Stiffness, complexity, cracking and stability of relativistic compacts stars

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Overview

- How ‘compact’ is a compact star?
  - Buchdahl bound
  - Electromagnetic analogue
  - Anisotropic analogue

- Compactness, stiffness, EOS and stability
  - Generalized Tolman VII solution
  - Stability analysis: Chandrasekhar’s method

- Complexity and stability

Stellar compactness

- For an assumed equation of state (EOS), the mass-radius ($M - R$) relationship of a self-gravitating compact stellar object can be generated. Alternatively, if the mass and radius of a compact star are known, its composition can be predicted.
- The current era of multi-messenger astronomy may provide a precise estimation of stellar observables like mass M and radius R of a compact star ('neutron star') which can help us in constraining the EOS.
- We ask the question: What is the maximum permissible compactness of a self-gravitating compact stellar object?
- Buchdahl bound: No uniform density stars with radii smaller than $9/8M$ can exist. For a stellar configuration in equilibrium, the Buchdahl bound implies $2M/R < 8/9$ [Phys. Rev., **116**, 1027 (1959)].

Electromagnetic generalization of Buchdahl bound

- What happens when the electromagnetic field is incorporated into the system?
- Note: In an Einstein-Maxwell system, the gravitational attraction is counterbalanced by the Coulomb repulsion, which prevents the body from collapsing to a point singularity.
- Our plan:
- Superimpose the electromagnetic field on uniform density fluid distribution.
- Generate new solutions which would reduce to uniform density Schwarzschild solution when the electric field is switched off.

Einstein-Maxwell system

We write the line element describing the interior of a static spherically symmetric charged fluid sphere:

$$ds_{-}^{2} = -e^{2\nu(r)}dt^{2} + e^{2\mu(r)}dr^{2} + r^{2}(d\theta^{2} + \sin^{2}\theta d\phi^{2}), \tag{1}$$

The energy-momentum tensor corresponding to matter and electromagnetic field have their respective expressions

$$T_{ij} = (p+\rho)u_i u_j + pg_{ij}, \quad E_{ij} = \frac{1}{4\pi}(F_i^l F_{jl} - \frac{1}{4}g_{ij}F^{lm}F_{lm}).$$

The Maxwell equations yield

$$E = \frac{e^{(\nu+\mu)}}{r^2}q(r), \tag{2}$$

where the total charge $q(r)$ contained within the sphere of radius r is defined as

$$q(r) = 4\pi\int_0^r \sigma r^2 e^{\mu} dr. \tag{3}$$

To solve the Einstein-Maxwell system, we assume the metric potential $\mu(r)$ in the Buchdahl-Vaidya-Tikekar form

$$e^{\mu} = \sqrt{\frac{1+f(r)}{1-\frac{r^2}{L^2}}}, \tag{4}$$

The spacetime metric of a static and spherically symmetric object in the presence of an electric field is then obtained as

$$ds_{-}^2 = -(1+f(r))^{\frac{1}{2}}\left(a - b\sqrt{1-\frac{r^2}{L^2}}\right)^2 dt^2 + \frac{1+f(r)}{1-\frac{r^2}{L^2}}dr^2 + r^2 d\Omega^2, \tag{5}$$

where a, b and L are constants which can be determined by matching the solution to the exterior R-N metric at the boundary. The solution reduces to the Schwarzschild interior solution when $f(r) = 0$. The model would be fully determined when $f(r)$ is prescribed.

- We assume $f(r) = k\frac{r^2}{L^2}$ which is the Vaidya and Tikekar (VT) ansatz [J. Astrophys. Astro., **3**, 325 (1982)] for the modelling of a relativistic compact star.
- The ansatz is motivated by a geometric property that $t = constant$ hypersurface of the associated spacetime, when embedded in a 4-Euclidean space, is not spherical but spheroidal. The parameter k indicates a departure from the sphericity of associated 3-space.
- In this set-up, the charge is obtained as

$$q^2(r) = \frac{kr^6\left[L^2(2-k) + k(7+4k)r^2\right]}{8(L^2+kr^2)^3}.$$

- Thus, the parameter k gets coupled to charge distribution. When the charge is set to zero, the solution goes over to the Schwarzschild uniform density fluid sphere.

