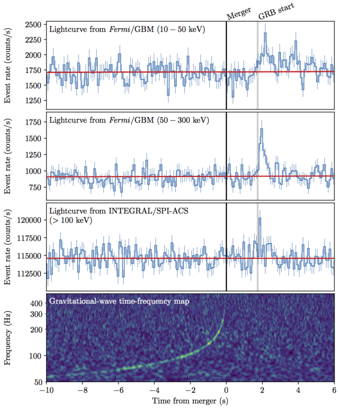
Recovery schemes in numerical GR MHD simulation of the post-merger system with a composition-dependent equation of state

Agnieszka Janiuk

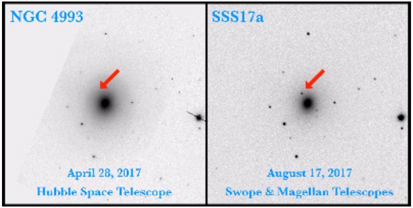
Center for Theoretical Physics
Polish Academy of Sciences
Warsaw

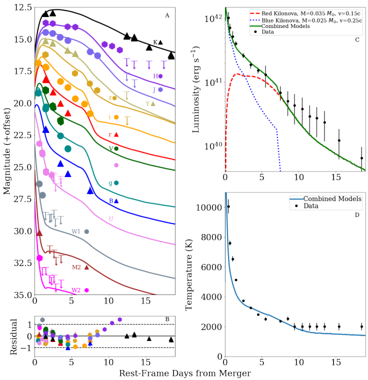
GW-GRB 170817

- First detection of gravitational wave signal accompanied with electromagnetic counterpart
- The GW waveform consistent with merger of two neutron stars, with a total mass of $2.82M_\odot$
- High energy emission in a form of a weak short GRB was detected about 1.74 second after the merger.



Kilonova observation

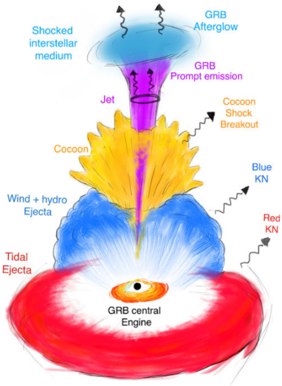




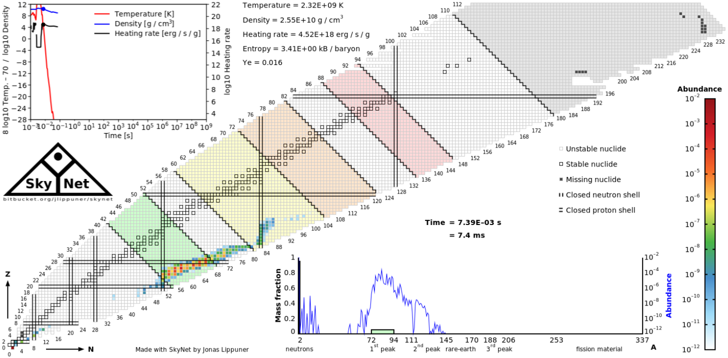
Rapidly fading electromagnetic transient in the galaxy NGC4993, is spatially coincident with GW170817 and a weak short gamma-ray burst (e.g., Smartt et al. 2017; Zhang et al. 2017, Coulter et al. 2017, Murguia-Berthier et al. 2017)

Kilonova

- NS-NS mergers eject material rich in heavy radioactive isotopes. Powering electromagnetic signal called a kilonova (e.g. Li & Paczynski 1998; Tanvir et al. 2013, Berger 2016).
- Dynamical ejecta from compact binary mergers, $M_{\rm ej} \sim 0.01 M_{\odot}$, can emit about $10^{40} - 10^{41}$ erg/s in a timescale of 1 week
- Subsequent accretion can provide bluer emission, if it is not absorbed by precedent ejecta (Tanaka, 2016)



