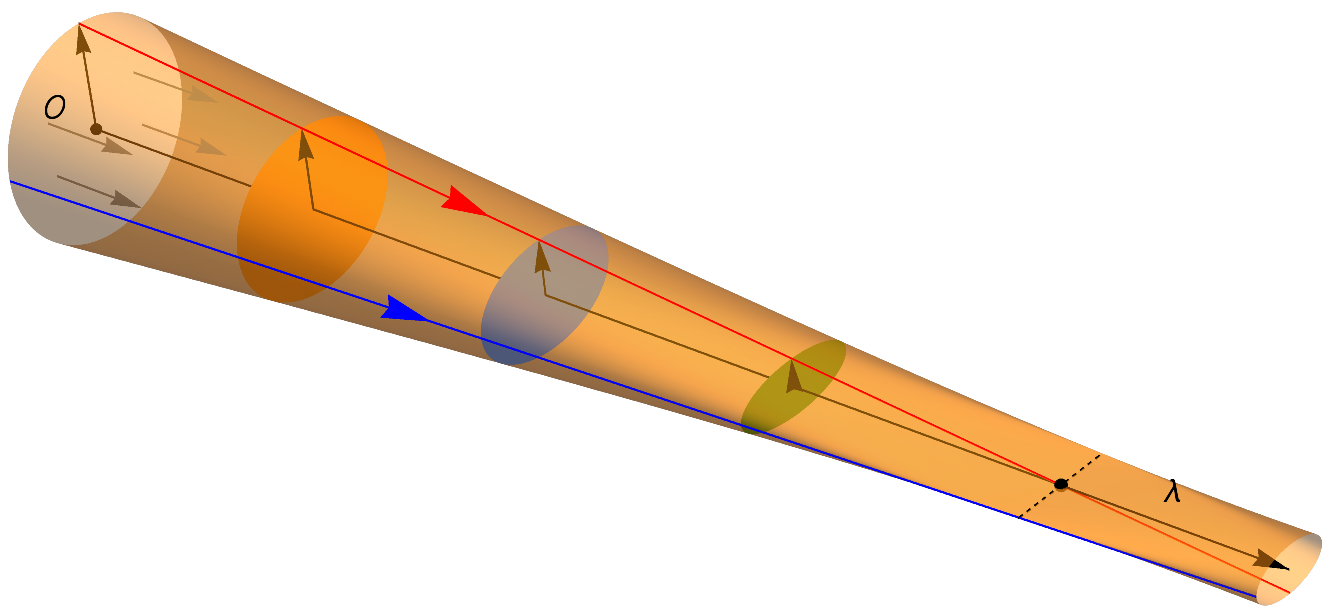

ODE for $A(\lambda)$

$$\frac{dA}{d\lambda} = A(\lambda)\,\theta(\lambda) \qquad \Longrightarrow A(\lambda) = A(\lambda_{\mathcal{O}})\,\exp\left(\int_{\lambda_{\mathcal{O}}}^{\lambda} \theta(\lambda)\,d\lambda\right)$$

