The 8th Conference of the Polish Society on Relativity

19 – 23 September 2022, Banach Center IMPAN, Warsaw, Poland

Strong lensing of gravitational waves - new opportunities for multimessenger astronomy

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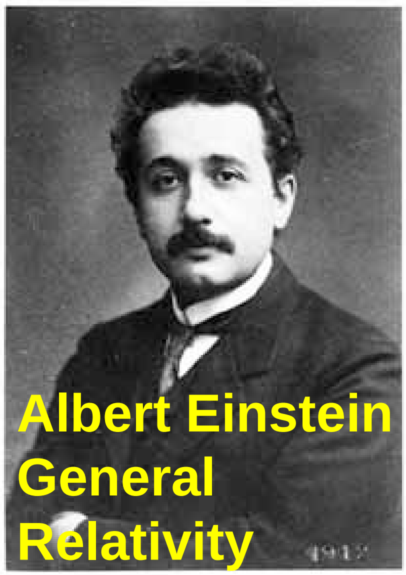
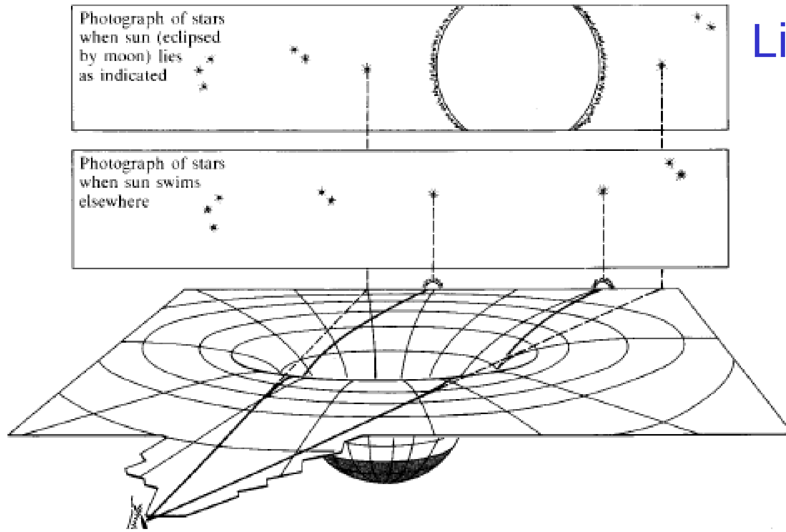
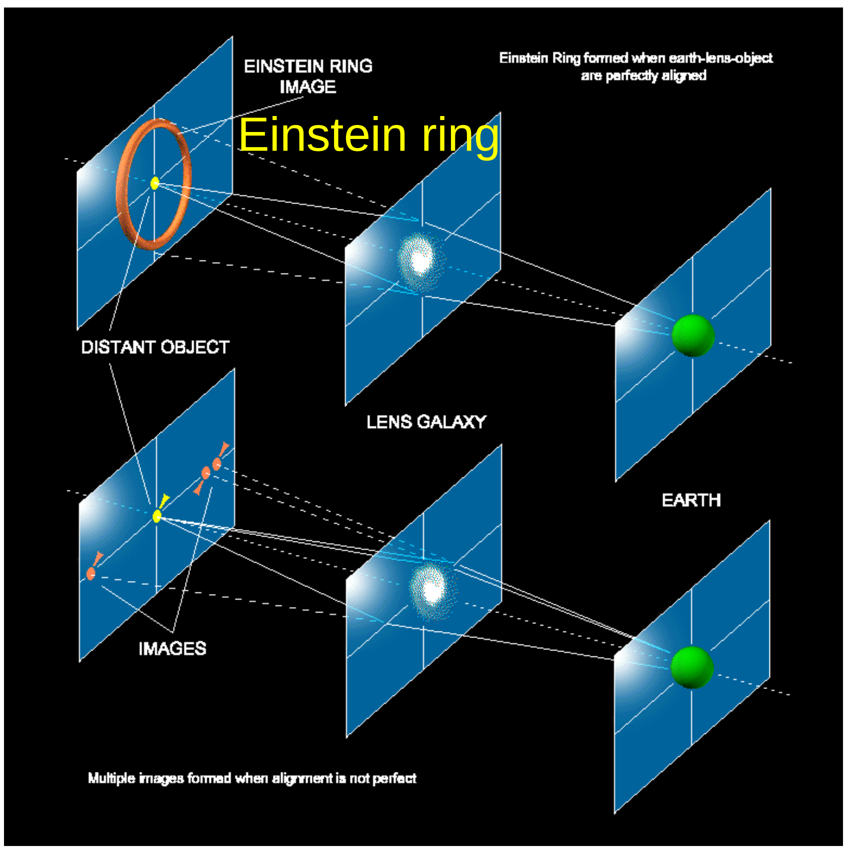




Light bending by the Sun

$$\boxed{\alpha = \frac{4GM}{c^2 R} = 1''.75}$$



<u>Consequences</u>:

gravitational lensing

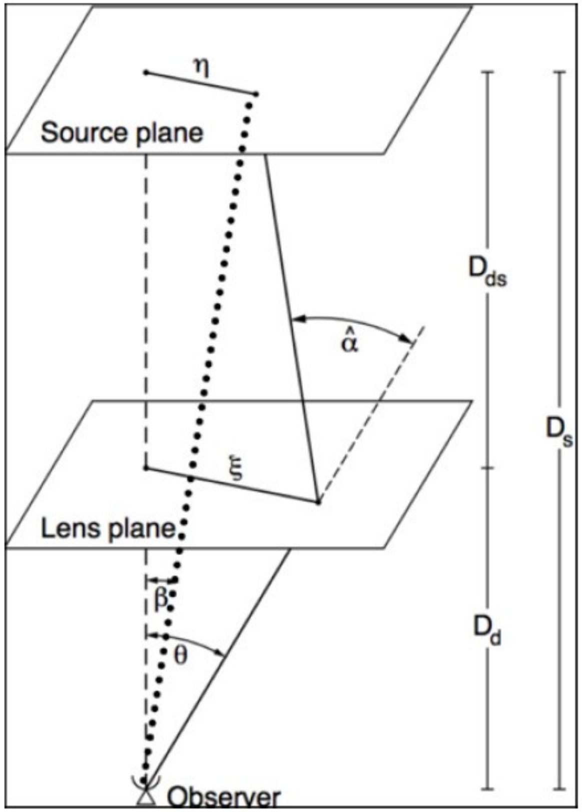
$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_L D_S}}$$

Eddington 1920

multiple images

Gravitational lensing – geometric optics

Light rays formalism



$$\hat{\boldsymbol{\alpha}}\left(\boldsymbol{\theta}\right) D_{ls} + \boldsymbol{\beta} D_s = \boldsymbol{\theta} D_s$$

$$\boxed{\boldsymbol{\theta} - \boldsymbol{\beta} - \nabla_{\boldsymbol{\theta}} \phi = 0}$$

$$A(\boldsymbol{\theta}) = \frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}} \qquad \mu(\boldsymbol{\theta}) = \frac{1}{\det A(\boldsymbol{\theta})}.$$

$$\boldsymbol{\alpha} = \nabla_{\boldsymbol{\theta}} \phi$$

magnification

Wavefront formalism

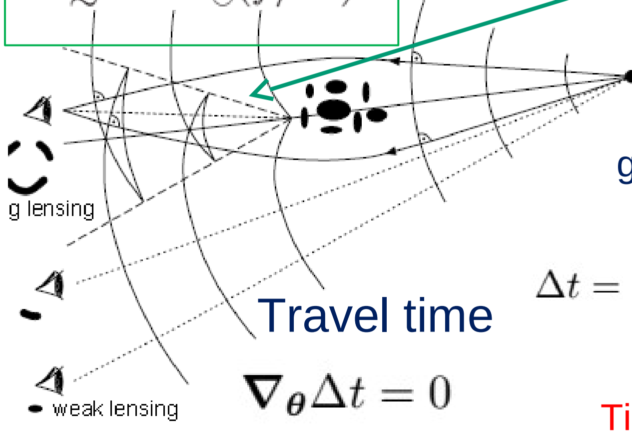
(Fermat principle)

breaks down when

$$\lambda \gtrsim GM/c^2$$

$$M \lesssim 10^5 \mathrm{M}_\odot (f/\mathrm{Hz})^{-1}$$

caustics



Travel time

$$\nabla_{\boldsymbol{\theta}} \Delta t = 0$$

Lens equation

$$\phi(\boldsymbol{\theta}) = \frac{D_{ls}}{D_l D_s} \frac{2}{c^2} \int \Phi(D_l \boldsymbol{\theta}, z) dz$$

Newtonian potential at lens plane

geometrical term

$$\Delta t = \frac{1+z_l}{c} \frac{D_{\mathrm{ol}} D_{\mathrm{os}}}{D_{\mathrm{ls}}} \left[\frac{(\boldsymbol{\theta} - \boldsymbol{\beta})^2}{2} - \phi(\boldsymbol{\theta}) \right]$$

Time delay distance

Fermat potential

$$\boxed{\theta_{\mathrm{E}} = 4\pi \frac{\sigma_{\mathrm{ap}}^2}{c^2} \frac{D_{\mathrm{ls}}}{D_{\mathrm{s}}} \left(\frac{\theta_{\mathrm{E}}}{\theta_{\mathrm{ap}}} \right)^{2-\gamma} f(\gamma)}$$

Einstein radius

Observables:

* image positions
and shape distortions

* time delay between images

* flux ratios
magnification ratios

Gravitational lensing in wave optics limit

Consider monochromatic wave

$$\psi(\boldsymbol{x},t) \;=\; \tilde{\psi}(\boldsymbol{x})e^{-2\pi i f t}$$

Propagation equation in presence of weak gravitational potential U
(Helmholtz equation)

$$(\boldsymbol{\nabla}^2+\omega^2)\tilde{\psi}=4\omega^2 U\tilde{\psi},$$

Schneider P, Ehlers J and Falco E E 1992 *Gravitational Lenses* (Berlin: Springer);
Takahashi R and Nakamura T 2003 Wave effects in the gravitational lensing of gravitational waves from chirping binaries *Astrophys. J.* **595** 1039–51

Amplification factor

$$F=\frac{\tilde{\psi}^{\mathrm{L}}}{\tilde{\psi}}$$

wave amplitude with lensing

without lensing

Diffraction integral

$$F(f,\boldsymbol{\beta})=\frac{1+z_{\rm l}}{c}\frac{D_{\rm ol}D_{\rm os}}{D_{\rm ls}}\frac{f}{i}\int d^2\boldsymbol{\theta}\,\exp\left[2\pi i f\Delta t(\boldsymbol{\theta},\boldsymbol{\beta})\right]$$

$$w=2\pi f(1+z_{\rm l})\frac{4GM(<\theta_{\rm Ein})}{c^3}$$

$w \lesssim 1$ Wave effects become important and significantly modify geometric optics predictions

$$|F(f,\boldsymbol{\beta})|^2=\frac{\pi w}{1-e^{-\pi w}}\left|{}_1F_1\left(\frac{i}{2}w,1;\frac{i}{2}w\hat{\beta}^2\right)\right|^2$$

Point mass

$$|F(f,\boldsymbol{\beta})|^2=\left|\sum_{n=0}^{\infty}\frac{\Gamma(1+n/2)}{n!}g(w,\hat{\beta})\right|^2$$

SIS lens

$$g(w,\hat{\beta})=\left(2we^{(3\pi/2)i}\right)^{n/2}{}_1F_1\left(-\frac{n}{2},1;\frac{i}{2}w\hat{\beta}^2\right)$$

$$w=2\pi f(1+z_{\rm l})\frac{1}{c}\left(\frac{4\pi\sigma^2}{c^2}\right)^2\frac{D_{\rm ol}D_{\rm ls}}{D_{\rm os}}$$

REVIEW Masamune Oguri 2019 *Rep. Prog. Phys.* **82** 126901

Strong gravitational lensing of explosive transients

Diffraction integral in geometric optics limit
i.e. only stationary points of time delay contribute

$$|F(f,\boldsymbol{\beta})|^2 \approx \sum_j |\mu(\boldsymbol{\theta}_j)| + 2\sum_{j<k} |\mu(\boldsymbol{\theta}_j)\mu(\boldsymbol{\theta}_k)|^{1/2} \cos\left[w\Phi(\boldsymbol{\theta}_j,\boldsymbol{\theta}_k) - \pi\Delta n_{jk}\right]$$

magnifications in g.o.

interference between images

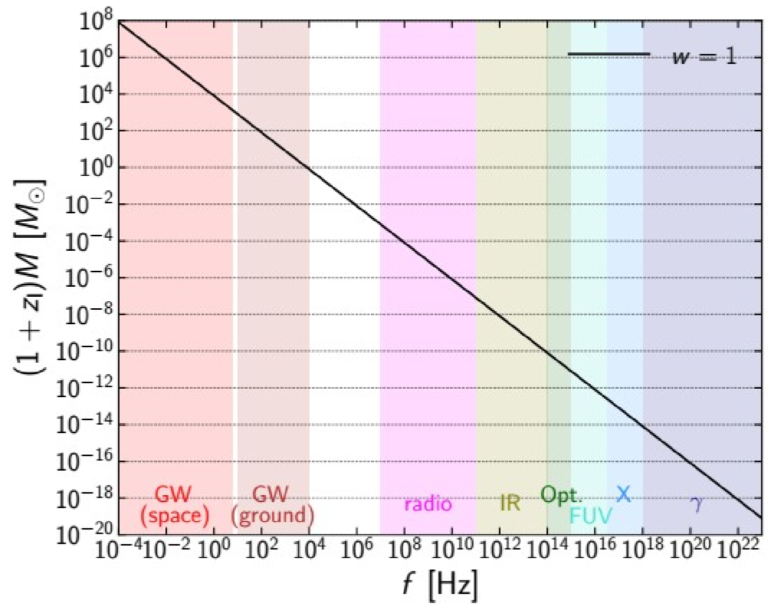


Figure 11. The relation between the frequency f and the (redshifted) mass $(1+z_l)M$ of a point mass lens for the dimensionless parameter $w = 1$, where w is defined in equation (44). The region below the solid line corresponds to the case that the gravitational lensing magnification is significantly suppressed due to the diffraction, which is one of wave optics effects.