Charged analogue of Buchdahl bound

Demanding that physical quantities such as isotropic pressure and energy density must not diverge at the centre, we arrive at compactness bound for a charged sphere

$$u = \frac{M}{R} = \frac{8/9}{(1+\sqrt{1-\frac{8\alpha^2}{9}})}, \tag{6}$$

where $\alpha^2 = Q^2/M^2$ and $Q = q(R)$ is the total charge.

- The above result provides an upper bound on $\alpha^2 \leq 9/8$ and $u \leq 8/9 < 1$. For $\alpha^2 = 0$, we regain $u \leq 4/9$.
- Note: Similar result was earlier obtained by Giuliani and Rothman [Gen. Relativ. Gravit. **40**, 1427 (2008)] by adopting a different technique.
- See: *An electromagnetic extension of the Schwarzschild interior solution and the corresponding Buchdahl limit*, Ranjan Sharma, Naresh Dadhich, Shyam Das and Sunil D. Maharaj, Eur. Phys. J C (2021) **81**, 79.

Anisotropic generalization of Buchdahl bound

Due to various physical processes, local anisotropy is expected to develop in the interior of a compact star [See: Bowers and Liang, Astrophys. J., **188**, 657 (1974); Herrera and Santos, Phys. Report **286**, 53 (1997)].

A particular anisotropic model:

We assume the line element describing the interior of an anisotropic star in the form

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \qquad (7)$$

The energy-momentum tensor for the anisotropic fluid distribution is assumed in the form

$$T^i_j = diag(\rho, \ -p_r, \ -p_t, \ -p_t), \qquad (8)$$

where p_r and p_t are the radial and tangential pressure, respectively.

- Utilizing the Vaidya and Tikekar ansatz as well as the Karmarkar's embedding condition [Proc. Ind. Acad. Sci. A, **27**, 56 (1948), a particular solution of the system was obtained by Das *et al* [Das, Sharma, Chakraborty and Baskey, Gen. Rel. Grav., **52**, 101 (2020)]:

$$e^{\nu} = \left[C + D\sqrt{(k-1)(r^2-L^2)}\right]^2. \quad (9)$$

$$e^{\lambda} = \frac{1-k(\frac{r^2}{L^2})}{1-\frac{r^2}{L^2}}, \quad (10)$$

$$\Delta = p_t - p_r = \frac{(k-1)kr^2\left[D(k-2)\sqrt{r^2-L^2}+C\sqrt{k-1}\right]}{(L^2-kr^2)^2\left[D(k-1)\sqrt{r^2-L^2}+C\sqrt{k-1}\right]}. \quad (11)$$

Anisotropic extension of Buchdahl bound

- Imposition of the requirement that central pressure must not diverge yields
$$u = \frac{2M}{R} \leq \frac{4(k-2)}{(5k-9)}, \tag{12}$$
which may be treated as a generalization of the Buchdahl bound for an anisotropic stellar configuration.
- By setting $k = 0$, one regains the usual Buchdahl bound $2M/R \leq 8/9$.
- See: Anisotropic generalization of Buchdahl bound for specific stellar models, Ranjan Sharma, A. Ghosh, S. Bhattacharya and S. Das, Eur. Phys. J C, (2021) 81:527.

Compactness, stiffness, EOS and stability

- Note: The parameter k in the previous cases quantifies charge or anisotropy.
- The curvature parameter k in the Vaidya-Tikekar model has a clear geometric interpretation (as discussed earlier) which can also be associated with the matter composition (EOS) of the star as well.
- Another interesting stellar model that can accommodate varying EOS is the Tolman VII solution [Phys Rev. **55**, (1939) 364]. It is an exact analytic solution to the Einstein field equations, which has found its application in describing the interior structure of compact objects like neutron stars.
- Making use of the Tolman VII solution, we intend to analyze the impact of such an EOS parameter on the compactness of a star.

Many versions of Tolman VII solution

- Tolman ansatz:

$$e^{-\lambda(r)} = 1 - u\zeta^2(5 - 3\zeta^2), \tag{13}$$

where the parameter u represents the stellar compactness $(\frac{M}{R})$ and $\zeta = \frac{r}{R}$. The constant R represents the stellar radius and M is the total mass enclosed within a radius R.