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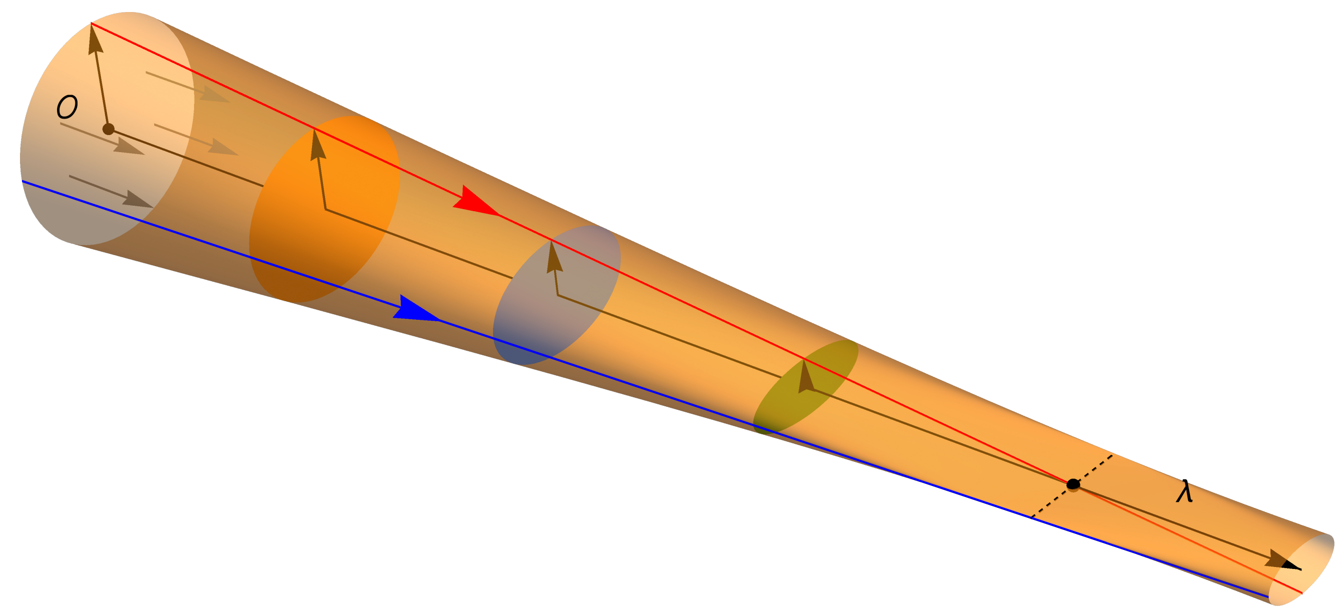

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up to the first focal point

$$\frac{d\theta}{d\lambda} = -\frac{\theta^2}{2} - \sigma_{AB}\,\sigma^{AB} - R_{\mu\nu}l^{\mu}l^{\nu} \qquad \Longrightarrow \theta(\lambda) \leq 0$$

up to the first focal point

Distance inequality

- Alternative baseline averaging via the trace of $\Pi^A{}_B$ [Räsänen 2014, Rosquist 1988, Ellis et al 1971…]:

$$\tilde{D}_{par} = \frac{2}{\Pi^A{}_A}$$

There is a similar theorem asserting that $\tilde{D}_{par} \geq D_{ang}$ if $\mathcal{O}$ and $\mathcal{E}$ sufficiently close