<u>Summary</u>
wave optics effects:
* diffraction
* interference

„Refsdal" supernova

„Refsdal supernova" discovered 11 Nov. 2014
Kelly et al. (2015) *Science* 347,1123

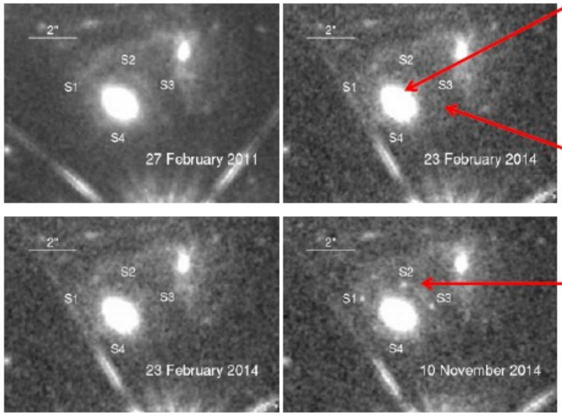
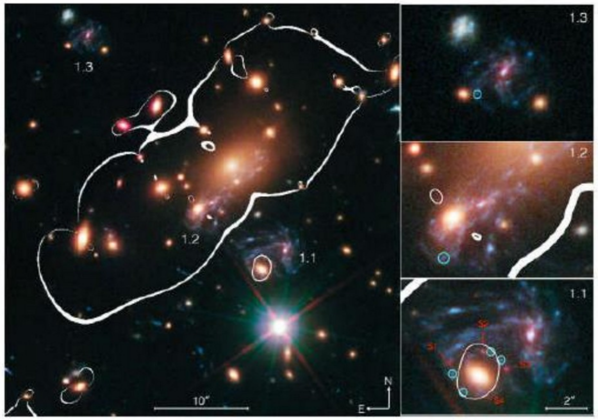


Fig. S4: Images of the lensing system from archival *HST* WFC3-IR observations in the *F140W* filter. All exposures obtained prior to 3 November 2014 show no evidence for variability at any of the positions associated with SN Refsdal.

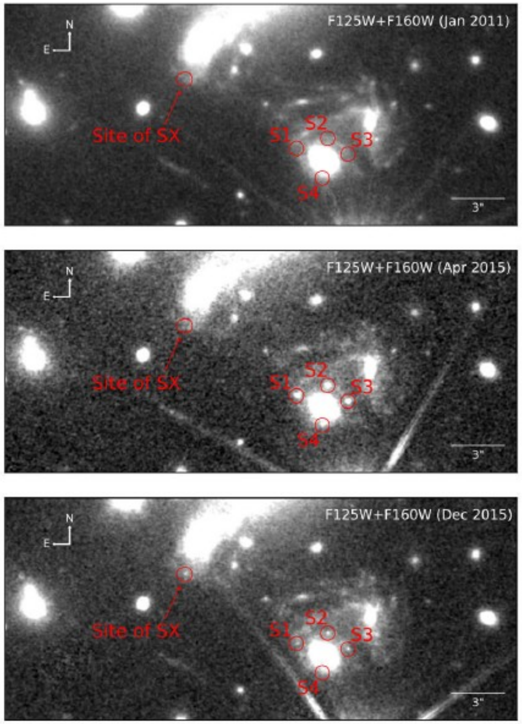
z=0.54 elliptical galaxy belonging to MACS J1149.6+2223 cluster

z = 1.49 source – spiral galaxy

host of SNII



future reappearance expected in ca. 1 yr



Kelly et al. (2016) *ApJL*

11 Dec. 2015
SNII found in SX image as predicted !!!

Great success of GR
(mass distribution modeling from strong lensing)

Success comparable to the greatest triumphs of celestial mechanics in XIX century (discovery of Neptune)

Selected for a Viewpoint in *Physics*

PRL **116,** 061102 (2016)

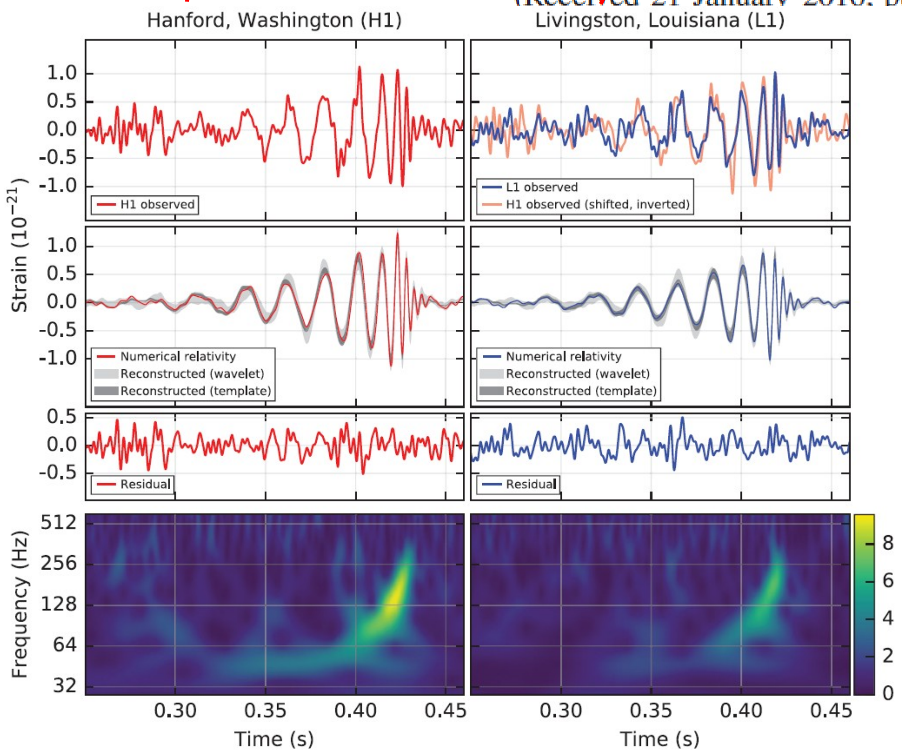
PHYSICAL REVIEW LETTERS

Observation of Gravitational Waves from a Binary Black Hol

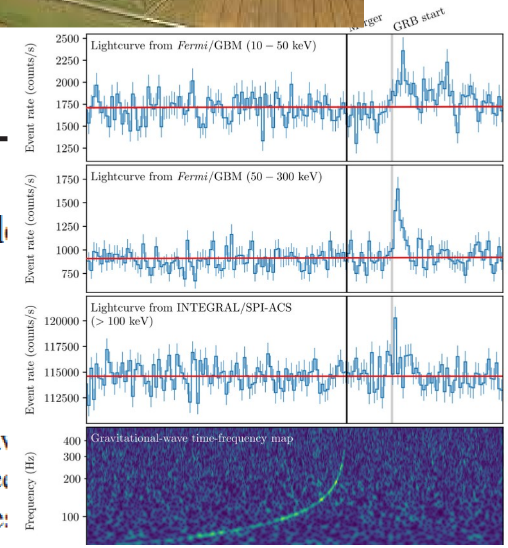
B. P. Abbott *et al.**

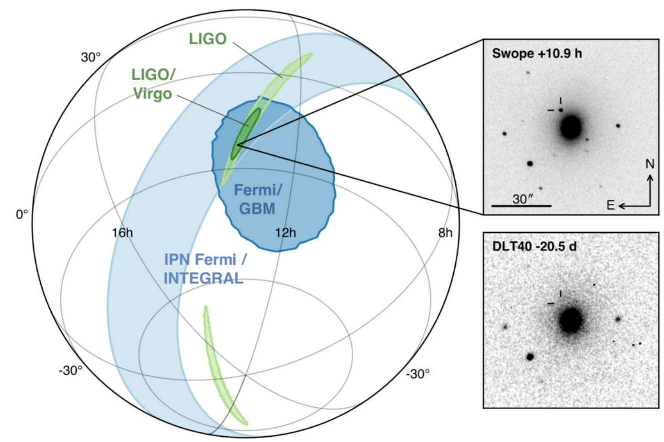
(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 21 January 2016; published 11 February 2016)



tors of the Laser Interferometer Grav
tional-wave signal. The signal swe
ave strain of 1.0×10^{-21}. It matche
r of a pair of black hol
a matched-filter sign
03 000 years, equivale
$^{160}_{180}$ Mpc correspondin



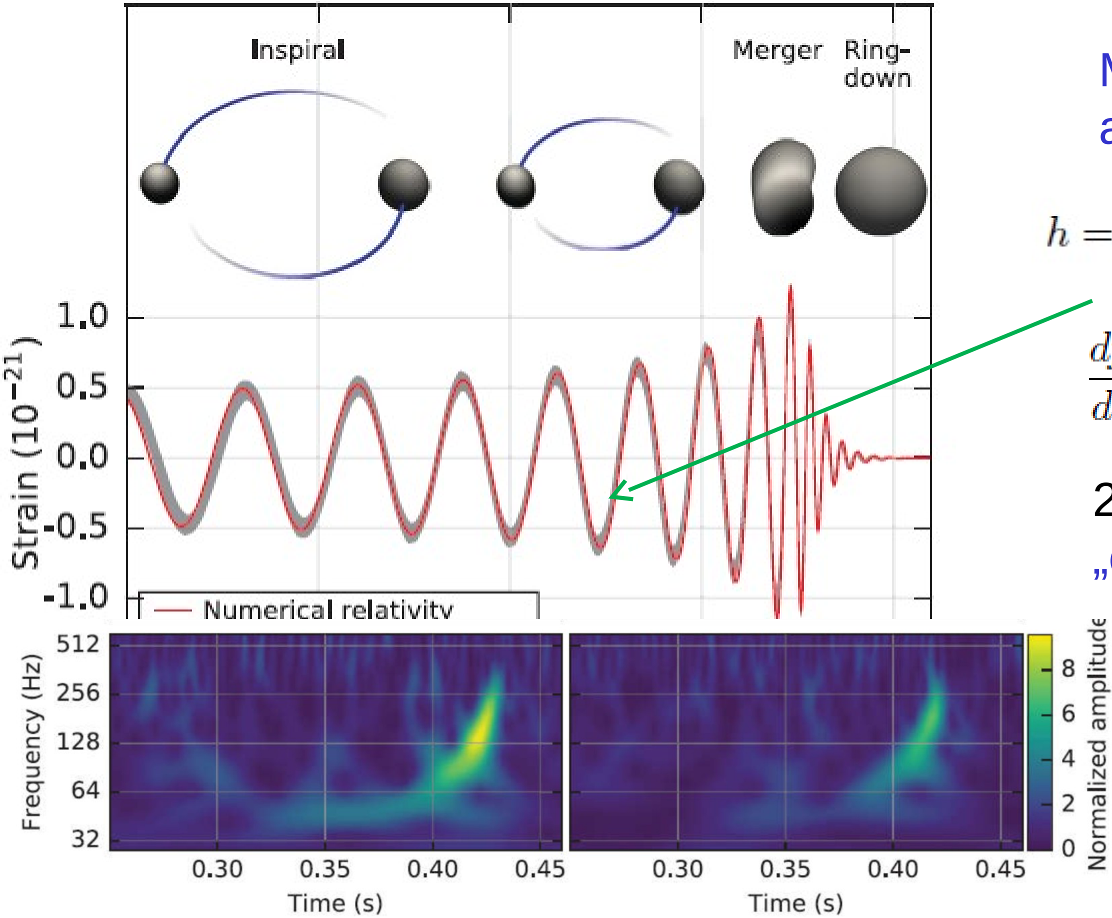


So far 90 registered gravitational signals in O1-O3 runs

GW150914 first ever
GW170817 first NS-NS
GW190426 first BH-NS

The idea of „standard sirens”

B. Schutz 1986
B.Schutz, A. Królak 1987



Measure the strain h(t)
and frequency drift df/dt

$$h = \frac{4\pi^{2/3}(G\mathcal{M})^{5/3}}{c^4 D} f(t)^{2/3} \cos\left[\int_0^t f(t')\,\mathrm{d}t'\right]$$

$$\frac{df}{dt} = \frac{96\pi^{8/3}}{5}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3} f^{11/3}$$

2 equations for 2 unknowns:
„chirp mass” *M* & distance D

$$\mathcal{M} = \frac{c^3}{G}\left(\frac{5}{96\pi^{8/3}}\frac{df}{dt}\right)^{3/5} f^{-11/5}$$

$$D = \frac{5c}{24\pi^2 h(t)}\frac{df(t)}{dt} f^{-3}\cos\left(\int_0^t f(t')dt'\right)$$

$$M_c \rightarrow (1+z) M_c$$

$$t \rightarrow (1+z)t$$

$$f \rightarrow \frac{1}{1+z} f$$

$$\frac{df}{dt} \rightarrow \frac{1}{(1+z)^2}\frac{df}{dt}$$

for cosmological sources

The distance inferred is the **luminosty distance**

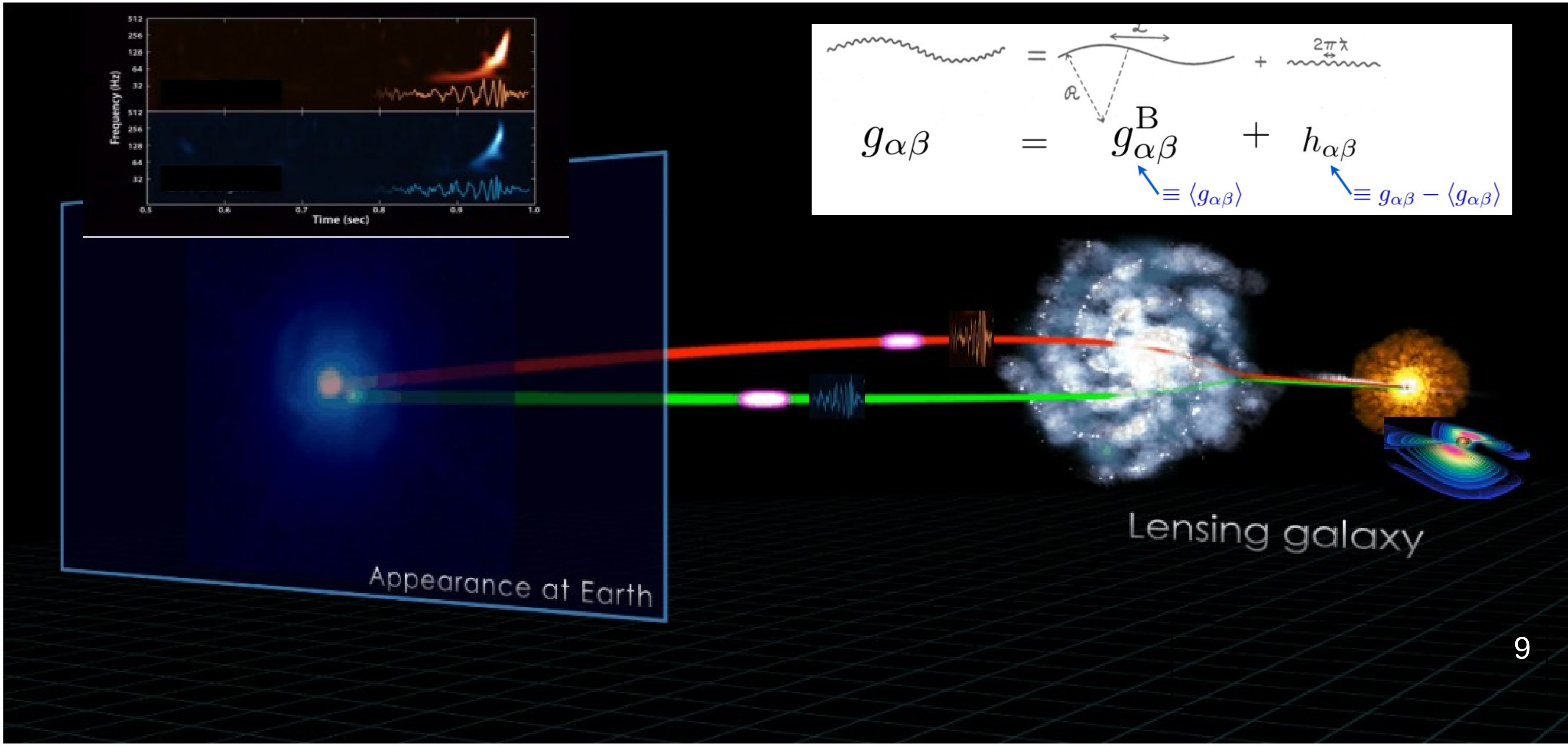
$$D_L = (1+z)\, D$$

Strong gravitational lensing of GW

- ⇒ GWs experience identically the same geometric-optics effects as EM waves:
  - gravitational redshift,
  - cosmological redshift,
  - gravitational lensing, ...

$$\Box h_{\alpha\beta} = 0 \text{ in vacuum}$$

Credit: Kip Thorne
Lorentz Lectures, Leiden 2009



What if LIGO's gravitational wave detections are strongly lensed by massive galaxy clusters?

arXiV: 1707.03412

Graham P. Smith,[1*] Mathilde Jauzac,[2,3,4,5] John Veitch,[1,6,7] Will M. Farr,[1,7]
Richard Massey,[2] Johan Richard[8]

Gravitational Waves Astrophysics: Early Results from GW
Searches and Electromagnetic Counterparts
Proceedings IAU Symposium No. 338, 2018
A.C. Editor, B.D. Editor & C.E. Editor, eds.