- The particular choice of the metric potential is equivalent to choosing an energy density distribution inside the star as

$$\rho(r) = \rho_c(1 - \zeta^2), \tag{14}$$

where ρ_c is the central energy density given by

$$\rho_c = \frac{15M}{8\pi R^3}. \tag{15}$$

Standard Tolman VII solution

The integration process determines the unknown metric potential as

$$e^{\nu(r)} = c_1 \cos^2\phi, \tag{16}$$

where

$$\phi = c_2 - \frac{1}{2}\log(\zeta^2 - \frac{5}{6} + \sqrt{\frac{e^{-\lambda}}{3u}}). \tag{17}$$

c_1 and c_2 are integration constants which get determined from the boundary conditions (continuity of metric functions across the boundary and vanishing of pressure at the boundary). The isotropic pressure is obtained as

$$p = \frac{1}{4\pi R^2}[\sqrt{3ue^{-\lambda}}\tan\phi - \frac{u}{2}(5 - 3\zeta^2)]. \tag{18}$$

The solution is a two-parameter $[M, \rho_c]$ family of solutions.

Generalized Tolman VII solution

Raghoonundun and Hobill [Phys. Rev. D **92**, (2015) 124005] considered a more generalized form of the density profile

$$\tilde{\rho}(r) = \rho_c[1 - \mu\left(\frac{r}{R}\right)^2], \tag{19}$$

where $\mu[0,1]$ is a free parameter representing the 'stiffness' of the EOS of the star [Phys. Rev. D **93**, 24033 (2016)]. The parameter may vary between $0 \leq \mu \leq 1$. In the extreme case of $\mu = 0$, one obtains an incompressible fluid sphere model and $\mu = 1$ corresponds to the original Tolman VII solution.

With the above energy density profile, the system of equations may be integrated to yield

$$e^{\tilde{\lambda}} = \frac{1}{1 - (\frac{8\pi\rho_c}{3})r^2 + (\frac{8\pi\mu\rho_c}{5R^2})r^4} = \frac{1}{1 - br^2 + ar^4}, \qquad (20)$$

$$e^{\frac{\tilde{\nu}(r)}{2}} = \tilde{c_1}\cos(\tilde{\phi}\xi(r)) + \tilde{c_2}\sin(\tilde{\phi}\xi(r)), \qquad (21)$$

where $\tilde{\phi} = \sqrt{\frac{a}{4}}$, $\rho_c = \frac{15M}{4\pi R^3(5-3\mu)}$, and

$$\xi(r) = \frac{2}{\sqrt{a}}\coth^{-1}\left(\frac{1+\sqrt{1-br^2+ar^4}}{r^2\sqrt{a}}\right). \qquad (22)$$

The solution now becomes a three-parameter $[M,\, \rho_c,\, \mu]$ family of solutions.

Improved Tolman VII solition

For the better agreement of the energy density profile with realistic neutron star EOS, Jiang and Yagi [Phys. Rev. D **99**, 124029 (2019)] have recently proposed an improved version of the Tolman VII solution, which is also a three-parameter [M, ρ_v, α] family of solutions. In this approach, the energy density is assumed to be of the form

$$\rho_{im}(r) = \rho_c[1 - \alpha\zeta^2 + (\alpha - 1)\zeta^4], \qquad (23)$$

where $\alpha[0, 2]$ is a free parameter. Note that the original Tolman VII solution is $\alpha \to 1$.

With this assumption, the solutions are obtained as

$$e^{-\lambda_{im}} = 1 - 8\pi R^2 \zeta^2 \rho_c \left(\frac{1}{3} - \frac{\alpha}{5}\zeta^2 + \frac{\alpha - 1}{7}\zeta^4\right), \tag{24}$$

$$e^{\nu} = c_1^{im} cos^2 \phi_{im}, \tag{25}$$

with

$$\phi_{im} = c_2^{im} - \frac{1}{2}\log\left(\zeta^2 - \frac{5}{6} + \sqrt{\frac{5e^{-}\lambda_{Tol}}{8\pi R^2 \rho_C}}\right). \tag{26}$$

- The advantages of these solutions are the following:
- The parameter μ is a measure of 'stiffness'.
- The parameter α can be linked to the EOS of the matter composition
- It will be interesting to examine the effects of such parameters on the stability of the configuration.

Stability analysis: Chandrasekhar's method

- In 1964, Chandrasekhar [Phys. Rev. Lett. **12**, 114; Astrophys. J **140**, 417 (1964)] first developed a method to study the stability of a star against radial oscillations. Bardeen *et al* [Astrophys. J. **145**, 505 (1966)] presented a variety of methods to analyze the stability of a star against radial oscillations.

