The most massive galaxy clusters

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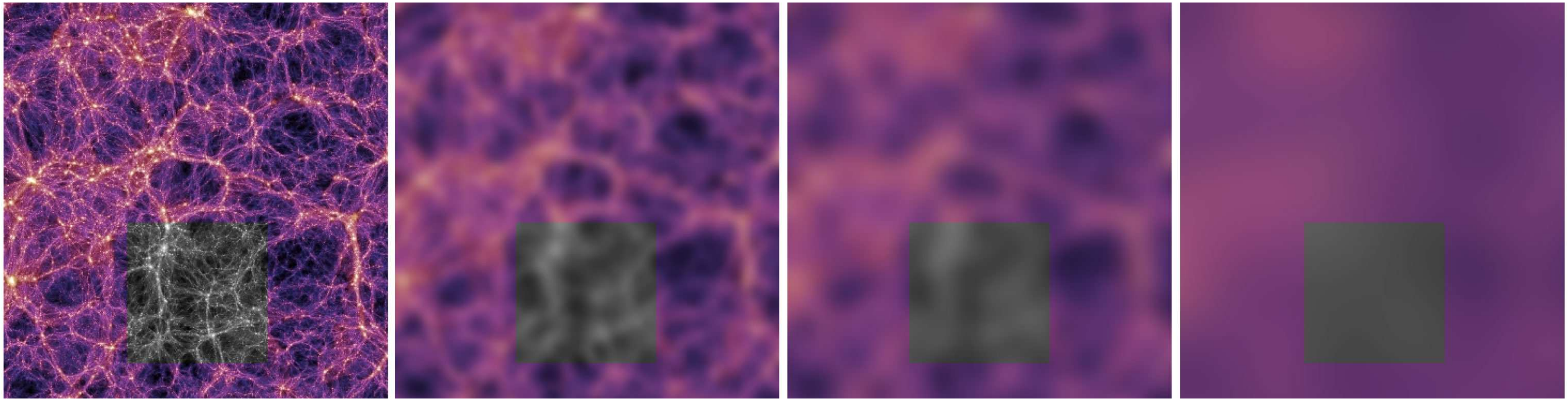
The Λ CDM model – challenges

C. Clarkson, G. Ellis, J. Larena, and O. Umeh, Rept. Prog. Phys. 74 (2011):

- **Averaging**: Coarse-graining of structure, such that small-scale effects are hidden to reveal large scale geometry and dynamics.
- **Backreaction**: Gravity gravitates, so local gravitational inhomogeneities may affect the cosmological dynamics. How this is calculated depends on the degree of coarse graining
- **Fitting**: How do we appropriately fit an idealized model to observations made from one location in a lumpy universe, given that this ’background’ does not in fact exist?



Falsifying the Λ CDM model

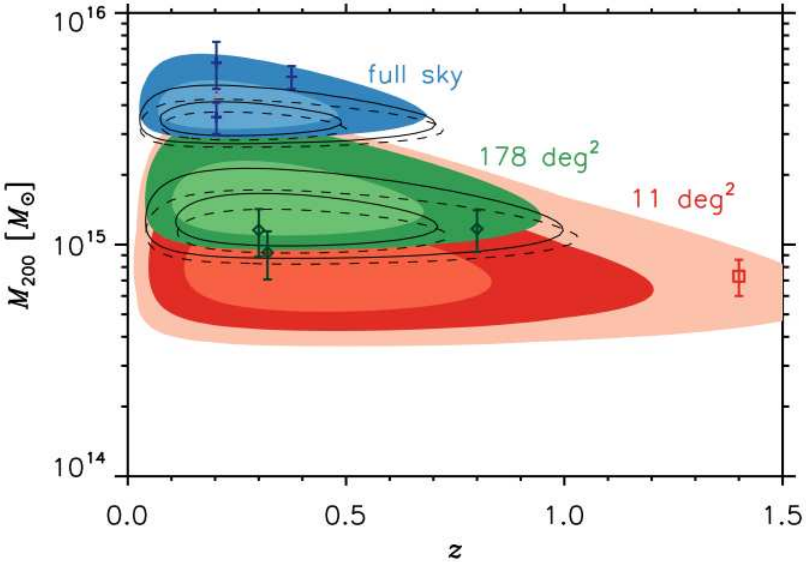
- The Clarkson-Bassett-Lu test (Clarkson, Bassett, Lu, 2008):