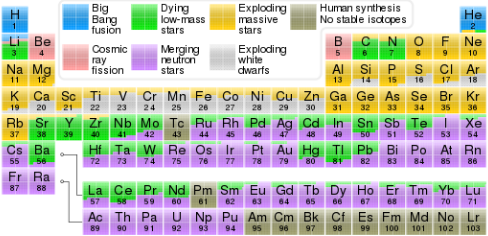
r-process nucleosynthesis

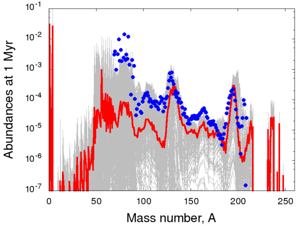


Nuclear reaction network calculation.

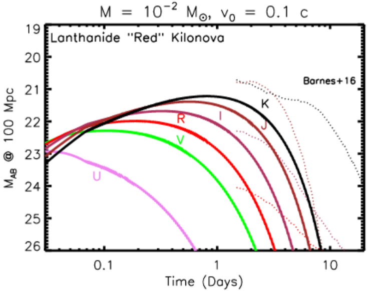
See e.g. (Janiuk A., 2014, A&A; 2019, ApJ) for studies of nucleosynthesis in black hole accretion disks as the GRB central engines)

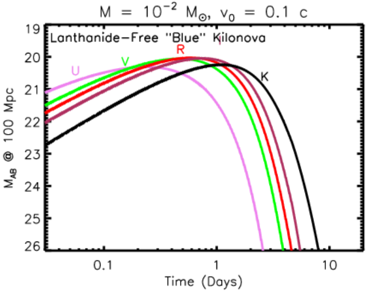
Origin of the heavy elements

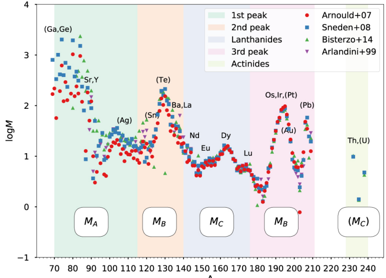




Kilonova colors







- 2nd peak

  $$Y_e \equiv \frac{n_e}{n_b} = (0.25, 0.5)$$

- 3rd peak

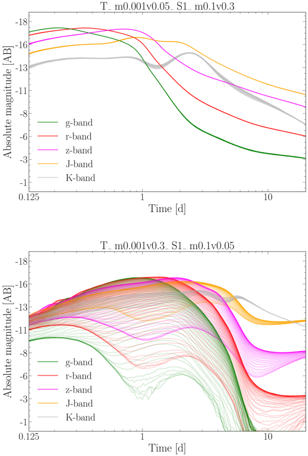
  $$Y_e = (0.15, 0.25)$$

  (Ji et al. 2019)

Radioactive decay power

- Day-timescale emission comes at optical wavelengths from lanthanide-free components, and followed by week-long emission with a spectral peak in the near-infrared
- Monte-Carlo radiative transfer code (Wollaeger et al. 2021)

Figure 3. from A Broad Grid of 2D Kilonova Emission Models
null 2021 APJ 918 10 doi:10.3847/1538-4357/ac0d03
http://dx.doi.org/10.3847/1538-4357/ac0d03



Our tool: solving GR MHD simulations

HARM code: High Accuracy Relativistic Magnetohydrodynamics (Gammie et al. 2003). The code provides solver for continuity and energy-momentum conservation and induction equations in GR:

$$\nabla_\mu(\rho u^\mu) = 0; \quad \nabla_\mu T^\mu_\nu = 0; \quad \nabla_\mu(u^\nu b^\mu - u^\mu b^\nu) = 0$$