DOI: 00.0000/X000000000000000X

England
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Strong-lensing of Gravitational V… Galaxy Clusters

Refsdal supernova reprise ?

irmingham, B15 2TT, England
enis Laval Cedex, France

Graham P. Smith,[1†] Christopher Berry,[1,2] Matteo Bianconi,[1]
Will M. Farr,[1,2] Mathilde Jauzac,[3,4,5,6] Richard Massey,[3]
Johan Richard,[7] Andrew Robertson,[4] Keren Sharon,[8]
Alberto Vecchio,[1,2] John Veitch.[9]

arXiV: 1803.07851

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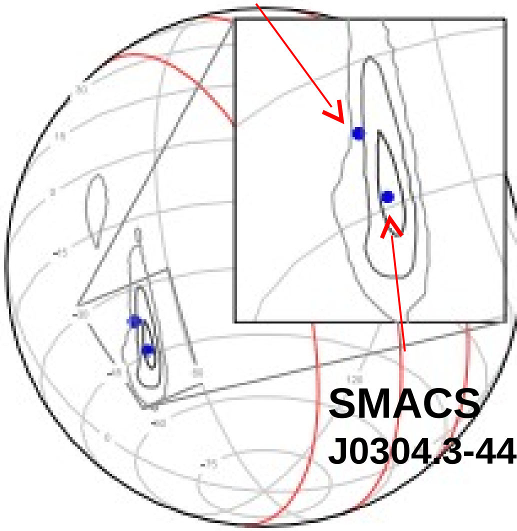
GCN: The Gamma-ray Coordinates Network (TAN: Transient Astronomy Network)

TITLE: GCN CIRCULAR NUMBER: 21692
SUBJECT: LIGO/Virgo G297595: Gemini and VLT observations by GLGW Hunters
DATE: 17/08/25 14:18:20 GMT
FROM: Graham P Smith at U of Birmingham <gps@star.sr.bham.ac.uk>
G. P. Smith (Birmingham), M. Jauzac (Durham), M. Bianconi (Birmingham), J. Richard (Lyon),
C. P. L. Berry (Birmingham), W. M. Farr (Birmingham), R. Massey (Durham), A. Robertson (Durham),
K. Sharon (Michigan), A. Vecchio (Birmingham), J. Veitch (Glasgow)



GW170814

The **Gravitationally-lensed Gravitational Wave Hunters collaboration**
report observations of strong-lensing galaxy clusters located within the sky localisation of the LIGO/Virgo trigger G297595 (LVC GCN Circ. 21474).

Reinterpreting Low Frequency LIGO/Virgo Events as Magnified Stellar-Mass Black Holes at Cosmological Distances.

arXiV: 1802.05273

Tom Broadhurst[1,2], Jose M. Diego[3], George Smoot III [4,5,6]

BH-BH detected by LIGO were so massive because they were all lensed

Very unlikely… !

Twin LIGO/Virgo Detections of a Viable Gravitationally-Lensed Black Hole Merger

arXiV: 1901.03190

Tom Broadhurst[1,2,3], Jose M. Diego[4], George F. Smoot[5,6,7,8]

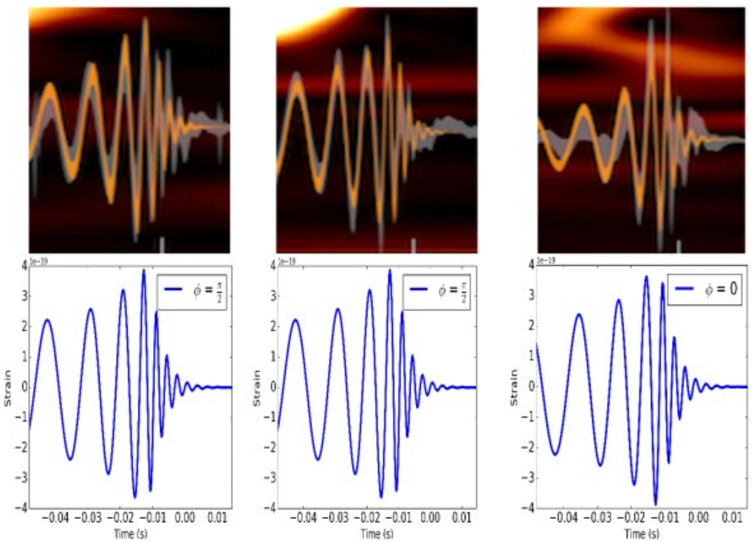


Figure 1: **Phase similarity:** The similar pair of detected waveforms of interest here are shown for GW170809 and GW170814 in the left and middle panels respectively[1]. These are compared with model waveforms below showing that a fixed phase, defined with respect to the moment of merger shown underneath in blue, and fixed chirp mass of $28.5 M_{\odot}$ (in the detector frame)

Nobel Prize 2006

GW170809 i GW170814

are two strong lensed signals …

SEARCH FOR GRAVITATIONAL LENSING SIGNATURES IN LIGO-VIRGO BINARY BLACK HOLE EVENTS

O.A. HANNUKSELA[1], K. HARIS[2], K.K.Y. NG[3,4], S. KUMAR[2,5,6], A.K. MEHTA[2], D. KEITEL[7], T.G.F. LI[1], P. AJITH[2,8]
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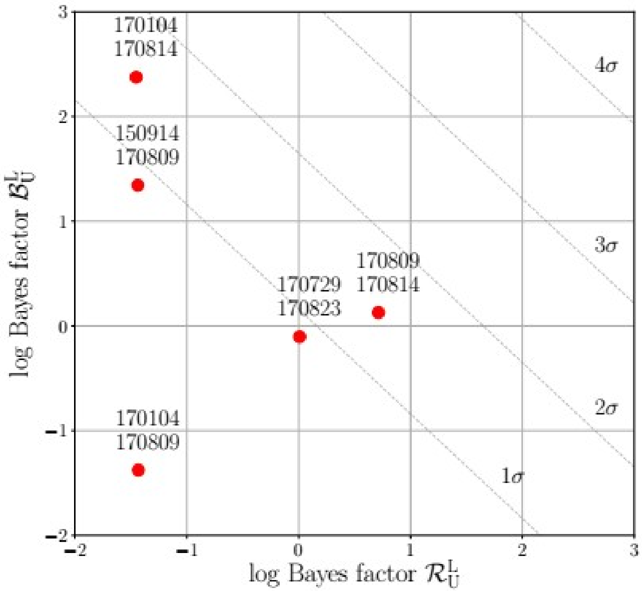
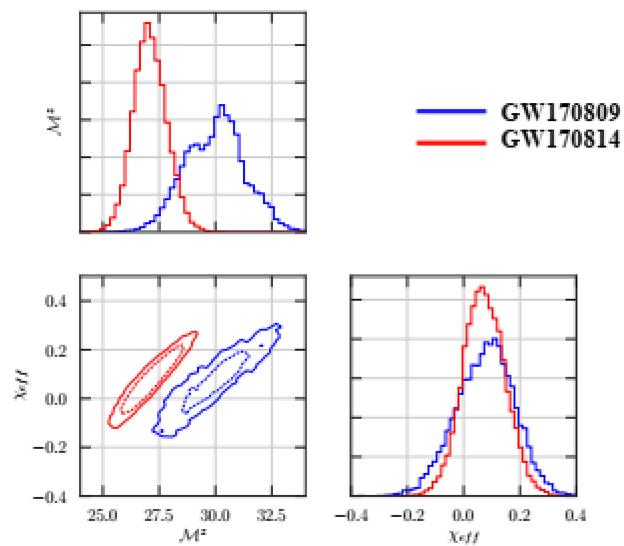
Draft version January 30, 2019

arXiV: 1901.02674

ABSTRACT

We search for signatures of gravitational lensing in the binary black hole events detected by Advanced LIGO and Virgo during their first two observational runs. In particular, we look for three effects: 1) evidence of lensing magnification in the individual signals due to galaxy lenses, 2) evidence of multiple images due to strong lensing by galaxies, 3) evidence of wave optics effects due to point-mass lenses. We find no compelling evidence of any of these signatures in the observed gravitational wave signals. However, as the sensitivities of gravitational wave detectors improve in the future, detecting lensed events may become quite likely.

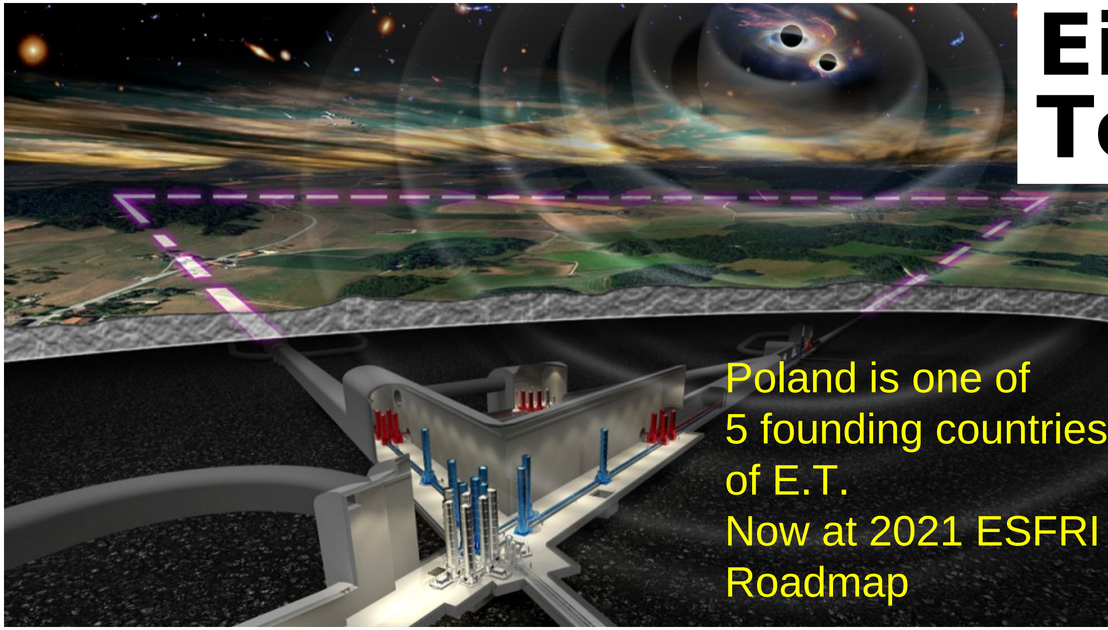




LVC rejects these claims …

arXiV: 2105.06384 latest results for O1-O3 runs

Einstein Telescope



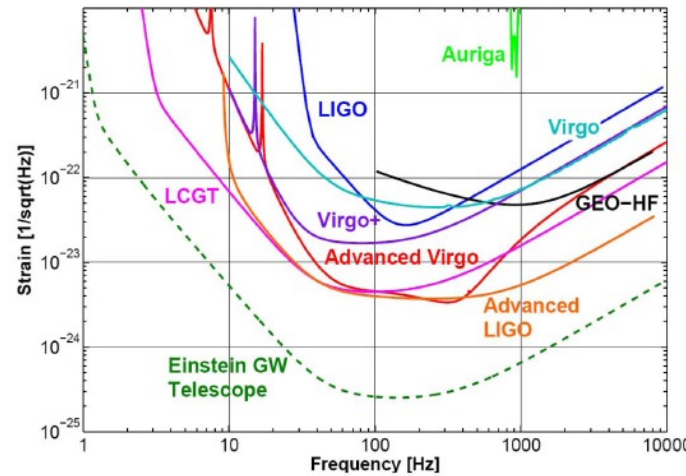
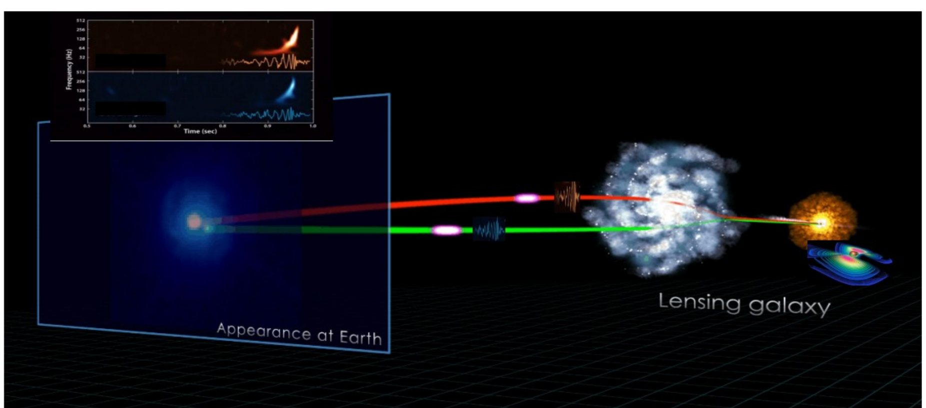


Figure 5: Sensitivities of gravitational wave detectors from the first to the third generation.

Table 2: Expected coalescence rates per Mpc^3 per Myr in the local universe ($z \simeq 0$). Also shown are predicted event rates in Advanced LIGO (aLIGO) and ET.

