Quantum simulations of loop quantum gravity

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Quantum Cosmos Lab

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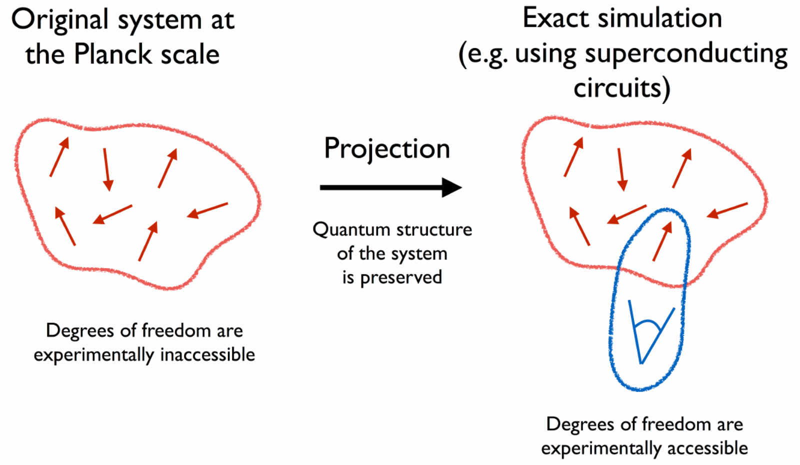
Quantum simulations of physics

Feynman, R. P. Simulating physics with computers. Int. J. Theor. Phys. 21, 467–488 (1982)