Energy tensor contains in general the gas and electromagnetic parts:

$$T^{\mu\nu} = T^{\mu\nu}_{\rm gas} + T^{\mu\nu}_{EM}$$

$$T^{\mu\nu}_{gas} = \rho h u^\mu u^\nu + p g^{\mu\nu} = (\rho + u + p) u^\mu u^\nu + p g^{\mu\nu}$$

$$T^{\mu\nu}_{EM} = b^2 u^\mu u^\nu + \frac{1}{2} b^2 g^{\mu\nu} - b^\mu b^\nu; \quad b^\mu = u_\nu {}^{*}F^{\mu\nu}$$

where u is internal energy, u^μ is four-velocity of gas, and $b^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} u_\nu F_{\rho\sigma}$. In force-free approximation, $E_\nu = u_\mu F^{\mu\nu} = 0$.
EOS in simplest case is that of ideal gas

$$p = K\rho^\gamma = (\gamma - 1)u$$

Initial Conditions

- After the two neutron stars merge, the remnant is a transient HMNS object, and then collapses to a black hole.
- BH is surrounded by a remnant accretion disk, that contains highly neutron-rich material.
- Initial conditions for our model invoke torus in the pressure equilibrium.
- Pressure maximum location, r_{max}, is our parameter, and the specific angular momentum is constant with radius (cf. Fishbone & Moncrief 1978)
- We adopt weak magnetic field in poloidal configuration, set by its vector potential $A_\phi = max(\rho/\rho_{max} - \rho_0, 0)$ and normalized to constant gas-to-magnetic pressure ratio $\beta = 100$.

Chemical composition and structure of the disk

- Due to high density, disk is opaque to photons.
- Neutrinos, created via β-reactions, electron-positron anihillation, and plasmon decay.
- Disk is composed of free nucleons, electron-positron pairs, and Helium nuclei.
- Magnetic fields transport angular momentum outwards, via turbulence. Disk becomes thinner, and accretion proceeds.
- Magnetic dissipation help drive unbound outflows from the disk surface.
- In addition, neutrino-driven wind ejects material. As the ejecta expand, they cool down, and heavy elements are synthesized via rapid neutron capture process.

Set of equations for evolving neutrino cooled disk

- Neutrinos carry away energy and lepton number, so they alter electron fraction and composition of ejected material.
- Dynamical simulations must consider the realistic equation of state (EOS) and impact of neutrinos in the optically thin and thick regions.

Lepton number conservation

$$\nabla_\mu(n_e u^\mu) = \mathcal{R}/m_b; \quad m_b = \rho/n_b; \quad Y_e = \frac{n_e}{n_b} = \frac{m_b n_e}{\rho}$$

where Y_e is the electron fraction.
Because the baryons dominate the rest-mass density, the baryon number conservation equation turns to regular continuity equation.
In the energy-momentum conservation equation, we will have a **source term** due to heating and cooling by neutrinos.

$$\nabla_\mu T^\mu_\nu = \mathcal{Q} u_\nu$$

Conserved variables

- GR MHD scheme is evolving conserved variables

$$\partial_t \mathbf{U}(\mathbf{P}) = -\partial_i \mathbf{F}^i(\mathbf{P}) + \mathbf{S}(\mathbf{P})$$

  which for $\mathbf{S} = 0$ depend only on fluxes at boundaries.
- $\mathbf{P}$ are the 'primitive variables, which are subject to the EOS. In non-relativistic MHD, both $\mathbf{P} \to \mathbf{U}$ and $\mathbf{U} \to \mathbf{P}$ have a closed-form solution. In GRMHD $\mathbf{U}(\mathbf{P})$ is a complicated, nonlinear relation. Inversion $\mathbf{P}(\mathbf{U})$ is calculated once or twice in every time step, numerically, by the recovery scheme.