Source	BNS	NS-BH	BBH
Rate ($\mathrm{Mpc}^{-1}\ \mathrm{Myr}^{-1}$)	0.1–6	0.01–0.3	2×10^{-3}–0.04
Event Rate (yr^{-1}) in aLIGO	0.4–400	0.2–300	2–4000
Event Rate (yr^{-1}) in ET	$\mathcal{O}(10^3$–$10^7)$	$\mathcal{O}(10^3$–$10^7)$	$\mathcal{O}(10^4$–$10^8)$



- Increased sensitivity great expectations
- Big catalogs of inspiral events up to cosmological distances
- Some of them would be gravitationally lensed

Discussed in papers

A. Piórkowska *et al.* JCAP10(2013)022 (NS-NS only)

M. Biesiada *et al.* JCAP10(2014)080 (full DCO: NS-NS, BH-NS, BH-BH)

X. Ding *et al.* JCAP12(2015)006 (relaxing intrinsic SNR=8 demand; magnification bias)

50 – 100 lensed events per year

BH-BH systems contribute 91 – 95%;
NS-NS systems 1 – 4%

The Astrophysical Journal, 874:139 (6pp), 2019 April 1 https://doi.org/10.3847/1538-4357/ab095c

CrossMark

How Does the Earth's Rotation Affect Predictions of Gravitational Wave Strong Lensing Rates?

Lilan Yang[1], Xuheng Ding[1], Marek Biesiada[2,3], Kai Liao[4], and Zong-Hong Zhu[1,2]
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Received 2018 September 3; revised 2019 January 10; accepted 2019 February 20; published 2019 April 1

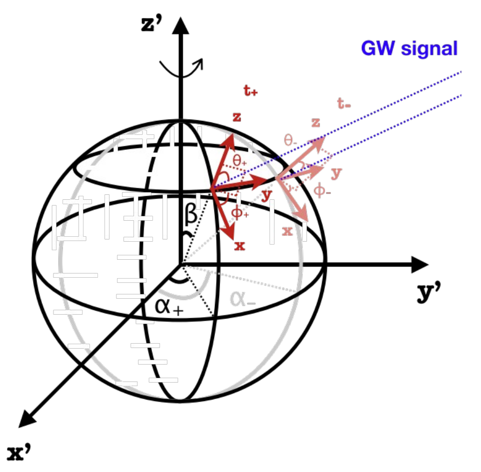

Table 1
Predictions of Yearly Lensed GW Event Rates for which Only I_- Image or Both I_- and I_+ Images are Magnified above the Threshold $\rho_0 = 8$

Metallicity Evolution Which Event Rate	High Only I_-	High I_- and I_+	Low only I_-	Low I_- and I_+
NS–NS				
Initial Design	0.7	0.4	0.6	0.4
Xylophone	1.4	1.1	1.2	0.7
BH–NS				
Initial Design	2.2	1.8	2.9	2.3
Xylophone	3.5	2.9	4.3	3.6
BH–BH				
Initial Design	106.6	94.3	130.3	115.4
Xylophone	143.5	128.0	177.6	159.2
Total				
Initial Design	109.5	96.5	133.8	118.1
Xylophone	148.4	132	183.1	163.5

Note. Results are shown for the standard model of DCO formation and two configurations of the ET. The "high" and "low" represent the "high-end" and "low-end" galaxy metallicity evolution.

In agreement with

Monthly Notices of the Royal Astronomical Society
MNRAS **476,** 2220–2229 (2018) doi:10.1093/mnras/sty411
Advance Access publication 2018 February 16

Gravitational lensing of gravitational waves: a statistical perspective

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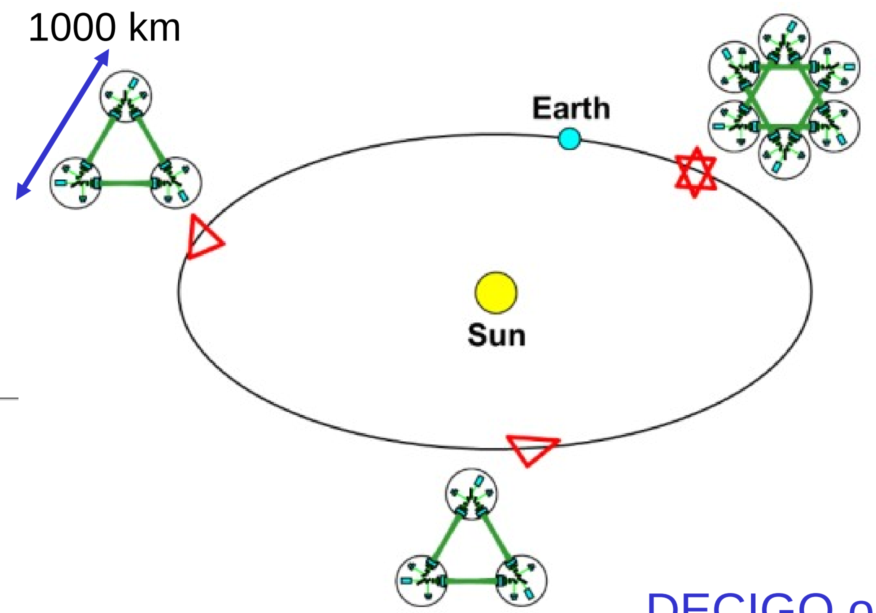
The future –
widen the frequency band of ground based detectors

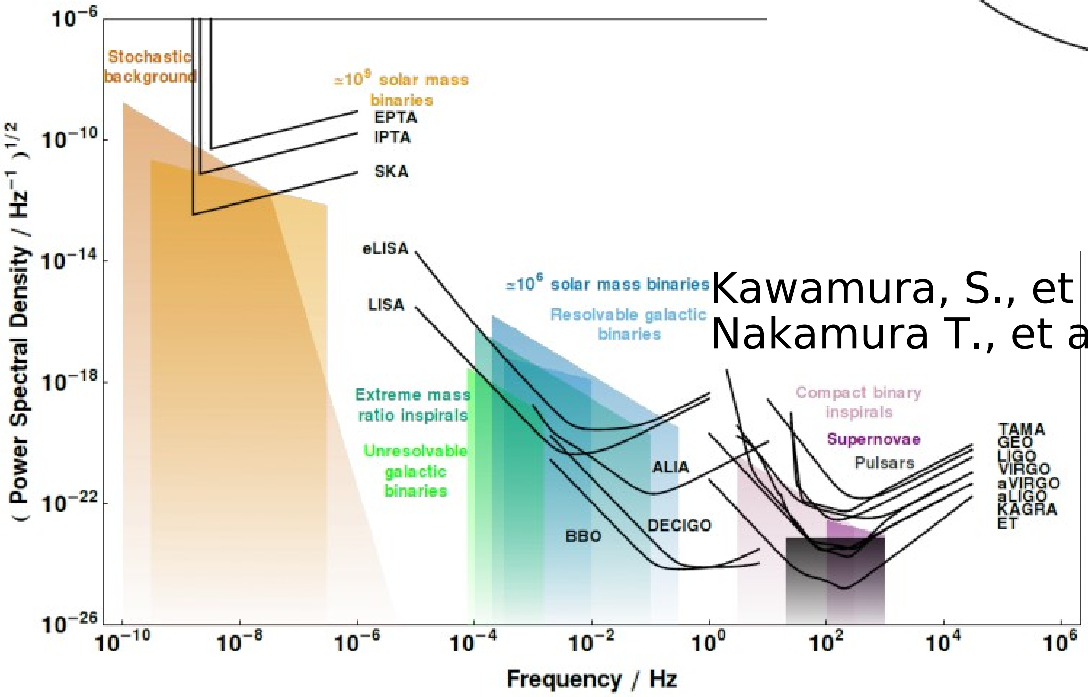
LISA – 0.1 mHz – 100 mHz

DECIGO – 1 mHz – 100 Hz

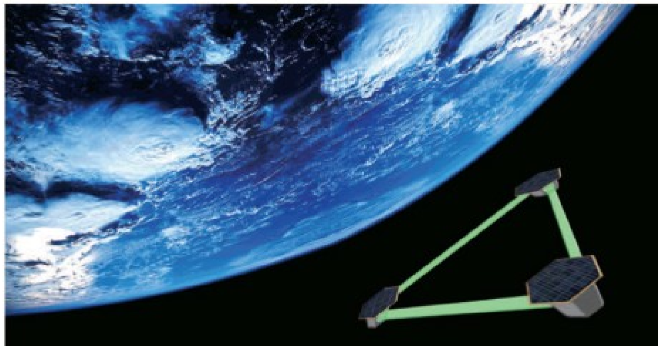


DECIGO orbit

Kawamura, S., et al. 2019, IJMPD, 28, 1845001
Nakamura T., et al. 2016, Prog. Theor. Exp. Phys. 093



Detector noise power spectrum density of different ground and space detectors



B-DECIGO smaller scale detector

The Astrophysical Journal, 908:196 (12pp), 2021 February 20 https://doi.org/10.3847/1538-4357/abd482

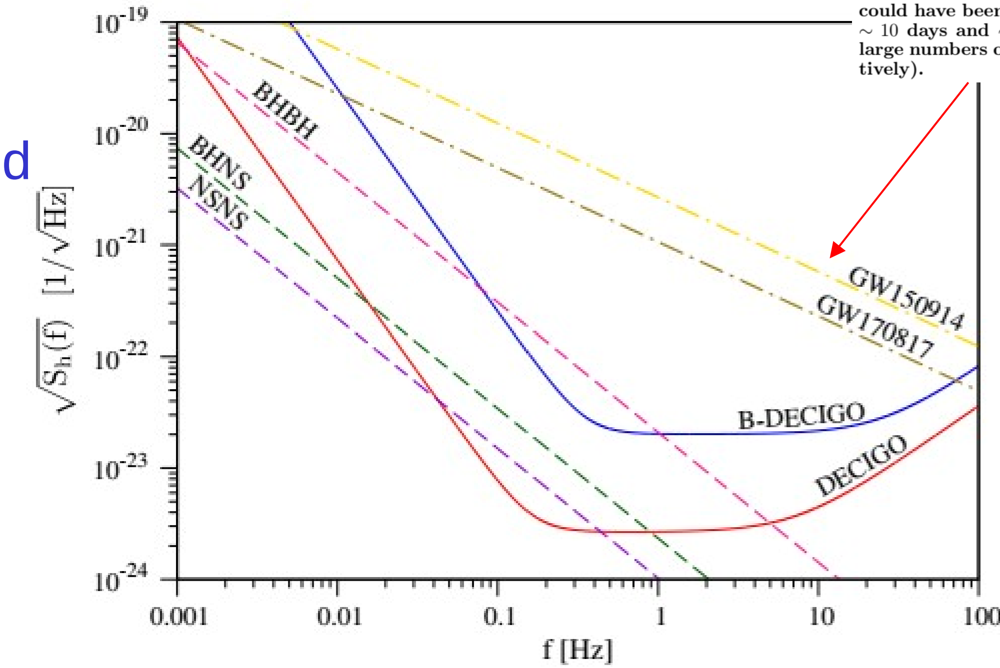
Inspiraling Double Compact Object Detection and Lensing Rate: Forecast for DECIGO and B-DECIGO

Aleksandra Piórkowska-Kurpas[1,2], Shaoqi Hou[3], Marek Biesiada[1,4], Xuheng Ding[3,5], Shuo Cao[1], Xilong Fan[3], Seiji Kawamura[6], and Zong-Hong Zhu[1,3]

tivity band. As discussed in details in Isoyama et al. (2018), time to coalescence can be estimated as: $t_c = 1.03 \times 10^6$ s $(\mathcal{M}_z/30.1\ M_\odot)^{-5/3}(f/0.1\ \mathrm{Hz})^{-8/3}$, which means that GW150914 and GW170817 could have been visible in (B-)DECIGO band for ~ 10 days and ~ 7 yrs prior to coalescence with large numbers of GW cycles (10^5 and 10^7, respectively).

DECIGO sensitivity will be significantly affetcted by unresolved BH-BH systems

B-DECIGO affected much less



These GW events could have been detected by DECIGO or B-DECIGO

ET: r_0 = 1917 Mpc

DECIGO r_0 = 6709 Mpc

B-DECIGO r_0 = 535 Mpc

50 lensed events per year
few lensed events per year

only BH-BH systems

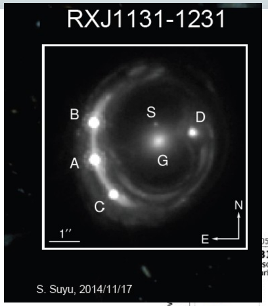
DECIGO and B-DECIGO

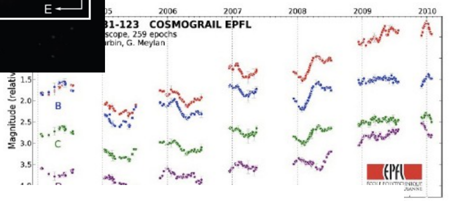
ARTICLE

DOI: 10.1038/s41467-017-01152-9 OPEN

Precision cosmology from future lensed gravitational wave and electromagnetic signals

Kai Liao[1,2], Xi-Long Fan[3], Xuheng Ding[1,4,5], Marek Biesiada[4,6] & Zong-Hong Zhu[1,4]





The standard siren approach of gravitational wave cosmology appeals to the direct luminosity distance estimation through the waveform signals from inspiralling double compact binaries, especially those with electromagnetic counterparts providing redshifts. It is limited by the calibration uncertainties in strain amplitude and relies on the fine details of the waveform. The Einstein telescope is expected to produce 10^4–10^5 gravitational wave detections per year, 50–100 of which will be lensed. Here, we report a waveform-independent strategy to achieve precise cosmography by combining the accurately measured time delays from strongly lensed gravitational wave signals with the images and redshifts observed in the electromagnetic domain. We demonstrate that just 10 such systems can provide a Hubble constant uncertainty of 0.68% for a flat lambda cold dark matter universe in the era of third-generation ground-based detectors.