Quantum computers and quantum systems

for one qubit

$$\dim\left(\mathcal{H}\right) = 2$$

for N qubits

$$\dim\left(\mathcal{H}^{\otimes N}\right) = 2^{N}$$

Quantum computers and quantum systems

for one qubit

$$\dim(\mathcal{H}) = 2$$

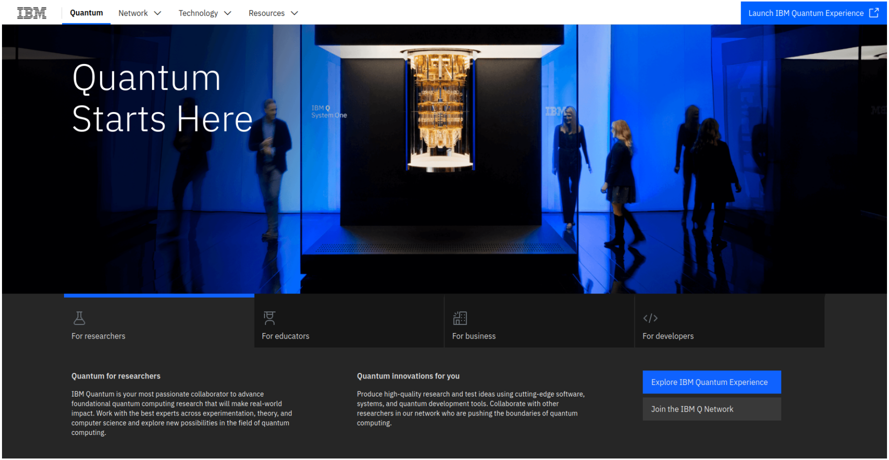
for N qubits

$$\dim\left(\mathcal{H}^{\otimes N}\right) = 2^N$$

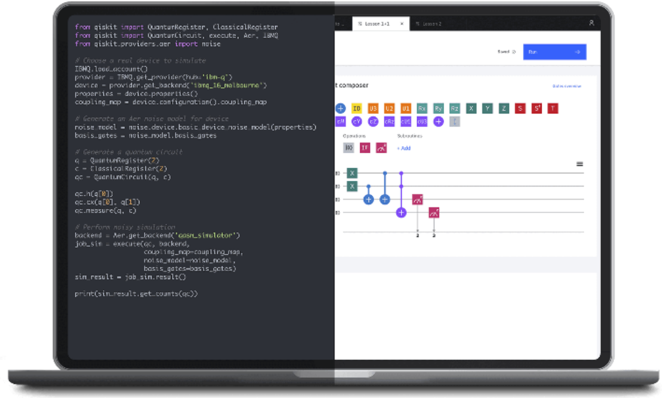
for $N = 50$

$$\dim\left(\mathcal{H}^{\otimes 50}\right) \simeq 10^{15}$$

Quantum computers IBM Q

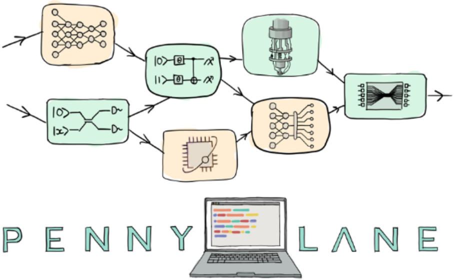


IBM - Qiskit library

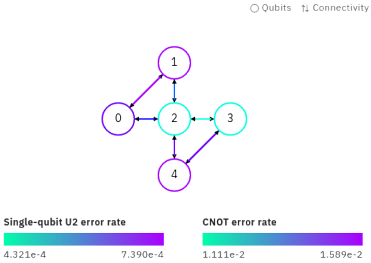


Xanadu - Penny Lane library

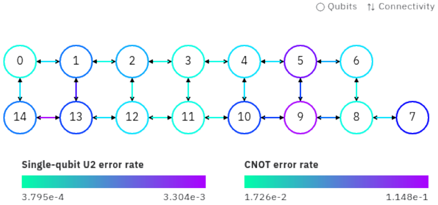
```python
import pennylane as qml
from pennylane import numpy as np

# create a quantum device
dev1 = qml.device('default.qubit', wires=1)

@qml.qnode(dev1)
def circuit(phi1, phi2):
  # a quantum node
  qml.RX(phi1, wires=0)
  qml.RY(phi2, wires=0)
  return qml.expval(qml.PauliZ(0))

def cost(x, y):
  # classical processing
  return np.sin(np.abs(circuit(x, y))) - 1

# calculate the gradient
dcost = qml.grad(cost, argnum=[0, 1])
```



Quantum processors



Yorktown quantum processor



Melbourne quantum processor

Quantum circuits

$|0\rangle$ —

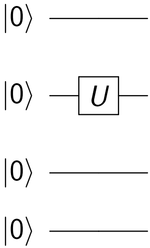
$|0\rangle$ —

$|0\rangle$ —

$|0\rangle$ —

Quantum circuits

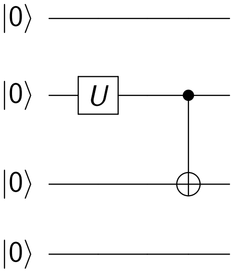
One-qubit gates:



$$U_3 = \begin{pmatrix} \cos(\theta/2) & -e^{i\lambda}\sin(\theta/2) \\ e^{i\phi}\sin(\theta/2) & e^{i\lambda+i\phi}\cos(\theta/2)\,, \end{pmatrix}$$

$$U_3|0\rangle = \cos(\theta/2)\,|0\rangle + e^{i\phi}\sin(\theta/2)\,|1\rangle$$

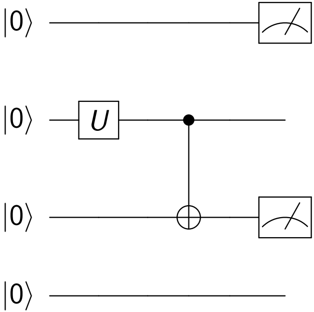
Two-qubits gates:



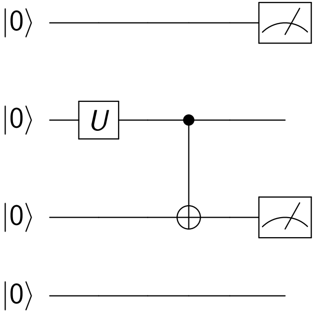
$$CX = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$CX|00\rangle = |00\rangle$$
$$CX|01\rangle = |01\rangle$$
$$CX|10\rangle = |11\rangle$$
$$CX|11\rangle = |10\rangle$$

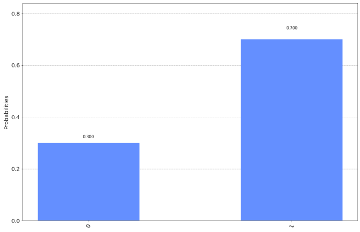
Measurements:



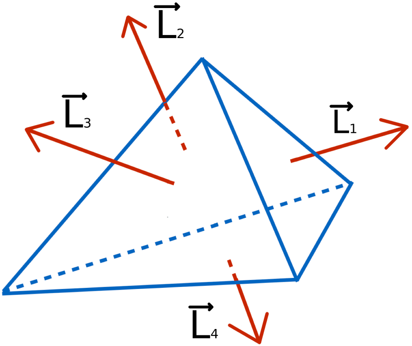
Measurements:



$\longrightarrow$

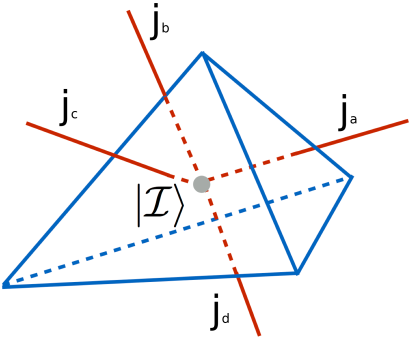


Quantum geometry



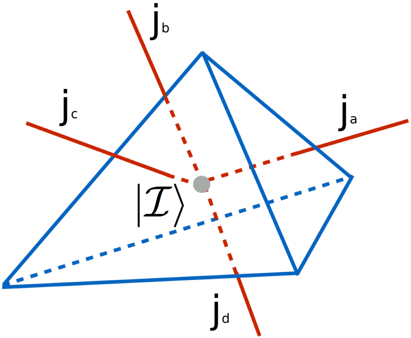
$$\vec{C} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \vec{L}_4 = 0$$

Quantum geometry



$$\left[L_a^i, L_b^j\right] = i\delta_{ab} l_0^2 \epsilon_k^{ij} L_a^k$$

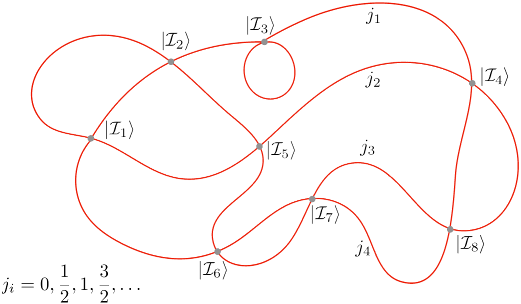
Quantum geometry



$$\left[L_a^i, L_b^j\right] = i\delta_{ab} l_0^2 \epsilon_k^{ij} L_a^k$$

$$\vec{C}|\mathcal{I}\rangle = 0; \quad |\mathcal{I}\rangle \in Inv_{SU(2)}\left(\mathcal{H}_{j_a} \otimes \mathcal{H}_{j_b} \otimes \mathcal{H}_{j_c} \otimes \mathcal{H}_{j_d}\right)$$