Conserved variables

Explicit form of primitive and conserved variables, fluxes, and source terms, is:

$$\mathbf{P} = [\rho,\ \mathcal{B}^k,\ \tilde{u}^i,\ Y_e,\ T]$$

$$\mathbf{U}(\mathbf{P}) = \sqrt{-g}[\rho u^t,\ T^t_t + \rho u^t,\ T^t_j,\ B^k,\ \rho Y_e u^t]$$

$$\mathbf{F}^i(\mathbf{P}) = \sqrt{-g}[\rho u^i,\ T^i_t + \rho u^i,\ T^i_j,\ (b^i u^k - b^k u^i),\ \rho Y_e u^i]$$

$$\mathbf{S}(\mathbf{P}) = \sqrt{-g}[0,\ T^\kappa_\lambda \Gamma^\lambda_{t\kappa} + \mathcal{Q} u_t,\ T^\kappa_\lambda \Gamma^\lambda_{t\kappa} + \mathcal{Q} u_i,\ 0,\ \mathcal{R}]$$

Here $B^i = \mathcal{B}^i/\alpha = {}^*F^{it}$ is magnetic field, and $\tilde{u}^\mu = (\delta^\mu_\nu + n^\mu n_\nu)u^\nu$ is the projected four-velocity, where the orthognal frame velocity is $n_\mu = [-\alpha, 0, 0, 0]$, $n^\mu = [1/\alpha, -\beta^i/\alpha]$, with lapse $\alpha = 1/\sqrt{g^{tt}}$, and shift function $\beta^i = -g^{ti}/g^{tt}$.
The fluid three velocity is then $v^i = \tilde{u}^i/\gamma = \tilde{u}^i/\alpha u^t$.

Recovery transformation

- No analytic expression of the primitive variables in terms of conserved variables
- Transformation between 'conserved' (momentum, energy density) and 'primitive' (rest mass density, internal energy) variables requires to solve a set of 5 non-linear equations
- Inversion is complex for a non-adiabatic relation of the pressure with density

Equations of state

There can be two main types of EOS

1. Analytic
   - Polytrope $p(\rho) = \kappa\rho^{\Gamma}$, $\epsilon(\rho) = \frac{\kappa}{\Gamma-1}\rho^{\Gamma-1}$
   - Ideal gas (gamma-law): $p(\rho) = (\Gamma - 1)\rho\epsilon$
2. Tabulated
   - 1-parameter $\epsilon(\rho)$, $P_{\epsilon}(\rho)$
   - 2-parameter $\epsilon(\rho, T)$, $P_{\epsilon}(\rho, T)$
   - 3-parameter $\epsilon(\rho, T, Y_e)$, $P_{\epsilon}(\rho, T, Y_e)$

It is also possible to construct hybrid EOS, with analytic and tabulated components, e.g. depending on temperature range

Recovery methods overview

We need to recover the primitive variables, as they are required to construct $T^{\mu\nu}$ and flux terms in fluid evolution. Recovery methods can lead to bounded or unbounded solution

- Newton-Raphson methods are unbounded. They have faster convergence but are less stable
- Bracketed root finding methods are slower, but more robust

A simple 1D method can be used for an analytic EOS.

- We find a root of equation, by means of NR method

$$f(p) = p - \bar{p}(\rho(\mathbf{U}, p), \epsilon(\mathbf{U}, p)) = 0$$

  where p is the current pressure guess, and $\bar{p}(\rho, \epsilon)$ is pressure found from the EOS.
- The derivative of df/dp is needed. Obtaining $\frac{\partial \bar{p}}{\partial \rho}$ and $\frac{\partial \bar{p}}{\partial \epsilon}$ is easy only when EOS is analytic.