$$\mathcal{C}(z) = 1 + H^2\left(DD'' - D'^2\right) + HH'DD' \quad , \quad D = (1+z)d_A$$

- The distance sum rule (Rasanen, Bolejko, Finoguenov, 2015):

$$k_S = -\frac{d_l^4 + d_s^4 + d_{ls}^4 - 2d_l^2 d_s^2 - 2d_l^2 d_{ls}^2 - 2d_s^2 d_{ls}^2}{4d_l^2 d_l^2 d_{ls}^2}$$

- The most massive objects in the Universe (Holz, Perlmutter, 2012)
'*If objects are found with excessively large masses, or insufficient objects are found near the maximum expected mass, this would be a strong indication of the failure of* Λ*CDM*'



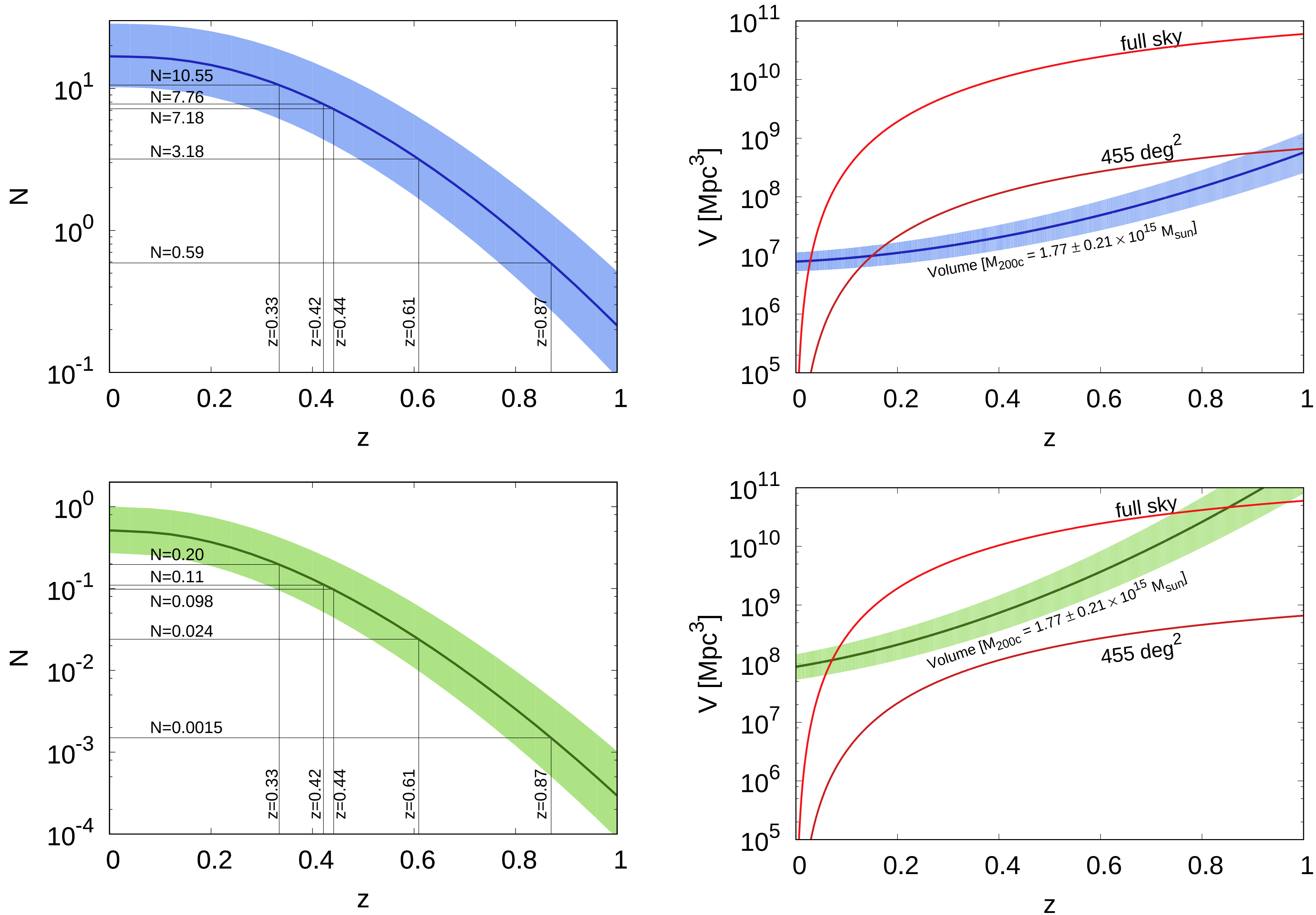
The silent universe mass function

The silent universe equations:

$$\begin{aligned}
\dot{\rho} &= -\rho\theta \\
\dot{\theta} &= -\frac{1}{3}\theta^2 - 4\pi G\rho - 6\sigma^2 + \Lambda \\
\dot{\sigma} &= -\frac{2}{3}\theta\sigma + \sigma^2 - \mathcal{E} \\
\dot{\mathcal{E}} &= -\theta\mathcal{E} - 4\pi G\rho\sigma - 3\sigma\mathcal{E}
\end{aligned}$$

The mass function of galaxy clusters:

$$\frac{\mathrm{dn}}{\mathrm{dlnM}} = \frac{\rho_0}{\mathrm{M}} f\left(\sigma_{\mathrm{M}}\right)\left|\frac{\mathrm{dln}\sigma_{\mathrm{M}}}{\mathrm{dlnM}}\right| \quad , \quad N = \int_{\mathrm{z_{min}}}^{\mathrm{z_{max}}} \mathrm{dz}\frac{\mathrm{dV}}{\mathrm{dz}} \int_{\mathrm{M_{min}}}^{\mathrm{M_{max}}} \mathrm{dM}\frac{\mathrm{dn}}{\mathrm{dM}}$$