Table 1 Relative uncertainties of three factors contributing to the accuracy of time-delay distance measurement

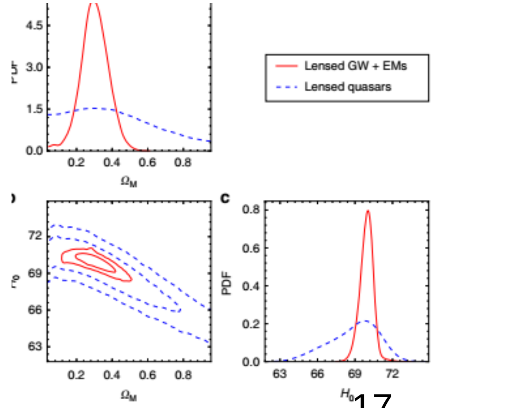
	$\delta\Delta t$	$\delta\Delta\psi$	δLOS
Lensed GW + EM	0%	0.6%	1%
Lensed quasar	3%	3%	1%

$\delta\Delta t$, $\delta\Delta\psi$, δLOS correspond to time delay, Fermat potential difference, and light-of-sight environment, respectively. We show the case for lensed gravitational wave (GW) + electromagnetic (EM) signals compared with standard technique in the EM domain using lensed quasars

Table 2 The average constraining power of 10 lensed gravitational wave + electromagnetic systems

	Flat ΛCDM (Ω_M fixed)	Flat ΛCDM		Flat ωCDM			Open ΛCDM		
	H_0	H_0	Ω_M	H_0	Ω_M	w	H_0	Ω_M	Ω_k
Uncertainty	0.37%	0.68%	27%	2.2%	36%	25%	1%	38%	±0.18

We concerns cosmological parameters in different scenarios: flat lambda cold dark matter (Flat ΛCDM) with or without dimensionless matter density Ω_M fixed, flat ωCDM where the dark energy equation of state ω is a free parameter, and open ΛCDM where cosmic curvature Ω_k is a free parameter. For the same number of lensed quasars, the power is weaker by a factor of ~4 according to the uncertainty propagation using Eq. (1) and Table 1



ARTICLE

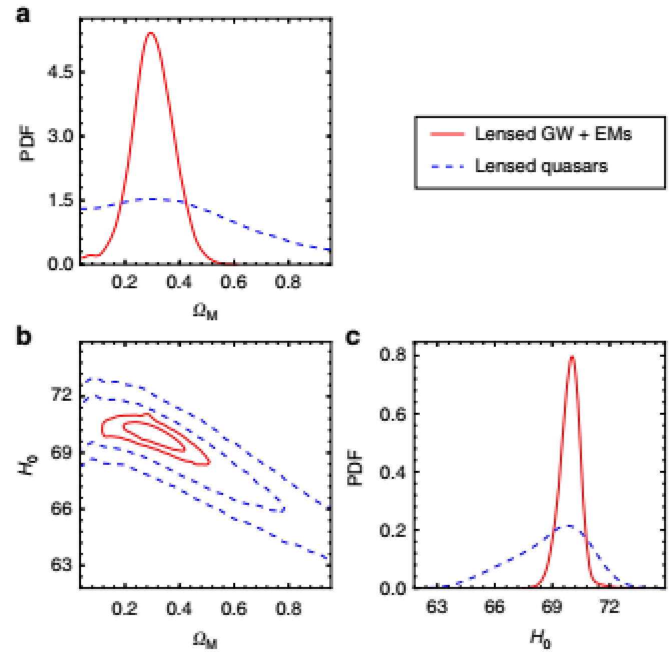
DOI: 10.1038/s41467-017-01152-9 OPEN

Precision cosmology from future lensed gravitational wave and electromagnetic signals

Kai Liao[1,2], Xi-Long Fan[3], Xuheng Ding[1,4,5], Marek Biesiada[4,6] & Zong-Hong Zhu[1,4]

The standard siren approach of gravitational wave cosmology appeals to the direct luminosity distance estimation through the waveform signals from inspiralling double compact binaries, especially those with electromagnetic counterparts providing redshifts. It is limited by the calibration uncertainties in strain amplitude and relies on the fine details of the waveform. The Einstein telescope is e... ctions per year, 50-100 of which will be le... tegy to achieve precise cosmography by ... from strongly lensed gravitational wave ... in the electro-magnetic domain. We der... Hubble constant uncertainty of 0.68% fo... the era of third-generation ground-based detectors.

Sub percent – percent accuracy of H_0 possible with 10 lensed GW+EM



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www.nature.com/scientificreports

Strongly gravitationally lensed type Ia supernovae: Direct test of the Friedman-Lemaître-Robertson-Walker metric

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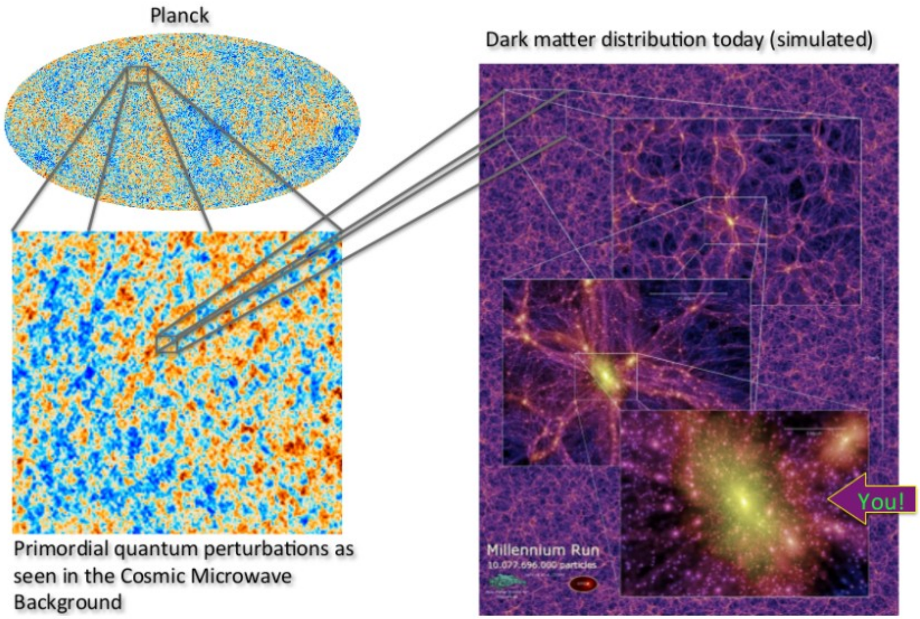
natureresearch

Direct test of the FLRW metric from strongly lensed gravitational wave observations

Shuo Cao[1], Jingzhao Qi[2], Zhoujian Cao[1], Marek Biesiada[1,3], Jin Li[4], Yu Pan[5] & Zong-Hong Zhu[1]

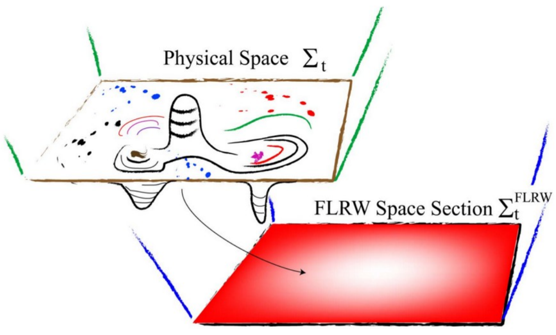
The assumptions of large-scale homogeneity and isotropy underly the familiar Friedmann-Lemaître-Robertson-Walker (FLRW) metric that appears to be an accurate description of our Universe. In this paper, we propose a new strategy of testing the validity of the FLRW metric, based on the galactic-scale lensing systems where strongly lensed gravitational waves and their electromagnetic counterparts can be simultaneously detected. Each strong lensing system creates opportunity to infer the curvature parameter of the Universe. Consequently, combined analysis of many such systems will provide a model-independent tool to test the validity of the FLRW metric. Our study demonstrates that the third-generation ground based GW detectors, like the Einstein Telescope (ET) and space-based detectors, like the Big Bang Observer (BBO), are promising concerning determination of the curvature parameter or possible detection of deviation from the FLRW metric. Such accurate measurements of the FLRW metric can become a milestone in precision GW cosmology.

Strong lensing and curvature of the Universe



Credit: F. Leclercq, A. Pisani , B.D. Wandeldt arXiv:1403.1260v1

Emerging spatial curvature



Buchert, Carfora, *Class. Quant. Grav.* 25 , 195001 (2008)

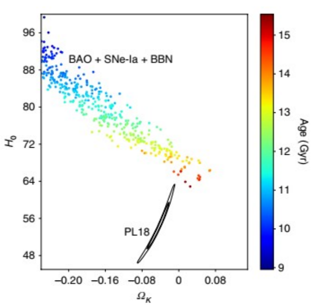
Formation of the large scale structure induces non-zero curvature at local scales

It is important to measure curvature with more local objects

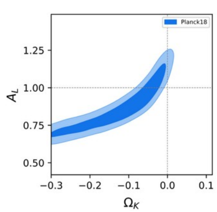
ARTICLES
https://doi.org/10.1038/s41550-019-0906-9
nature astronomy

Planck evidence for a closed Universe and a possible crisis for cosmology

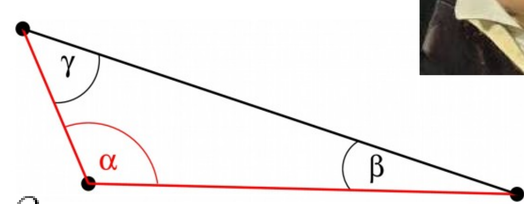
Eleonora Di Valentino[1], Alessandro Melchiorri[2*] and Joseph Silk[3,4,5]



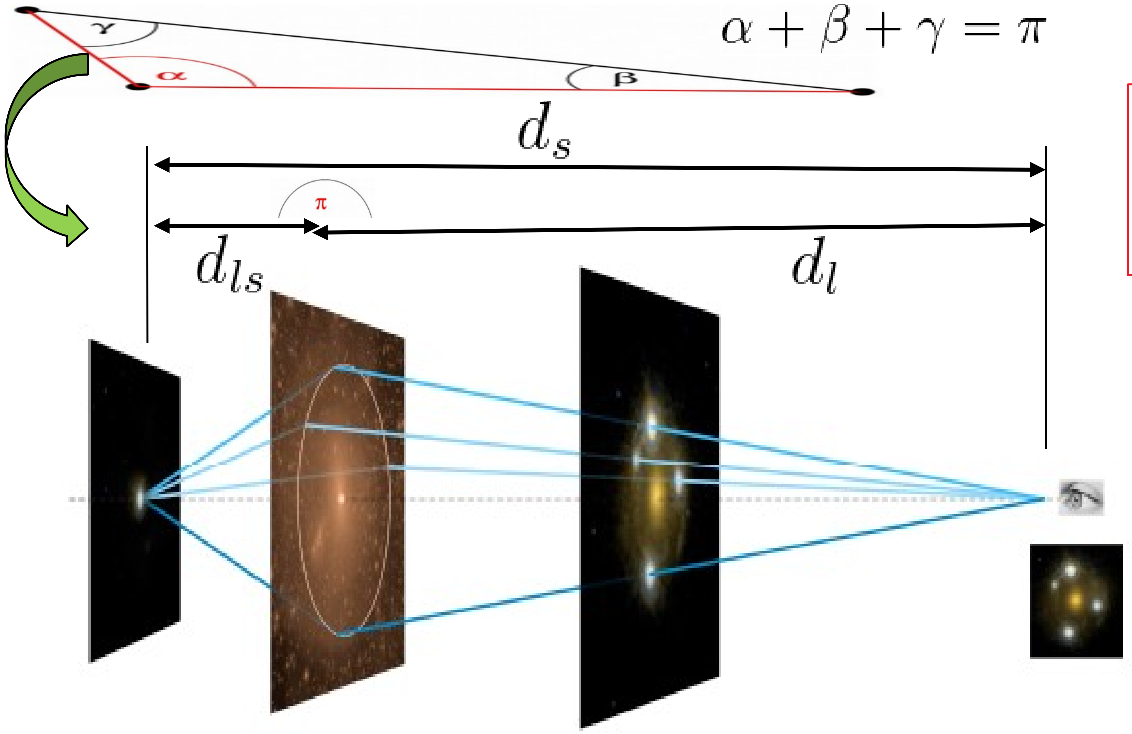
the preference at 3.4σ for a closed Universe



Carl Friedrich Gauss



$$\alpha + \beta + \gamma = \pi$$



$$d_{ls} = d_s - d_l$$ rule valid in flat FLRW metric

Strong lenisng systems offer us „degenerated triangles"

Distance sum rule – valid in any FLRW metric

$$d_{ls} = \sqrt{1 + \Omega_k d_l^2}\; d_s - \sqrt{1 + \Omega_k d_s^2}\; d_l$$

$$\Omega_k(z_l, z_s) = \frac{d_l^4 + d_s^4 + d_{ls}^4 - 2d_l^2 d_s^2 - 2d_l^2 d_{ls}^2 - 2d_s^2 d_{ls}^2}{4 d_l^2 d_s^2 d_{ls}^2}$$

This is a function of two redshifts, but within the FLRW metric it should be just a single number !

One can obtain Ω_k if

d_l, d_s, d_{ls} are known

Observations:
z_l , z_s – known

Images -- > d_{ls} / d_s

Time delays -- > $d_l d_s$ / d_{ls}

So: d_l is measurable
=================

d_s – match by redshift some standard candle (or ruler)

If we observe strongly lensed GW signals and their EM counterparts

Advantages:

very precise measurement of time delay
(however affected by microlensing and l.o.s. contamination)

$$D_{\Delta t} \equiv \frac{D_l^A D_s^A}{D_{ls}^A} = \frac{c}{1+z_l}\frac{\Delta t_{i,j}}{\Delta\phi_{i,j}}.$$

Time delay distance – Fermat potential reconstructed from EM images of lensed host galaxy

$$\frac{d_{ls}}{d_s} = \frac{D_{ls}^A}{D_s^A} = \frac{\theta_E}{4\pi}\frac{c^2}{\sigma_{ap}^2}\left(\frac{\theta_E}{\theta_{ap}}\right)^{\gamma-2} f(\gamma, M_E)^{-1}$$

EM images + spectroscopy of the lens

$$D_l = (1+z_l) D_{\Delta t}\frac{D_{ls}^A}{D_s^A}.$$

Sources are standard sirens
(up to magnification factor, which can be assessed from flux ratio and lens reconstruction)

$$D_s = \frac{1}{1+z_s} D_s^L$$

$$\Omega_k(z_l, z_s) = \frac{1}{4}\frac{(1+z_l)^2 d_{\Delta t}^2}{d_s^4} + \frac{1}{4}\frac{(1+z_l)^2}{d_{\Delta t}^2 (d_{ls}/d_s)^4} + \frac{1}{4}\frac{1}{(1+z_l)^2 d_{\Delta t}^2} - \frac{1}{2}\frac{1}{(d_{ls}/d_s)^2 d_s^2} - \frac{1}{2}\frac{1}{d_s^2} - \frac{1}{2}\frac{1}{(1+z_l)^2 d_{\Delta t}^2 (d_{ls}/d_s)^2}$$

Cosmic curvature in terms of measureable quantities

Uncertainty budget: covariances by propagating uncorrelated measurement uncertainties of the observables $\gamma, \theta_E, \theta_{ap}, \Delta t, \Delta\psi$ (LOS), d_s^L, and F

$$\delta\Omega_k(z_l, z_s) \sim (\delta\gamma, \delta\theta_E, \delta\theta_{ap}, \delta(\Delta t), \delta(\Delta\psi(LOS)), \delta d_s^L, \delta F)$$

	$\delta\theta_E$	$\delta\sigma ap$	$\delta\gamma$
Image configuration	1%	5%	1%
	$\delta\Delta t$	$\delta\Delta\psi$	$\delta\Delta\psi(LOS)$
Time delay	0%	$\sim(\delta\theta E,\delta\gamma)$	1%
	δd_s^L (SNR)	δd_s^L (WL)	δF (SL + ML)
Lensed GW	$2/\rho net$	$0.05z$	10% (50%)