Let us consider a perturbation of the metric for a spherically symmetric star

$$ds^2 = -e^{\nu(r,t)}dt^2 + e^{\lambda(r,t)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \qquad (27)$$

where

$$\nu(r,t) = \nu_0(r) + \delta\nu(r,t) \qquad (28)$$

$$\lambda(r,t) = \lambda_0(r) + \delta\lambda(r,t), \qquad (29)$$

where ν_0 and λ_0 denote equilibrium configuration and $\delta\nu(r,t)$ and $\delta\lambda(r,t)$ are small perturbations from this configuration.

We also perturb the energy density ρ and pressure p

$$\rho(r,t) = \rho_0(r) + \delta\rho(r,t), \quad (30)$$
$$p(r,t) = p_0(r) + \delta p(r,t). \quad (31)$$

The small perturbation in radial parameter r is assumed to be

$$\delta r = u_n(r) e^{\frac{\nu_0(r)}{2}} e^{i\omega_n t}/r^2, \quad (32)$$

where $u_n(r)$ and ω_n are the amplitude and frequency of the n-th normal mode, respectively.

Utilizing energy conservation, baryon number conservation and Einstein's field equations, we arrive at the dynamical equation governing the stellar pulsation in its n-th normal mode ($n = 0$ gives the fundamental mode) which has the Sturm-Liouville's form

$$P(r)\frac{d^2u_n(r)}{dr^2} + \frac{dP}{dr} + [Q(r) + {\omega_n}^2 W(r)]u_n(r) = 0, \tag{33}$$

where the functions $P(r)$, $Q(r)$ and $W(r)$ are expressed in terms of the equilibrium configuration of the star given by

$$P(r) = \gamma p_0 e^{\frac{(\lambda_0+3\nu_0}{2}} r^{-2}, \tag{34}$$

$$Q(r) = e^{\frac{(\lambda_0+3\nu_0)}{2}}[\frac{{p_0'}^2}{r^2(p_0+\rho_0)} - \frac{4p_0'}{r^3} - \frac{8\pi p_0}{r^2}(\rho_0+p_0)e^{2\lambda_0}], \tag{35}$$

$$W = e^{\frac{(3\lambda_0+\nu_0)}{2}} r^{-2}(\rho_0+p_0), \tag{36}$$

For the fundamental mode of oscillation ($n = 0$), the pulsation equation takes the form

$$\omega_0^2 \int_0^R exp[\frac{1}{2}(3\lambda_0 + \nu_0)](p_0 + \rho_0)\frac{u_0^2}{r^2} dr \quad = \int_0^R exp(\frac{1}{2}(3\nu_0 + \lambda_0)) \left(\frac{p_0 + \rho_0}{r^2}\right) ([-\frac{2}{r} + \frac{d\nu_0}{dr} - \frac{1}{4}(\frac{d\nu_0}{dr})^2 + 8\pi p_0 \, exp(\lambda_0)] u^2 + \frac{dp_0}{d\rho_0}\left(\frac{du_0}{dr}\right)^2) dr, \tag{37}$$

where we substituted the varying adiabatic index

$$\gamma = \frac{p + \rho}{p} \frac{dp}{d\rho}.$$

A stellar configuration will be stable if ω is real. Since the integration of the left-hand side of the equation (37) is always positive definite, for stability, we need to show that the right-hand side of this equation is positive.

To integrate the right-hand side, we employ the method given by Bardeen *et al* [Astrophys. J. **145**, 508 (1966)]. We take a trial solution for u_0 as

$$u_0 = re^{\frac{\nu_0(r)}{2}} \tag{38}$$

with the following boundary conditions:

1. $$u_0 \approx r^3 \quad as \quad r \rightarrow 0, \tag{39}$$
2. The Lagrangian change in pressure(Δp) at the surface ($r = R$) must vanish which implies

$$\frac{du_0}{dr} \rightarrow 0 \quad as \quad r \rightarrow R. \tag{40}$$

Numerical analysis

To analyze the stability of the Tolman VII solution, we evaluate the right-hand side of (37). We analyze the stability of a given configuration in two different ways:

1. (i) By keeping the radius of the star fixed (we take $R = 10$ km).
2. (ii) By keeping the mass of the star fixed (we take $M = 1.4 M_{\odot}$).