Tabulated EOS used in HARM_COOL code

- In the non-adiabatic regime, we employ a tabulated equation of state, where thermodynamic variables (e.g. pressure, energy, and speed of sound) and chemical potentials of species (incl. neutrinos) are given as a function of density, temperature and electron fraction.
- Linear interpolation is done to find variables over the computational grid (Janiuk et al., 2019; cf. O'Connor & Ott 2010)
- High density regime: fluid consists of free n, p, e^+, e^-, alpha particles and photons.
- Low density regime: nuclear statistical equilibrium (NSE)

Recovery schemes: 2D and 3D

- In a 2D scheme, there are two independent variables, e.g, v^2 and specific enthalpy W. Temperature is obtained from the tables, solving $h = h(\rho, T, Y_e)$, and pressure and temperature are also obtained by Newton-Raphson method for W and v^2.
- Alternatively, the system of GR MHD equations is reduced to 3 equations, which have three unknowns.
  The chosen independent variables can be: γ, T, and $W = h\rho\gamma^2$. Pressure is interpolated from EOS tables, as $P(\rho, T, Y_e)$.

2D recovery scheme of Noble et al. (2006)

The scheme reduces the dimensionality of the recovery problem by making use of certain scalar quantities that can be computed from the conservatives. This 2D scheme solves simultaneously the set of two equations:

$$f_1: \quad \tilde{Q}^2 = v^2(\mathcal{B}^2 + W)^2 - \frac{(Q_\mu \mathcal{B}^\mu)^2(\mathcal{B}^2 + 2W)}{W^2}$$

$$f_2: \quad Q_\mu n^\mu = -\frac{\mathcal{B}^2}{2}(1 + v^2) + \frac{(Q_\mu \mathcal{B}^\mu)^2}{2W^2} - W + p(u, \rho)$$

2D recovery scheme of Noble et al. (2006)

The independent variables used in this scheme are defined as:
$Q_\mu = -n_\mu T^\nu_\mu = \alpha T^t_\mu$;
where $\tilde{Q}^\nu = j^\nu_\mu Q^\mu$ is energy-momentum density in the normal observer frame,
and $D = -\rho n_\mu u^\mu = \alpha \rho u^t = \gamma \rho$;
is mass density in the observer's frame,
and $\mathcal{B}^i = \alpha B^i = \alpha {}^*F^{it}$ is magnetic 3-vector.
and $w = \rho + u + p$ with $W = w\gamma^2$ is enthalpy
(note it is original notation)
To solve this 2-D set of equations by means of Newton-Raphson method, the Jacobian matrix with $\frac{\partial f_1}{\partial(v^2)}$, $\frac{\partial f_2}{\partial(v^2)}$, $\frac{\partial f_1}{\partial W}$, and $\frac{\partial f_2}{\partial W}$ is needed. Note that this scheme does not require an analytic EOS, and derivatives of pressure wtr. to ρ, v^2, and u may be computed from tables using finite difference method.

3D recovery scheme of Siegel et al. (2018)

The system is extended to solve 3 equations, on W, z, and T, by adding a constraint on the internal energy given by EOS tables.

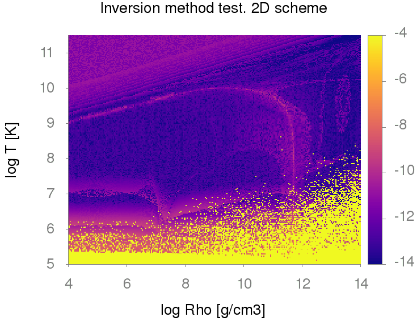
$$f_1: \quad [\tau + D - z - B^2 + \frac{B^i S_i}{2z^2} + \rho]W^2 - \frac{B^2}{2} = 0$$

$$f_2: \quad [(z + B^2)^2 - S^2 - \frac{2z + B^2}{z^2}(B^i S_i)^2]W^2 - (z + B^2)^2 = 0$$

$$f_3: \quad \epsilon - \epsilon(\rho, T, Y_e) = 0$$

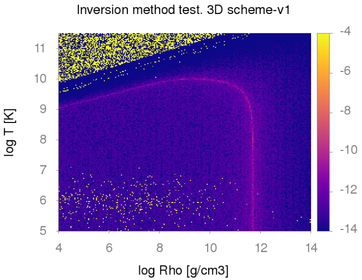
Here the temperature is employed directly as an unknown through $\epsilon(W, zT) = h - 1 - \frac{P}{\rho} = \frac{z - DW - \rho W^2}{DW}$ and does not require inversion of the EOS. Notice here different notation: S_i is energy-momentum density, W is Lorentz factor, z is enthalpy, and $\tau = -(n_\mu n_\nu T^{\mu\nu} + D)$.