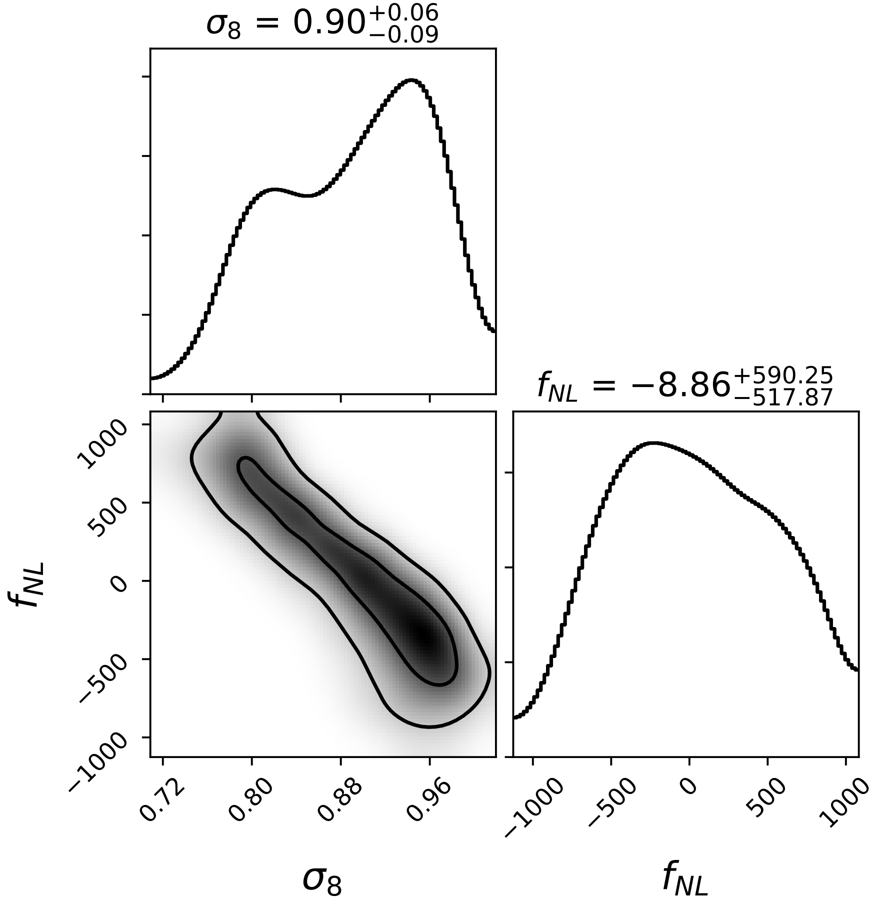
Bolejko, Ostrowski, 2020

Apparent non-gaussianities

- We keep all the cosmological parameters fixed, except for the σ_8 and f_{NL}

- We look at the mass range: $10^{15} - 10^{16}\mathrm{M}_{\mathrm{sun}}$

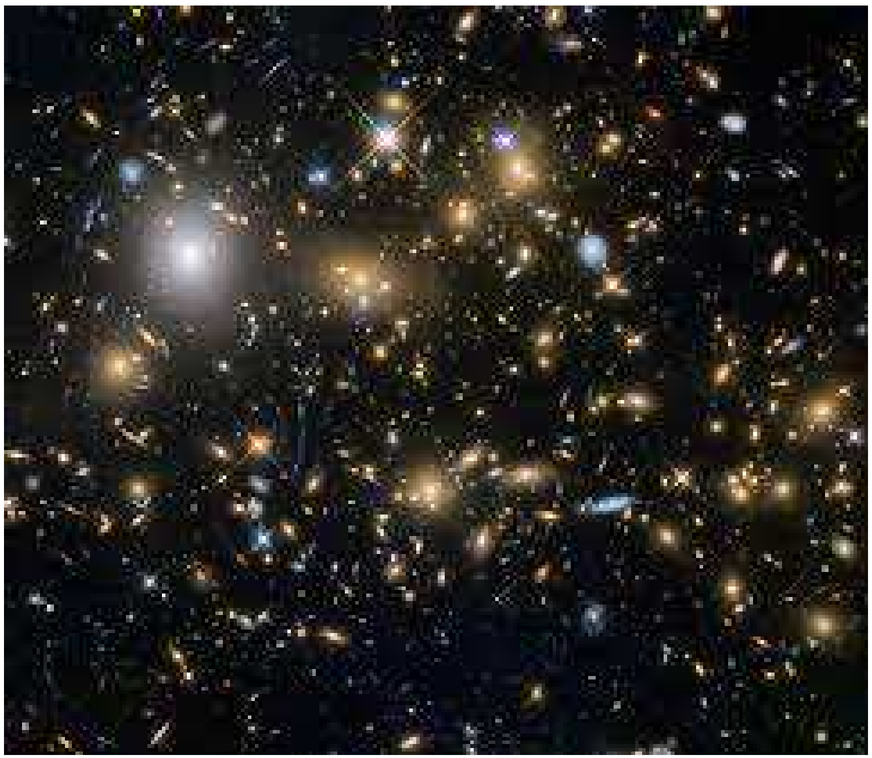
- If our mass function of the galaxy clusters is correct, then the N–body–derived mass function would bias the σ_8 and f_{NL}