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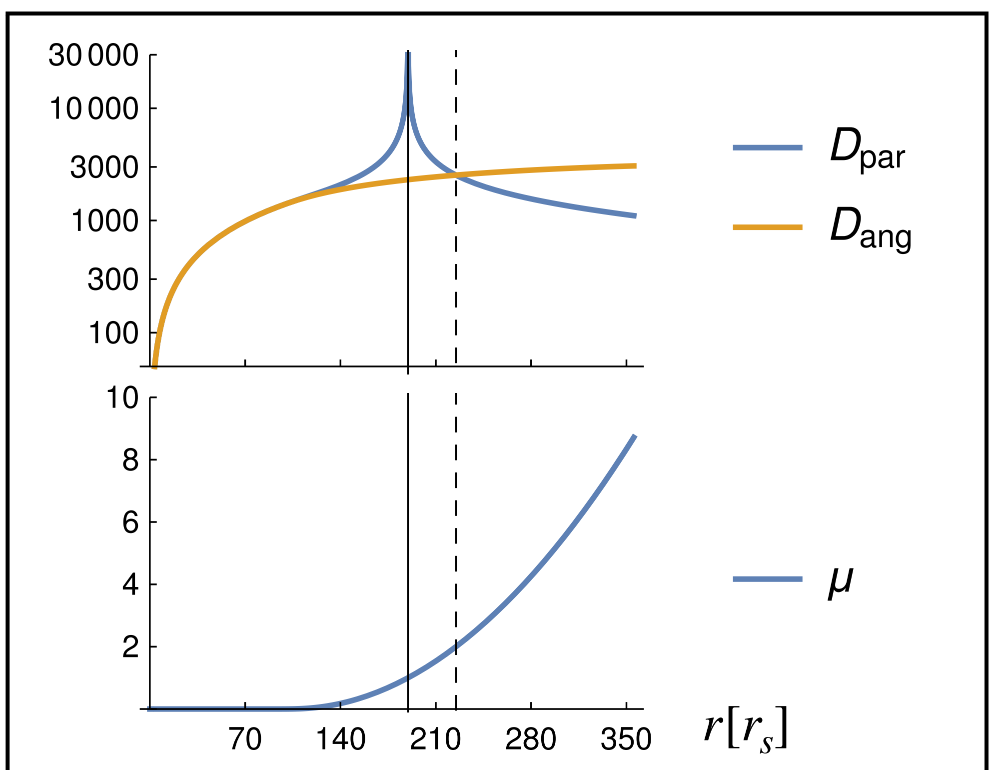
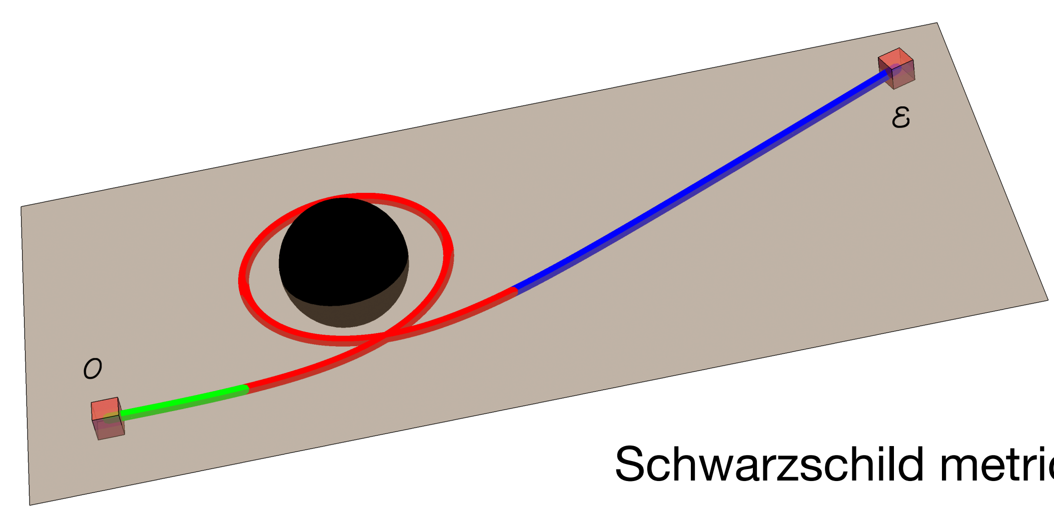
- Past the first focal point the inequality may not hold even if NEC holds



J. Serbenta, M. K., Class. Quantum Grav. **39** 155002 (2022)



Schwarzschild metric

Distance inequality

Distance inequality

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  Milky Way gravitational potential: 1-10 Mpc at least

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- [Räsänen 2014] - proposed consistency tests of FLRW metric based on comparison of D_{par} and D_{ang}

  We propose a simple test of NEC + light propagation in GR: the sign of the difference between D_{par} and D_{ang}

  Wrong sign would be difficult to explain within GR as we understand it today

Summary

- In GR parallax distance D_{par} is different from angular diameter distance D_{ang} (and the related luminosity distance D_{lum})

- Their relative difference μ (distance slip) carries information about the matter density along the line of sight. Very small effect though.

- Kinematic invariance of μ

- Sign of μ related directly to the null energy condition (NEC)

Thank you!