Convergence tests



- Parameters:
  $Y_e = 0.1$, $\gamma = 2$, $p_{gas}/p_{mag} = 10^5$.
- Conserved variables derived in Kerr metric, then primitives perturbed by a factor of 1.05.
- Variables recovered through the 2D scheme compared to the unperturbed, to calculate $Err = \Sigma_{k=0,NPR}(P_k - \bar{P_k})^2$

Convergence tests



- Method 1: we compute specific internal energy from state vector x and conservatives as in Eq. (25) in Cerda-Duran et al. (2008), and solve f_3
- Method 2: we compute pressure from state vector x and conservatives and solve f_3

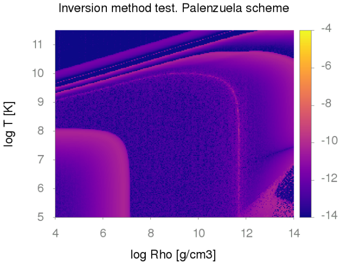
Palenzuela method

The scheme is solving 1D equation, for the rescaled variable

$$\chi = \frac{\rho h \gamma^2}{\rho \gamma}$$

Other quantities are also rescaled, accordingly, to give Lorentz factor (cf. Palenzuela et al., 2015), and we give the brackets for χ:

$$2 - 2\frac{Q_\mu n^\mu + D}{D} - \frac{\mathcal{B}^2}{D} < \chi < 1 - \frac{Q_\mu n^\mu + D}{D} - \frac{\mathcal{B}^2}{D}$$



The equation

$$f(\chi) = \chi - \tilde{\gamma}(1 + \tilde{\epsilon} + \frac{\tilde{P}}{\tilde{\rho}}) = 0$$

is solved, with $\tilde{P} = P(\tilde{\rho}, \tilde{\epsilon}, Y_e)$ found in tables.

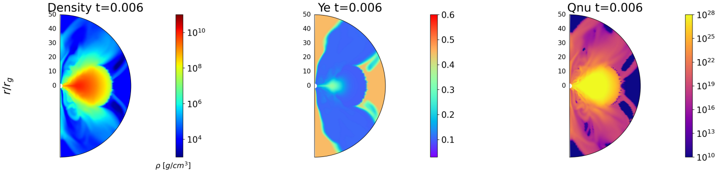
Neutrino transport

- We employ the neutrino leakage scheme that computes a gray optical depth estimate along radial rays for electron neutrinos, electron antineutrinos, and heavy-lepton neutrinos (nux), and then computes local energy and lepton number loss terms.
- Source code of the scheme downloaded from *https://stellarcollapse.org*. Details: O'Connor & Ott (2010), Ott et al. (2012).

Evolving density and electron fraction

Initial tests within an optically thin regime for neutrinos.

$$\tau(r,\theta,\phi) = \int_r^R \sqrt{\gamma_{rr}}\bar{\kappa}_{\nu_i} dr' < 2/3$$



Now, we calculate both $\mathcal{Q}$ and $\mathcal{R}$ which are energy loss rate and net rate of emission/absorption of neutrinos per unit volume, and they enter to the source terms in GR MHD equations.

Work in progress!!!

Conclusions

- GR MHD simulations have been widely used to model engines of gamma ray bursts. Now implemented to the case study of post-merger system and kilonova source
- Numerical calculations are complex and need to cover physics of dense nuclear matter. Their performance is sensitive to the chosen recovery schemes
- Proper source terms need to be added in the system of equations to describe evolution of neutrino losses, coupled with composition changes

Thank you for attention!