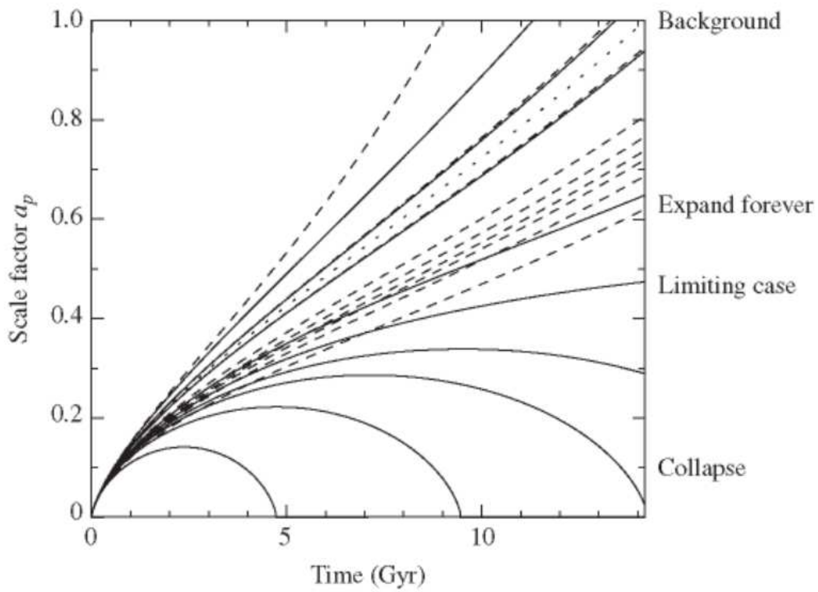


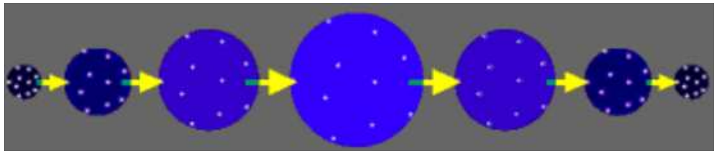
Bolejko, Ostrowski, 2020

The turnaround radius: $\theta = 0$

Initially expanding overdensity reaches the turnaround and then collapses







NASA, ESA, STScI/AURA, J. Blakeslee, H. Ford; Press, Schechter, 1971; http://astronomy.swin.edu.au

The turnaround radius (Pavlidou, Tomaras, 2014):

$$R_{max} = \left(\frac{3GM}{\Lambda}\right)^{1/3}$$

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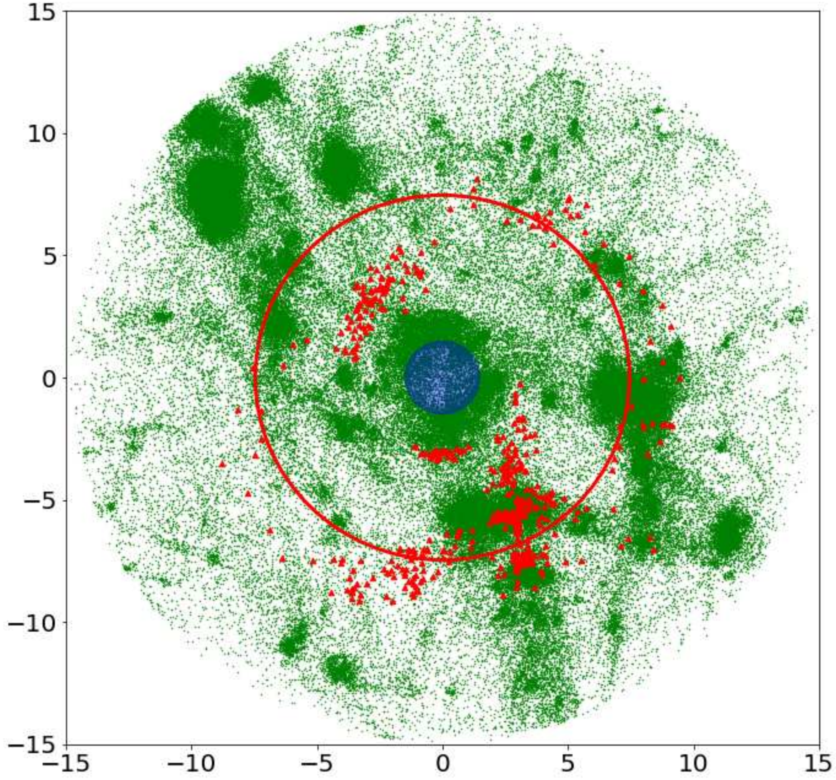