Mock catalog:
100 lensed GW for ET
1000 lensed GW for BBO

Merger rates – from StarTrack pop syn code masses and orientations sampled randomly

Lenses – elliptical galaxies, VDF from Schechter function fitted to SDSS DR3

Reconstruction and magnification uncertainties from the budget above

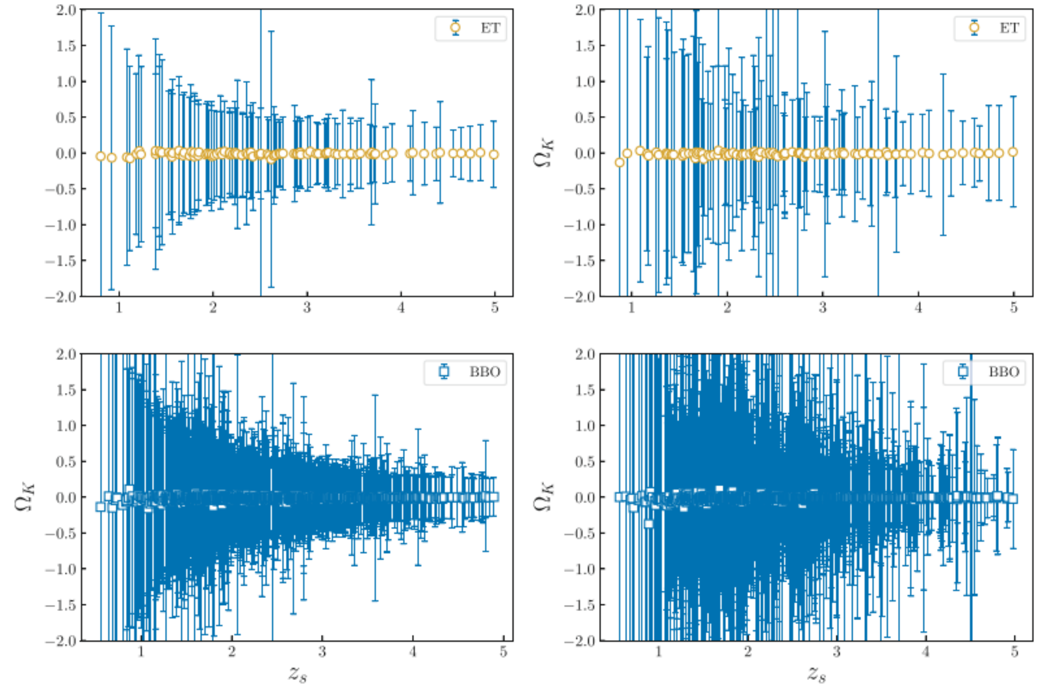


Figure 1. An example of the simulated measurements of the cosmic curvature from future observations of lensed GWs. We simulated 100 lensed GW signals detectable by the ET (upper panel) and 1000 signals detectable by the BBO (lower panel). Left and right panel respectively show the results with 10% and 50% uncertainty in the amplification factor measurements.

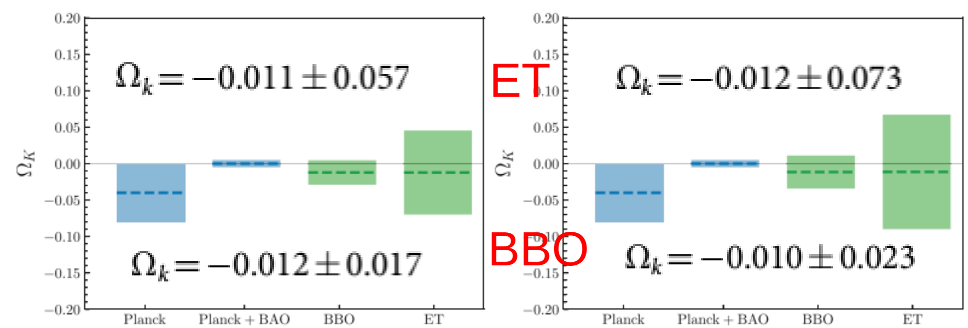


Figure 2. Statistical summary of simulated predictions of the Ω_k parameter measurements (inverse variance weighting) from future observations of lensed GWs. Left and right panel respectively show the results with 10% and 50% uncertainty in the amplification factor measurements. Predictions for the ET and the BBO are confronted with constraints achievable from the CMB and BAO measurements.

Dark matter (satellite) halo **mass deficit?**

1. **Dark matter cores** of kpc size are preferred by observed circular velocities in dwarf/low-surface-brightness (LSB) galaxies, while simulations suggest cusps [Moore 1994; Burkert 1995, …].

   (core/cusp problem)

2. **Non-observation of very massive satellite halos** predicted by simulations in our Milky Way [M.Boylan-Kolchin et al. 2011, 2012] and others [Ferrero et al. 2011].

   (too-big-to-fail problem)

3. Given the long lifetime of dwarfs, some globular/star clusters are expected to be destroyed, or sink to the center **if their host halos are cuspy** [J. Binney & S.Tremaine 2008, F. Contenta et al. 2017, P. Boldrini et al. 2018, …].

   (GC timing problem)

More heat/entropy needed in halo centre (if confirmed)

Heat needed to make a kpc dark core: $10^{53} - 10^{55}$ erg

- Baryonic effects? heated by supernova / in-falling clumps.

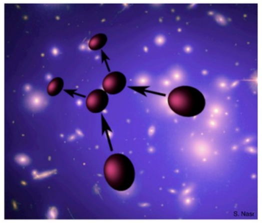
  Each supernova deposits $\sim 10^{51}$ erg in interstellar medium *[e.g. Madau, Shen, Governato 2014, …]*

- Self-interacting dark matter? (also decaying / fuzzy dark matter, ….)

Observational evidence for self-interacting cold dark matter

D.N. Spergel and P J. Steinhardt [astro-ph/9909386]

Infalling dark matter is scattered before reaching the center of the galaxy so that the orbit distribution is isotropic rather than radial. These collisions increase the entropy of the dark matter phase space distribution and lead to a dark matter halo profile with a shallower density profile.



O(1) scattering per (central) particle → $\frac{\sigma_{\rm SI}}{m_{\rm DM}} \sim 0.5 - 10{\rm cm}^2/{\rm g}$

- **Self-interacting dark matter (SIDM)?**

  - Stronger self-scattering **needed** for (dwarf-sized) halos

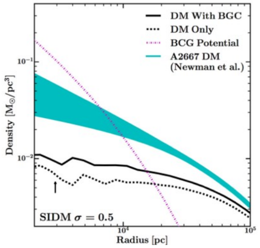
    [O. D. Elbert et al. 2016, K. Bondarenko 2016,….]

    $\frac{\sigma_{\rm SI}}{m_{\rm DM}} \sim 0.5 - 10{\rm cm}^2/{\rm g}$ at dwarf scales of DM velocity ~ 10 km/s

  - Weaker self-scattering **favored** by cluster merging/halo profiles etc.

    [O. D. Elbert et al. 2016, K. Bondarenko 2016,….]

    $\frac{\sigma_{\rm SI}}{m_{\rm DM}} \leq 0.2 - 1{\rm cm}^2/{\rm g}$ at cluster scales of DM velocity ~ 1000km/s



Plane GW Traveling Through Homogeneous Matter

- **Fluid:**
  - GW shears the fluid, (rate of shear) = $\sigma_{jk} = \frac{1}{2}\dot{h}^{\rm GW}_{jk}$
  - no resistance to shear, so no action back on wave
  - Viscosity $\eta \sim \rho v s = ({\rm density})({\rm mean\ speed\ of\ particles})({\rm mean\ free\ path})$ produces stress $T_{jk} = -2\eta\sigma_{jk} = -\eta\dot{h}^{\rm GW}_{jk}$ **NOTE:** s must be $< \lambda$
  - Linearized Einstein field equation: $\Box h^{\rm GW}_{jk} = -16\pi(T_{jk})^{\rm TT} = 16\pi\eta\dot{h}^{\rm GW}_{jk}$
  - Wave attenuates: $h^{\rm GW}_{jk} \sim \exp(-z/\ell_{\rm att})$ where $\ell_{\rm att} = \frac{1}{8\pi\eta} = \frac{1}{8\pi\rho v s}$

In the fluid with shear viscosity $h_{\alpha,{\rm visc}} = h_\alpha {\rm e}^{-\beta D/2}$

attenuated wave leads to biased luminosity distance $D_{\rm L,eff}(z,\beta) = D_{\rm L}(z){\rm e}^{\beta D(z)/2}$

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Measuring the viscosity of dark matter with strongly lensed gravitational waves

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$$h_{\alpha,\mathrm{visc}} = h_\alpha \mathrm{e}^{-\beta D/2}$$

attenuated wave leads to biased luminosity distance

$$D_{\mathrm{L,eff}}(z,\beta) = D_{\mathrm{L}}(z)\mathrm{e}^{\beta D(z)/2}$$

lensed transitents (GW, SNIa) signal leads to very precise determination of „time delay distance”

$$\Delta t_{i,j} = \frac{D_{\Delta t}(1+z_1)}{c}\Delta\phi_{i,j}$$

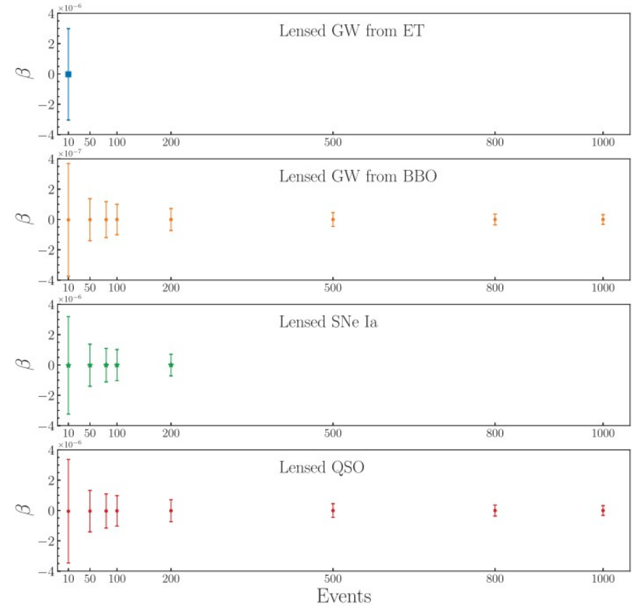
$$D_{\Delta t}(z_l, z_s) \equiv \frac{D_A(z_l)D_A(z_s)}{D_A(z_l, z_s)}$$

To be determined by unlensed standard sirens

$$D_{\Delta t} = \frac{D_{\mathrm{L}}(z_l)D_{\mathrm{L}}(z_s)}{(1+z_l)^2 D_{\mathrm{L}}(z_s) - (1+z_s)(1+z_l)D_{\mathrm{L}}(z_l)}$$

$$\chi^2 = \sum_{i=1}^{i} \frac{\left(D^{lens}_{\Delta t,i}(z_{l,i}, z_{s,i}) - D^{unlens}_{\Delta t,i}(z_{l,i}, z_{s,i}; \beta)\right)^2}{\sigma^2_{i,\mathrm{lens}} + \sigma^2_{i,\mathrm{unlens}}}$$

objective function fitted for beta



relation with DM physical parameters

$$\frac{\langle \sigma_\chi \rangle}{m_\chi} = \frac{6.3\pi G \langle v \rangle}{c^3 \beta}$$

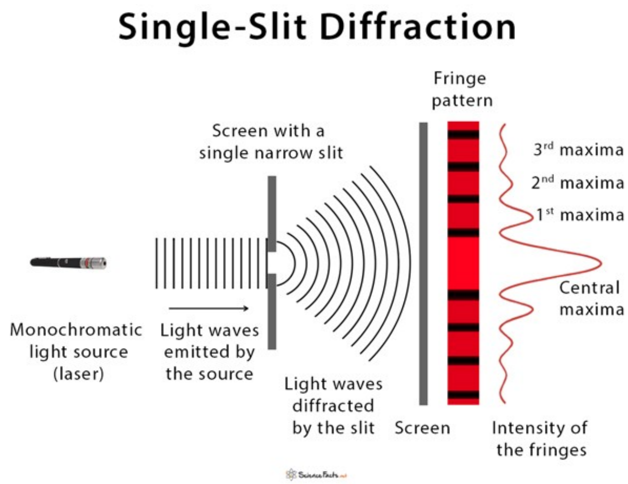
Table 1. Summary of the constraints obtained from different observations. Type I and II, respectively, correspond to the two cases of self-interacting DM in galaxies and galaxy clusters.

Data	$\Delta\beta$	$\Delta(\sigma_\chi/m_\chi)(I)$	$\Delta(\sigma_\chi/m_\chi)(II)$
GW (lensed; ET) + GW (unlensed)	10^{-6} Mpc^{-1}	10^{-4} cm^2 g^{-1}	10^{-3} cm^2 g^{-1}
GW (lensed; BBO) + GW (unlensed)	10^{-8} Mpc^{-1}	10^{-6} cm^2 g^{-1}	10^{-5} cm^2 g^{-1}
QSO (lensed; LSST) + GW (unlensed)	10^{-7} Mpc^{-1}	10^{-5} cm^2 g^{-1}	10^{-4} cm^2 g^{-1}
SNe Ia (lensed; LSST) + GW (unlensed)	10^{-6} Mpc^{-1}	10^{-4} cm^2 g^{-1}	10^{-3} cm^2 g^{-1}

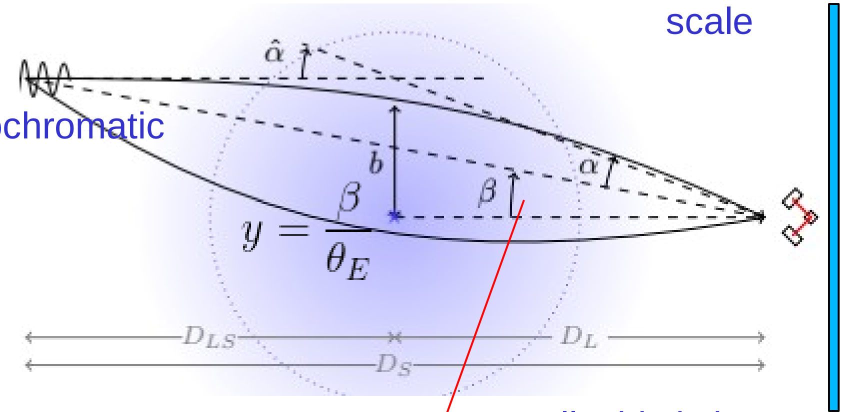
1 cm^2 g^{-1} = 1.8 barn GeV^{-1}

it would be able to DM viscosity and differentiate cluster and small scale scenarios