Table: Compactness bound below which a stellar configuration remains stable (Tolman VII solution with $\mu = 1$). The radius is kept fixed at $R = 10$ km.

$Mass(M_\odot)$	$compactness(M/R)$	$Integral$
1.4	0.2065	$+ve$
1.6	0.236	$+ve$
1.8	0.2655	$+ve$
2	0.295	$+ve$
2.2	0.3245	$+ve$
2.4	0.354	$+ve$
2.5	0.3695	$+ve$
2.6	0.3835	$-ve$

Table: Compactness bound below which a stellar configuration remains stable (Tolman VII solution with $\mu = 1$). The mass is kept fixed at $M = 1.4M_{\odot}$.

$Radius(R)(Km.)$	$Compactness(M/R)$	$Integral$
10	0.2065	$+ve$
9	0.2294	$+ve$
8	0.2581	$+ve$
7	0.295	$+ve$
6	0.3441	$+ve$
5.3	0.3896	$-ve$

In table 1 and 2, it is interesting to note that the configuration in this model remains stable for maximum compactness ~ 0.38 which is below the Buchdal bound $(\frac{M}{R} < \frac{4}{9})$.

Stability range in case of generalized Tolman VII solution

The generalized Tolman VII solution admits a wide range of values of the stiffness parameter μ and hence the solution permits us to analyze the impact of 'stiffness' on the stability of a given configuration.

Table: Compactness bound below which a stellar configuration remains stable (Generalized Tolman VII solution with $\mu = 0.1$). The radius is kept fixed at $R = 10$ km.

$Mass(M)(M_\odot)$	$Compactness(\frac{M}{R})$	$Integral$
1.4	0.2065	$+ve$
1.8	0.2655	$+ve$
2.2	0.3245	$+ve$
2.6	0.3835	$+ve$
2.8	0.413	$+ve$
3	0.4425	$+ve$
3.1	0.45725	$-ve$

Table: Compactness bound below which a stellar configuration remains stable (Generalized Tolman VII solution with $\mu = 0.6$). The radius is kept fixed at $R = 10$ km.

$Mass(M)(M\odot)$	$Compactness(\frac{M}{R})$	$Integral$
1.4	0.2065	$+ve$
1.8	0.2655	$+ve$
2.2	0.3245	$+ve$
2.6	0.3835	$+ve$
2.8	0.413	$+ve$
2.9	0.42775	$-ve$

Table: Compactness bound below which a stellar configuration remains stable (Generalized Tolman VII solution with $\mu = 0.9$). The mass is kept fixed at $M = 1.4M_{\odot}$.

Radius(R)(K.m.)	*Compactness(M/R)*	*Integral*
10	0.2065	$+ve$
9	0.229	$+ve$
8	0.258	$+ve$
7	0.295	$+ve$
6	0.344	$+ve$
5	0.413	$-ve$

Table: Stiffness parameter (μ) and the critical bound on compactness $(M/R)_{max}$.

μ	$(M/R)_{max}$
0.1	0.44075
0.6	0.417
0.9	0.3938
1	0.37245

Stability range in case of improved Tolman VII solution

The improved Tolman VII solution admits a wide range of values of the EOS parameter α. A similar analysis yields the following:

Table: Relationship between the EOS parameter α and critical bound on compactness $(M/R)_{max}$.

α	$u_{critical}$
0.8	0.439362
1	0.382407
1.5	0.338525

- The results show a correlation between stiffness vis-a-vis EOS of the matter composition and the stability of the configuration.
- The most compact object has a homogeneous distribution of matter i.e., a constant density star with $\mu = 0$. As inhomogeneity in the matter distribution increases, the critical upper bound on compactness decreases.

Complexity and stability

- The idea of ‘Complexity; within a self-gravitating system stems from the basic understanding that the simplest system is a homogeneous fluid distribution having isotropic pressure. Such a system is assigned to have zero complexity.
- However, an anisotropic system (principal stresses unequal) with inhomogeneous energy density distribution can have zero complexity provided the terms implying density inhomogeneity and anisotropic pressure cancel each other.