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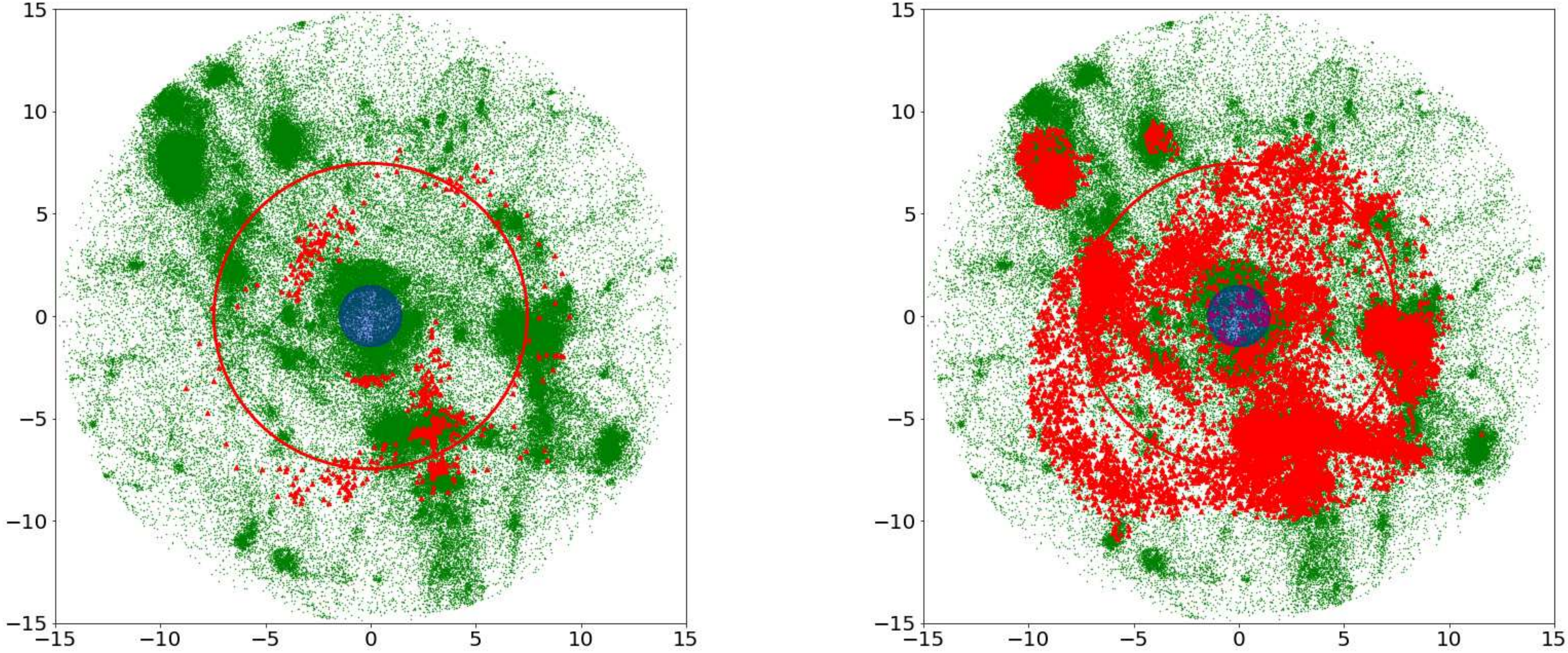