Diffractive GW microlensing



GW
monochromatic

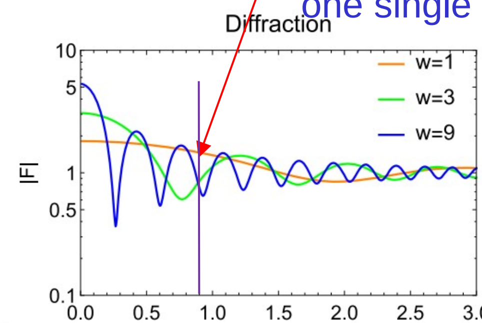


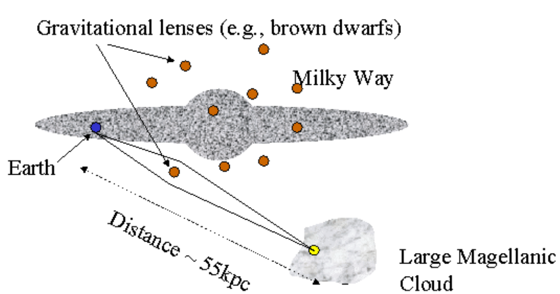
Screen
– astronomical scale

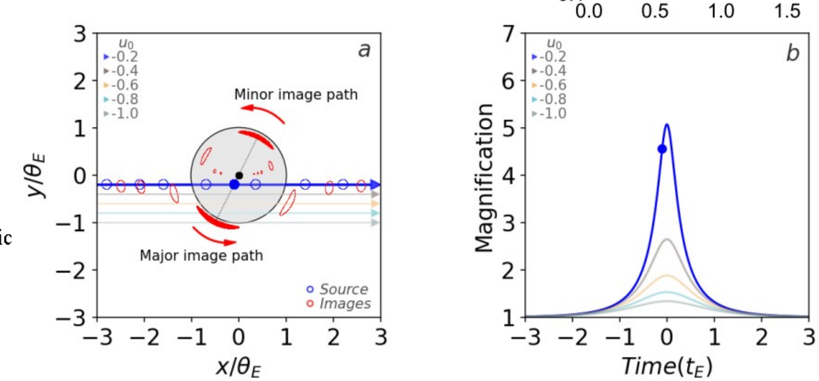
In case of light we observe fringes on the screen

Usually this is just one single „y"

Idea:
B.Paczyński 1986







The Astrophysical Journal, 875:139 (6pp), 2019 April 20 https://doi.org/10.3847/1538-4357/ab1087

The Wave Nature of Continuous Gravitational Waves from Microlensing

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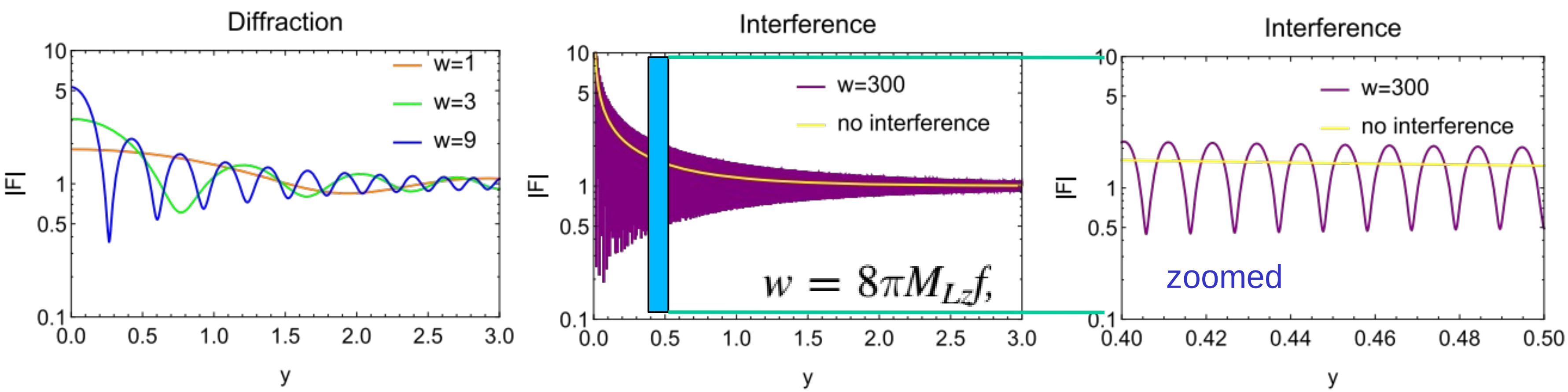

Figure 1. Relative amplitude of a lensed continuous gravitational wave as a function of the source position for the point mass lens. Left: diffraction in wave optics description for different values of the *w* parameter. Middle: interference pattern of images in the geometric optics limit of $w = 300$. Right: zoomed-in of the interval $y \in [0.4, 0.5]$. In the geometric optics limit, amplification of light from lensed images is shown in yellow for comparison.

We summarize the criteria of detectability of the fringes for a point mass lens:

1. $w > 1$ to have enough amplification variation;
2. $y < 3$ to detect fringes before they are damped;
3. $\Delta t > t_f$ to see a fringe pattern.

$$t_f = \frac{R_E}{v_{\rm eff}\, w}$$

timescale of observability of diffraction fringes

THE ASTROPHYSICAL JOURNAL, 875:139 (6pp), 2019 April 20 https://doi.org/10.3847/1538-4357/ab1087

The Wave Nature of Continuous Gravitational Waves from Microlensing

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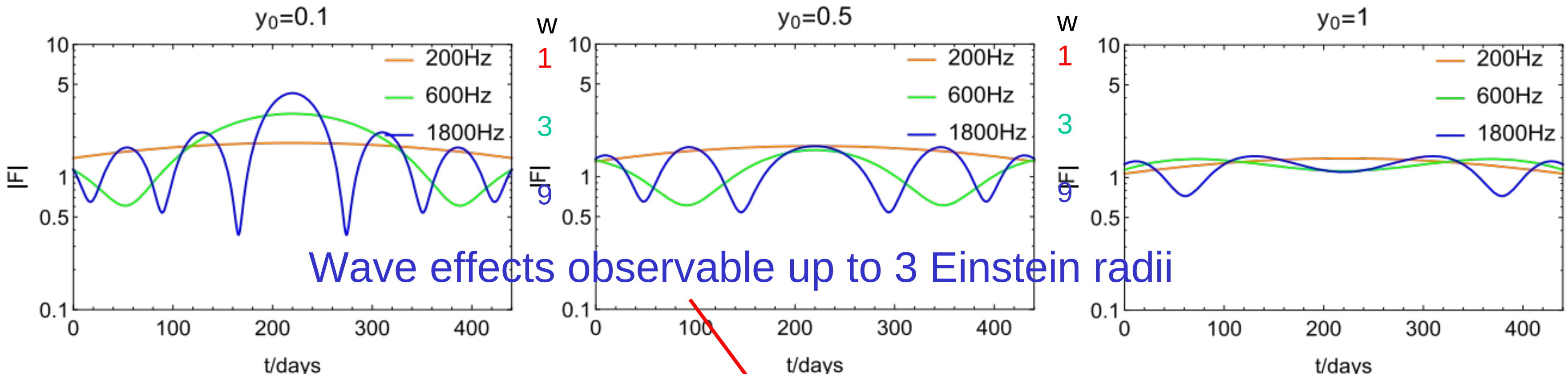

Figure 3. Amplitude variation of a monochromatic GW in a time interval corresponding to two Einstein crossing times for a 40 $M_\odot$ lens. The closest approach time corresponds to t_E. Diffraction patterns are shown for three frequencies of the GW: $f = 200$ Hz, 600 Hz, and 1800 Hz. Panels from left to right correspond to three values of the source position at the closest encounter $y_0 = 0.1$, 0.5, and 1, respectively.

$$t_E \approx 34.7\ \text{days}\ \sqrt{4\frac{D_l}{D_s}\left(1-\frac{D_l}{D_s}\right)\left(\frac{D_s}{8\ \text{kpc}}\right)}^{1/2} \times \left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{v_{\rm eff}}{200\ \text{km s}^{-1}}\right)^{-1},$$

220 days for a 40 $M_\odot$ lens

source

$$f = 600\ \text{Hz},\quad w = 3,\quad t_f = 73.3\ \text{days}$$

Galactic bulge 10^9 NS
Globular clusters 10^3 NS

boosts optical depth

$$\tau = \int_0^{D_s} n(D_l)\pi R_E^2 dD_l.$$

disc lenses $v_{\rm rot} \approx 220\ \text{km s}^{-1}$

$$\boxed{\tau_{\rm GW} \sim f_l\, y_{\max}^2 \times 10^{-7} \sim f_l \times 10^{-6}}$$

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Gravitational Microlensing by the Globular Cluster Stars

by

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ongoing work with
Orest Dorosh & Adam Zadrożny

M22 projected location close to Galactic bulge

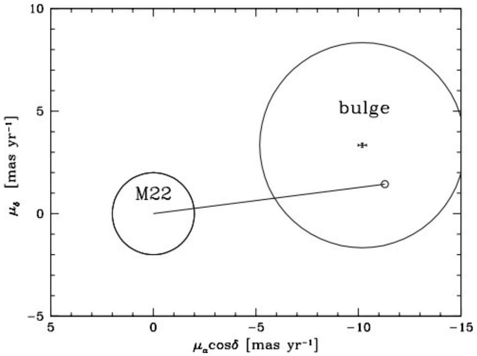
Velocity dispersion of glob. cluster stars

$$\tau = \frac{\sigma^2}{c^2}\frac{2\pi}{\phi}\left(1 - \frac{D_l}{D_s}\right) \approx 2.4\ 10^{-5}\left(\frac{\sigma}{10\ \mathrm{km\ s^{-1}}}\right)^2\left(\frac{1'}{\phi}\right)$$

$$\tau_{\mathrm{GW}} \sim f_l\ y_{\max}^2\ (1/\phi) \times 10^{-5} \sim f_l\ (1/\phi) \times 10^{-4}$$

angular distance from the cluster center

for stars it is an obstacle,
for GWs crowding is not a problem !



The Astrophysical Journal Letters, 744:L18 (4pp), 2012 January 10 doi:10.1088/2041-8205/744/2/L18

THE FIRST CONFIRMED MICROLENS IN A GLOBULAR CLUSTER*

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diffractive lensing might become a tool of looking for IMBHs in cluster centers

PHYSICAL REVIEW D **101,** 064011 (2020)

Interference of lensed GW – beat patterns

Gravitational wave interference via gravitational lensing: Measurements of luminosity distance, lens mass, and cosmological parameters

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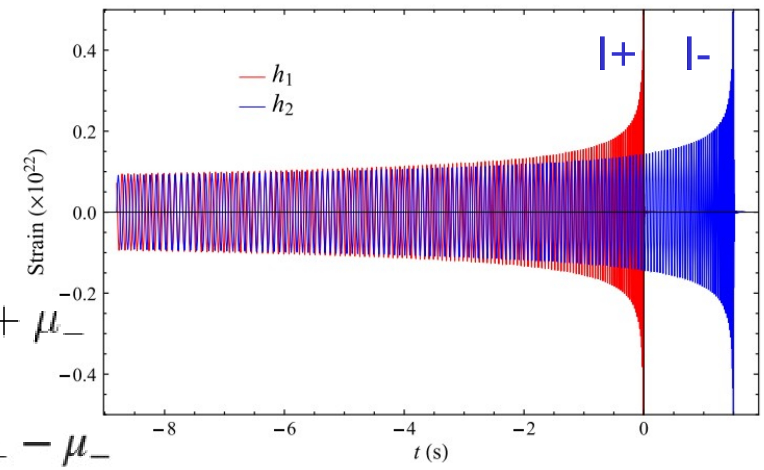
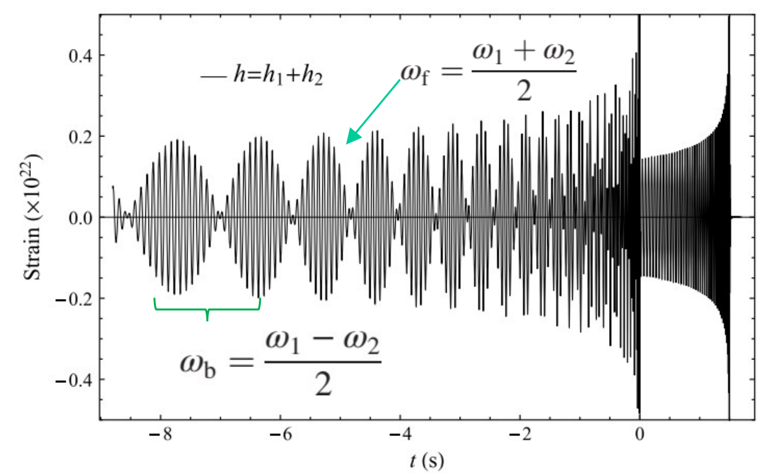
Assumptions:
1. we observe GW signals from both images I+, I- simultaneously for some time (ET – restrictive, DECIGO, LISA – typical)
2. lens: point mass or SIS

$$
\begin{aligned}
h &= \mu_+[A^+\cos(\omega_1 t+\phi_1)+A^\times \sin(\omega_1 t+\phi_1)] \\
&\quad + \mu_-[A^+\cos(\omega_2 t+\phi_2)+A^\times \sin(\omega_2 t+\phi_2)] \\
&= \mu_s\Big[A^+\cos(\omega_{\rm f} t+\phi_{\rm f})\cos(\omega_{\rm b} t+\phi_{\rm b}) \\
&\quad + A^\times \cos\left(\omega_{\rm f} t+\phi_{\rm f}-\frac{\pi}{2}\right)\cos(\omega_{\rm b} t+\phi_{\rm b})\Big] \\
&\quad + \mu_d\Big[A^+\cos\left(\omega_{\rm f} t+\phi_{\rm f}+\frac{\pi}{2}\right)\cos\left(\omega_{\rm b} t+\phi_{\rm b}-\frac{\pi}{2}\right) \\
&\quad + A^\times\cos(\omega_{\rm f} t+\phi_{\rm f})\cos\left(\omega_{\rm b} t+\phi_{\rm b}-\frac{\pi}{2}\right)\Big],
\end{aligned}
$$

$$\mu_s = \mu_+ + \mu_-$$

$$\mu_d = \mu_+ - \mu_-$$





One can use h(t) = h_1(t) + h_2(t) as template in matched filtering

μ_s/μ_d can be infered

What can be extracted from beat patterns

$$\frac{\mu_+}{\mu_-} = \frac{1+\mu_s/\mu_d}{1-\mu_s/\mu_d}$$

matched filtering

Point lens of mass M

$$r = \frac{\mu_+}{\mu_-}$$

redshifted lens mass can be extracted just from GW signal

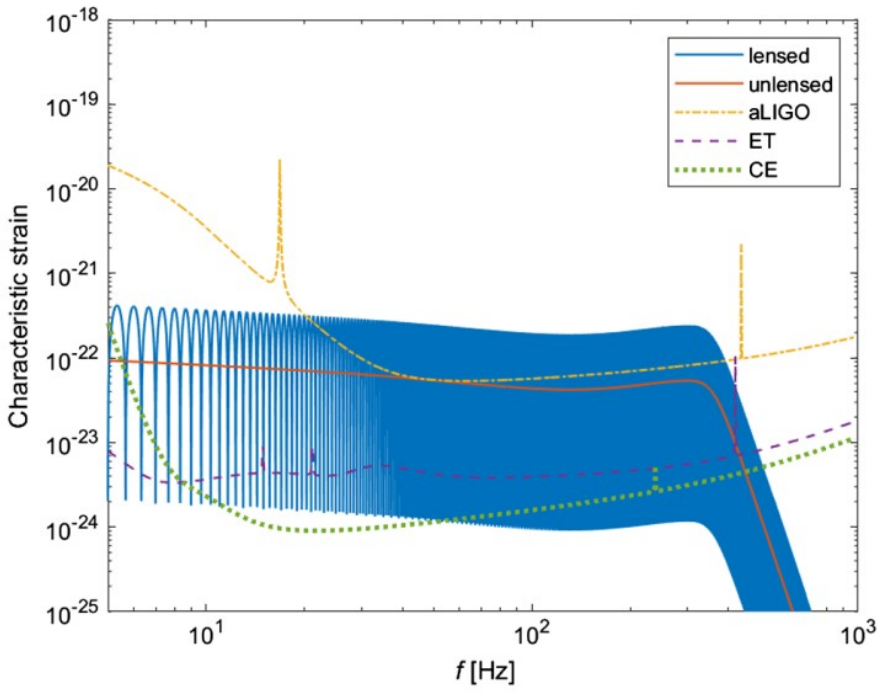
$$M_l(1+z_l) = \frac{c^3}{2G}\Delta t \left[\frac{r-1}{\sqrt{r}} + \ln r\right]^{-1}$$