- Following Herrera's [Phys. Rev. D. **97** (2018) 044010] concept of 'cracking' method, Abreu *et al* [Class. Quantum Grav. **24** (2007) 4631] have formulated a criterion which determines a potentially stable or unstable region. It is observed that the region having $-1 \leq v_{st}^2 - v_{sr}^2 \leq 0$ would be a potentially stable region whereas the region for which $0 < v_{st}^2 - v_{sr}^2 \leq 1$ is expected to be potentially unstable, where $v_{st}^2 = \frac{dp_t}{d\rho}$, $v_{sr}^2 = \frac{dp_r}{d\rho}$.
- Ratanpal [IOP SciNotes, **I** (2020) 025207] has recently shown that a spherically symmetric anisotropic matter distribution would be potentially stable provided the gradient of anisotropy $p_t - p_r$ remains an increasing function of the radial variable r.
- We provide new criteria for the stability of a spherically symmetric anisotropic star with zero 'complexity'.

Mathematical framework

Consider a spherically symmetric distribution of anisotropic fluid expressed by the line element

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{41}$$

The energy-momentum tensor of the anisotropic fluid distribution is assumed to be

$$T^i_j = \mathrm{diag}(\rho,\ -p_r,\ -p_t,\ -p_t), \tag{42}$$

Making use of Herrera's approach, we define the 'complexity' factor of the stellar configuration as

$$Y_{TF} = 8\pi\Pi - \frac{4\pi}{r^3}\int_0^r \tilde{r}^3\rho'\tilde{d}r, \tag{43}$$

where $\Pi = p_r - p_t$ is the anisotropy. Vanishing of the 'complexity' implies

$$\Pi = \frac{1}{2r^3}\int_0^r \tilde{r}^3\rho'\tilde{d}r. \tag{44}$$

Stability criterion for zero ‘complexity’

Theory: *For a static, spherically symmetric, anisotropic distribution of matter with decreasing energy density function, if the ‘complexity’ at all interior points of the distribution vanishes, the configuration will be potentially stable if the tangential pressure remains greater than the radial pressure at each radial point of the configuration.*

Proof: We differentiate equation (43) with respect to radial variable r and obtain

$$\frac{dY_{TF}}{dr} = 8\pi \frac{d\Pi}{d\rho}\frac{d\rho}{dr} - 4\pi[-\frac{3}{r^4}\int_0^r \tilde{r}^3 \rho' \tilde{d}r + \rho']. \quad (45)$$

Vanishing 'complexity' implies $Y_{TF} = 0 = Y'_{TF}$, where a prime $'$ denotes differentiation with respect to r. Equation (45) then takes the form

$$\frac{d\Pi}{d\rho} = \frac{-\frac{3}{r^4}\int_0^r \tilde{r}^3 \rho' \tilde{d}r + \rho'}{2\frac{d\rho}{dr}}. \quad (46)$$

Using equation (44) in (46), we obtain

$$\frac{d\Pi}{d\rho} = \frac{1}{2\rho'}[-\frac{3}{r^4}2r^3\Pi + \rho'], \quad (47)$$

which is equivalent to

$$v_{st}^2 - v_{sr}^2 = \frac{1}{2}[\frac{6}{r}\frac{\Pi}{\rho'} - 1] \quad (48)$$

Since a potentially stable region is characterized by the condition $-1 \leq v_{st}^2 - v_{sr}^2 \leq 0$, in our case, we must have

$$\frac{1}{2}[\frac{6}{r}\frac{\Pi}{\rho'} - 1] \leq 0, \tag{49}$$

which implies that we must have

$$\Pi < \rho' \frac{r}{6}. \tag{50}$$

Since, one assumes $\rho' < 0$ for a physically reasonable stellar configuration, condition (50) yields

$$p_r < p_t. \tag{51}$$

Summary

- We have constructed a charged stellar solution which is a generalization of the homogeneous density distribution in which the parameter k gets coupled to the charge distribution. When the charge is set to zero, the solution goes over to the Schwarzschild uniform density fluid sphere. The solution facilitates the computation of the charged analogue of the Buchdahl compactness limit.
- We have also obtained an anisotropic generalization of the Buchdahl bound. For a physically reasonable stellar configuration, the bound was obtained by demanding that the central pressure must not diverge.
- Making use of the generalized Tolman VII solution, we have analyzed the stability of a given relativistic stellar configuration in the context of its stiffness.
- Finally, for an anisotropic stellar configuration possessing zero 'complexity', we have proposed a stability criterion.

Thank you!