Hansen, Hassani, Lombriser, Kunz, 2020

The maximum volume: $\langle\theta\rangle = 0$

- **Instead of looking for the zero-expansion surface, we can look for the maximum volume**

- **Hamiltonian constraint in the synchronous, co-moving coordinates:**

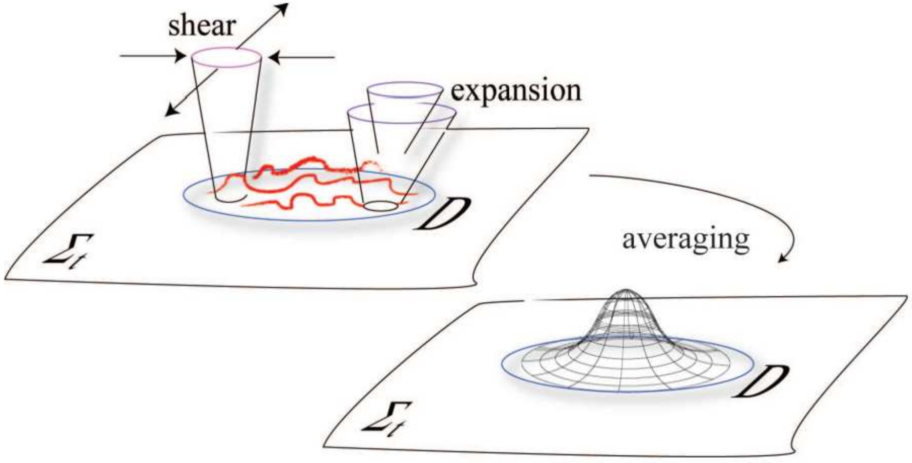
$$\frac{1}{3}\theta^2 = 8\pi G\rho + \sigma^2 - \frac{1}{2}\mathcal{R} + \Lambda$$

- **Applying the averaging operator:**

$$\langle\mathcal{A}\rangle = \frac{1}{V_{\mathcal{D}}}\int_{\mathcal{D}} \mathrm{d}\mu \ , \ \langle\theta\rangle_{\mathcal{D}} = \frac{\partial_t V_{\mathcal{D}}}{V_{\mathcal{D}}}$$

and the non-commutation formula:

$$\partial_t\langle\mathcal{A}\rangle_{\mathcal{D}} - \langle\partial_t\mathcal{A}\rangle_{\mathcal{D}} = \langle\mathcal{A}\theta\rangle_{\mathcal{D}} - \langle\mathcal{A}\rangle_{\mathcal{D}}\langle\theta\rangle_{\mathcal{D}}$$



Buchert, Carfora, 2008

The maximum volume: $\langle\theta\rangle = 0$

- To calculate the evolution of volume we have to close the averaged equations - we use the relativistic Zel'dovich approximation (extrapolated 1st order Lagrangian perturbation theory)

- To find the maximum volume we insert the turnaround condition:

$$\langle\theta\rangle_{\mathcal{D}} = \frac{\partial_t V_{\mathcal{D}}}{V_{\mathcal{D}}} = 0$$

- The resulting formula reads:

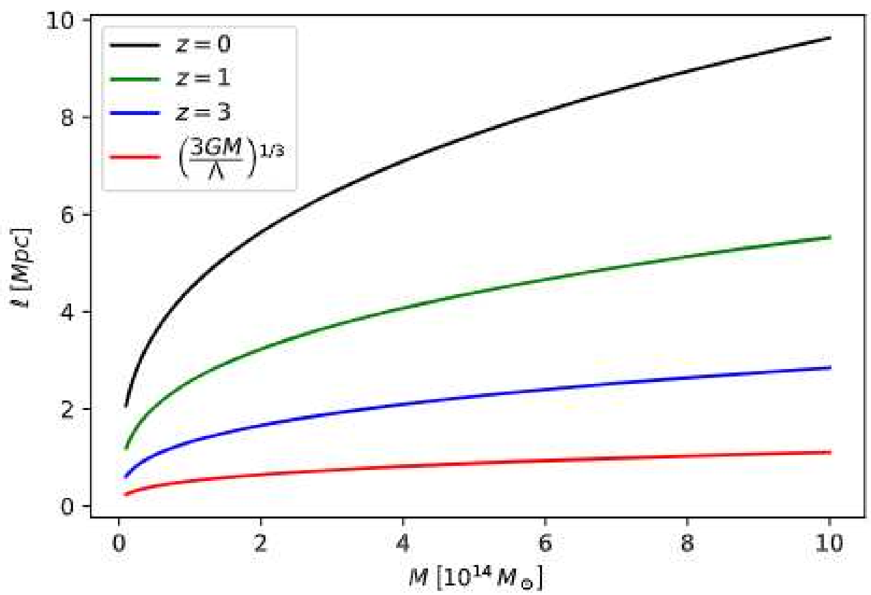
$$V_{max} = \frac{M}{\rho_H \left(1 + 3H\left(\frac{\dot{q}}{q}\right)^{-1}\right)}$$

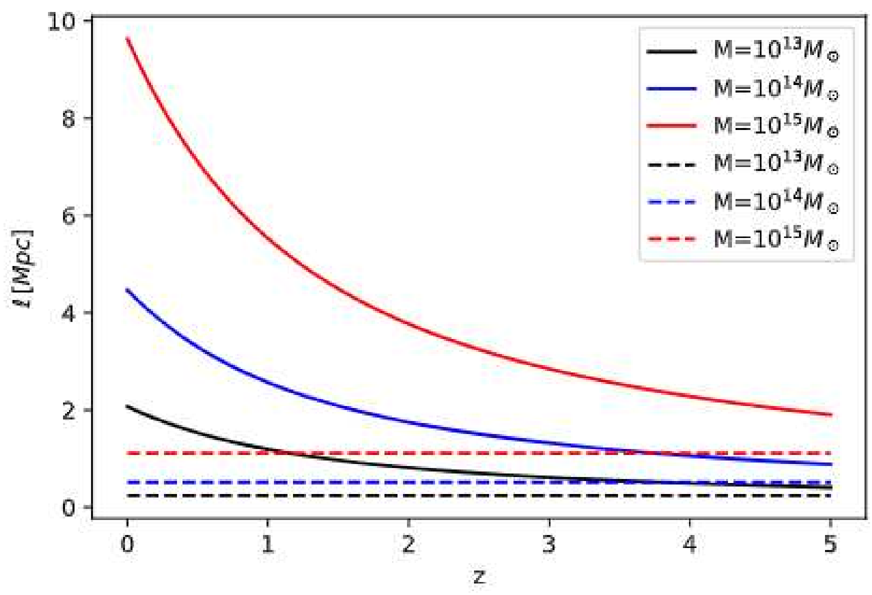
where:

$$\ddot{q} + 2H\dot{q} - 4\pi G \rho_H q = 0$$

The maximum volume: $\langle\theta\rangle = 0$

- The maximum volume of the collapsing object is a function of its mass and the background parameters (it changes with redshift)
- It does not assume any form of symmetry
- It can be used as a test for the ΛCDM cosmological model





Summary

- The most massive galaxy clusters can be used as cosmological probes and to test the ΛCDM models

- Relativistic structure formation methods, including exact solutions and perturbation theories, provide a useful theorethical insight, complementary to the N-body simulations

- Further details can be found in:

  Bolejko and Ostrowski, PRD (2020), Roukema and Ostrowski, JCAP (2020), Ostrowski, AppB (2020), Ostrowski and Delgado Gaspar (2022)