SIS lens

with measured $r = \frac{\mu_+}{\mu_-}$

$$\mu_+ = \frac{r}{1+r} \qquad \mu_- = \frac{1}{1+r}$$

$$D_{\Delta t} \equiv \frac{D_l D_{ls}}{D_s} = \frac{c^5 \Delta t}{32\pi^2 \sigma_v^4 (1+z_l)} \frac{r-1}{r+1}$$



Time delay distance can be extracted without Fermat potential reconstruction

Conclusions

- New generation of ground-based detectors (ET) will considerably enhance the statistics of GW events – lensed signals will be detected
- Space-borne detectors sensitive at lower GW frequencies will create new synergies with ground-based detectors.
- Detection of inspiralling DCO in DECIGO for days and months before they enter LIGO/Virgo or ET band – first constraints on chirp mass, spin and location. Large numer of cycles – possibility to constrain LIV theories, massive gravitons, non-standard gravity theories.
- Lensed GW signals accompanied by EM counterparts will be important for precision cosmology
- Wave phenomena – diffraction and interference of GWs from lensed images create new opportuinites
- Lensing of GW in DECIGO band – unresolved multiple images in adiabatic inspiral phase, will allow to detect interference effects – beat patterns.

PRL **118,** 091102 (2017) PHYSICAL REVIEW LETTERS week ending 3 MARCH 2017

Speed of Gravitational Waves from Strongly Lensed Gravitational Waves and Electromagnetic Signals

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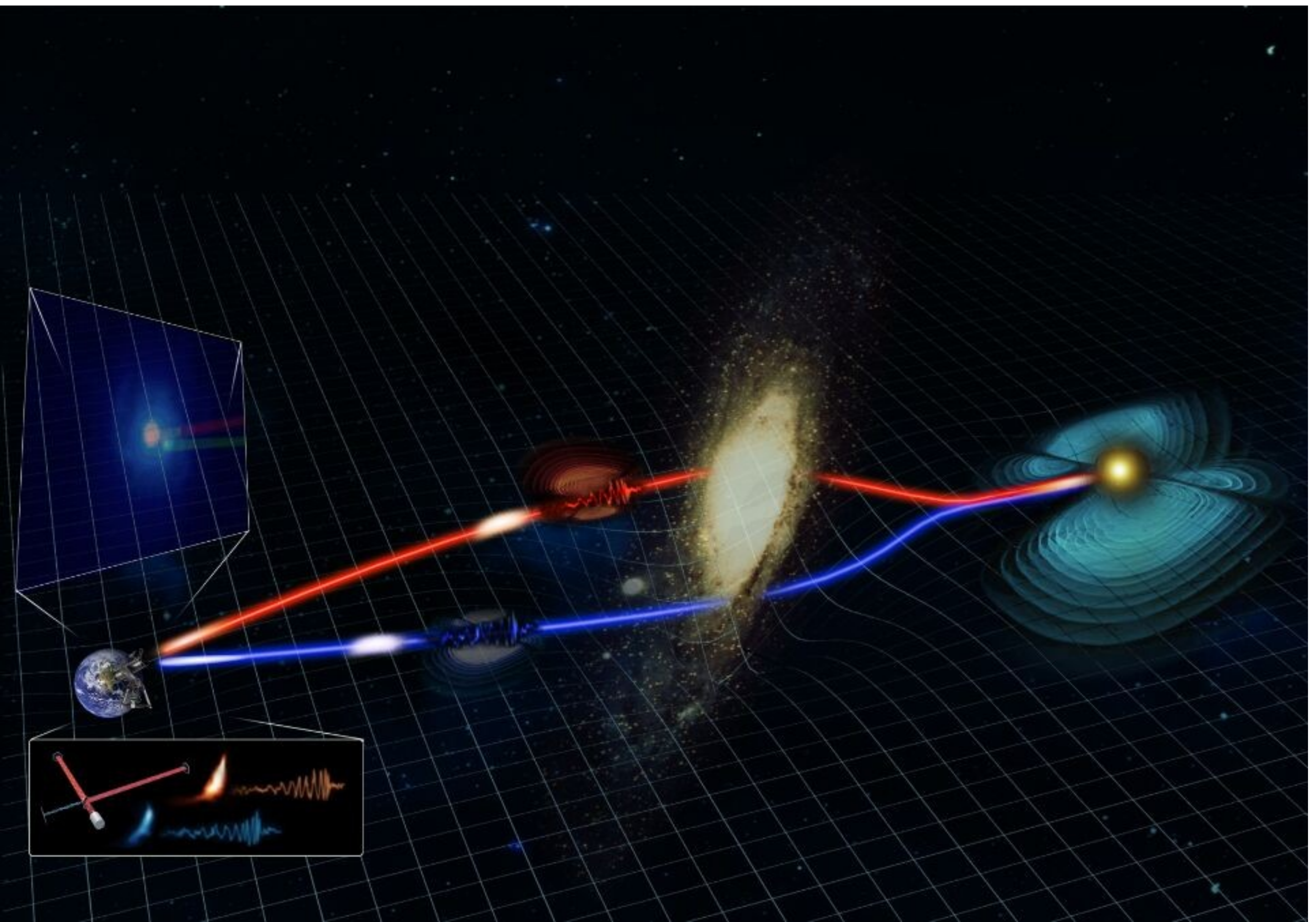


See also

Multimessenger time delays from lensed gravitational waves

Tessa Baker and Mark Trodden
Phys. Rev. D 95, 063512 (2017)

Differences in lensing time delays between images registered in EM and GW enable to test the speed of gravitational waves

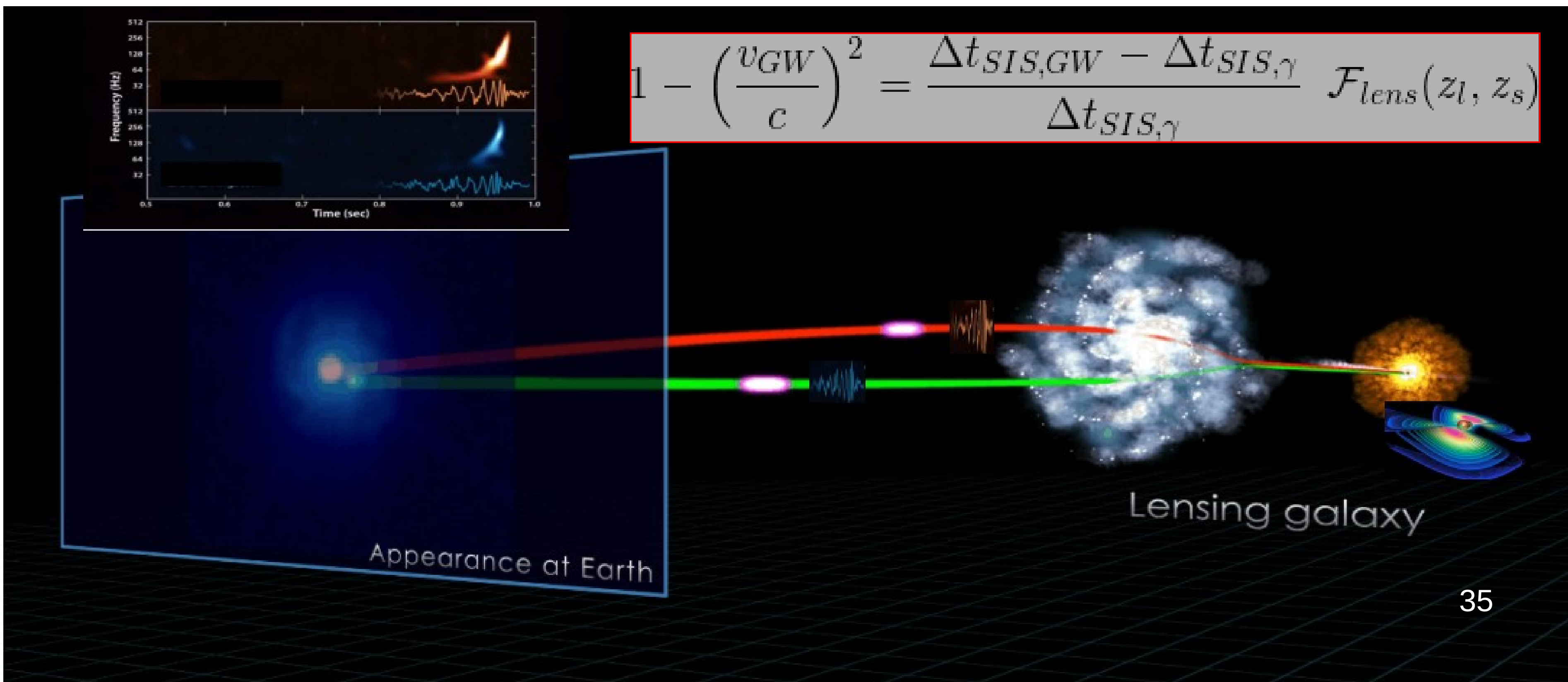
General Idea

Classically GW propagates with the speed of light c, but in some theories of modified gravity it can propagate with v_{GW} different from c

In a strongly lensed event time delays between images in GW Δt_{GW} and in EM Δt_{γ} would be different.

<u>This method is **free** from any assumptions regarding intrinsic timelag</u> between EM and GW signal emission.

$$1 - \left(\frac{v_{GW}}{c}\right)^2 = \frac{\Delta t_{SIS,GW} - \Delta t_{SIS,\gamma}}{\Delta t_{SIS,\gamma}} \; \mathcal{F}_{lens}(z_l, z_s)$$



The idea of seeking for the difference in time delays of lensed EM and GW signals as a way of testing the speed of GWs is independent of many pre-assumptions inherent to other alternative methods.

• It is free from any assumptions concerning moments of emission of GW and EM signals (intrinsic time lag)

•Does not rely on detailed analysis of waveforms – only on detection trigger

•It does not really depend even on lens model (SIS served for illustration) – if GW and EM signals propagate at different speeds (detectable with this technique) it would be revealed as a difference of time delays anyway (EM and GW traverse the same lensing potential whatever it is).

•Only for qualitative interpretation of such a difference, lensing potential should be known precisely – this can be achieved with dedicated follow up study of the lensing system in the optical.

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Mon. Not. R. Astron. Soc. **396,** 946–950 (2009) doi:10.1111/j.1365-2966.2009.14748.x

Gravitational lensing time delays as a tool for testing Lorentz-invariance violation

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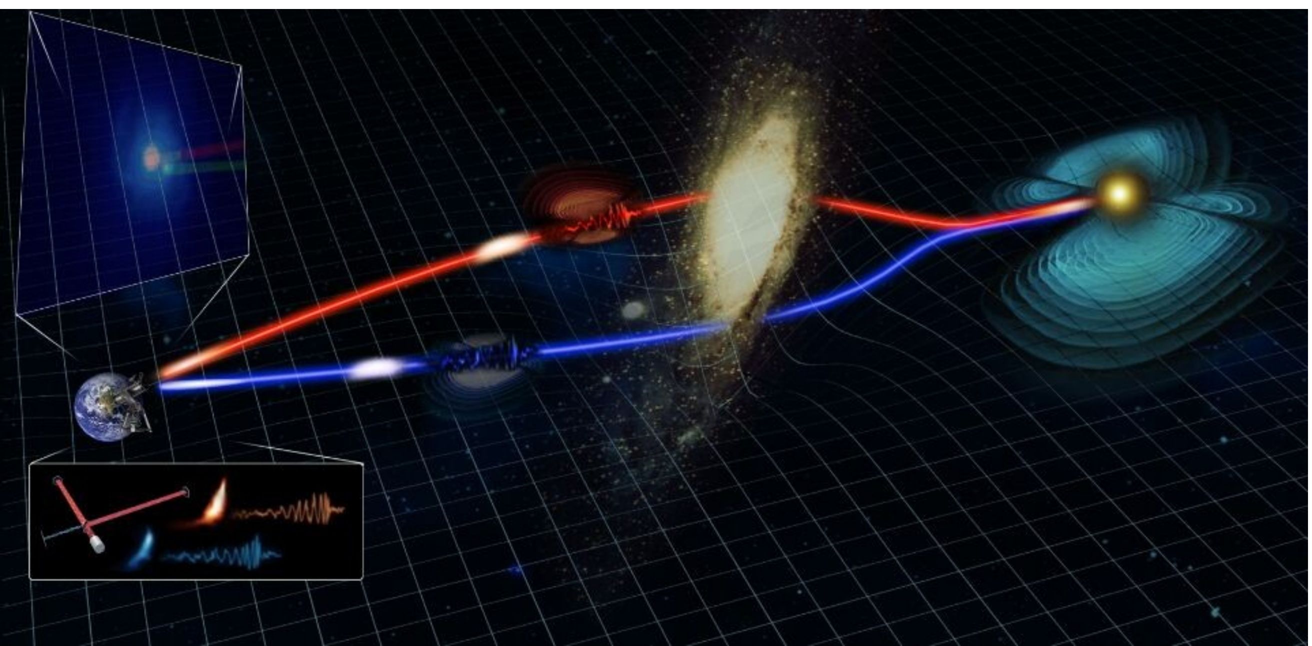

The same idea allows to test Lorentz Invariance Violation (modified dispersion relation)

Thank you !