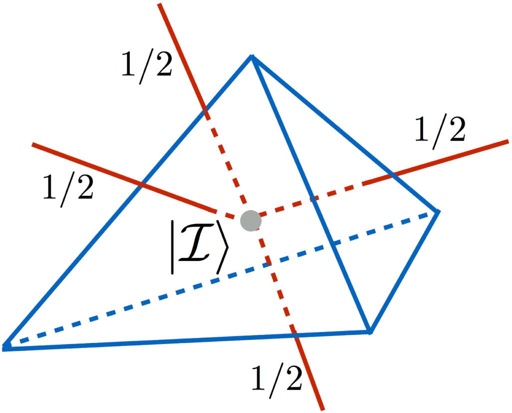
Spin-network states



$$|\mathcal{I}_n\rangle \in Inv_{SU(2)}\left(\mathcal{H}_{j_a} \otimes \mathcal{H}_{j_b} \otimes \mathcal{H}_{j_c} \otimes \mathcal{H}_{j_d}\right)$$

$$|\Gamma, j_l, \mathcal{I}_n\rangle = \bigotimes_n \mathcal{I}_n$$

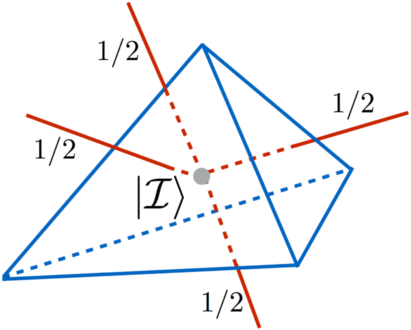
Single node



$$|\mathcal{I}\rangle \in Inv_{SU(2)}\left(\mathcal{H}_{1/2} \otimes \mathcal{H}_{1/2} \otimes \mathcal{H}_{1/2} \otimes \mathcal{H}_{1/2}\right)$$

$$|\mathcal{I}\rangle = \cos\frac{\theta}{2}|\iota_0\rangle + e^{i\phi}\sin\frac{\theta}{2}|\iota_1\rangle$$

Single node on qubits



$$|\mathcal{I}\rangle = \cos\frac{\theta}{2}|\iota_0\rangle + e^{i\phi}\sin\frac{\theta}{2}|\iota_1\rangle =$$

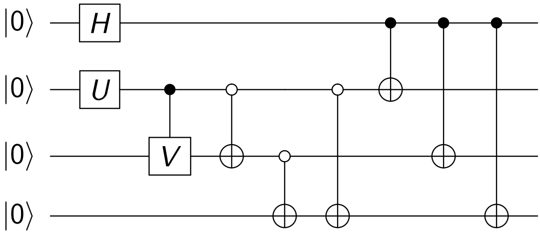
$$\frac{c_1}{\sqrt{2}}(|0011\rangle + |1100\rangle)+$$

$$\frac{c_2}{\sqrt{2}}(|0101\rangle + |1010\rangle)+$$

$$\frac{c_3}{\sqrt{2}}(|0110\rangle + |1001\rangle)$$

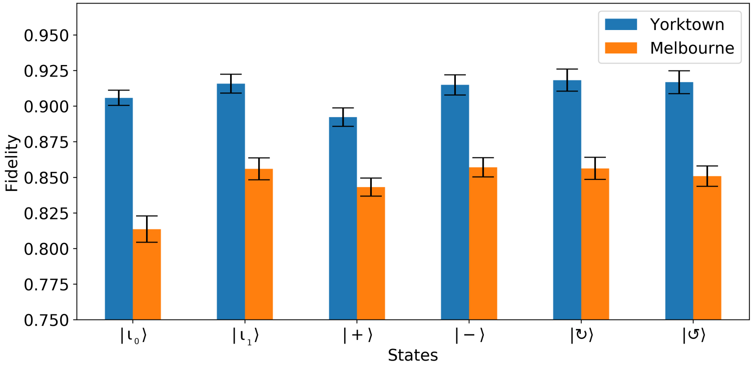
Single node on qubits

$$|\mathcal{I}\rangle = \frac{c_1}{\sqrt{2}}(|0011\rangle + |1100\rangle) + \frac{c_2}{\sqrt{2}}(|0101\rangle + |1010\rangle) + \frac{c_3}{\sqrt{2}}(|0110\rangle + |1001\rangle)$$



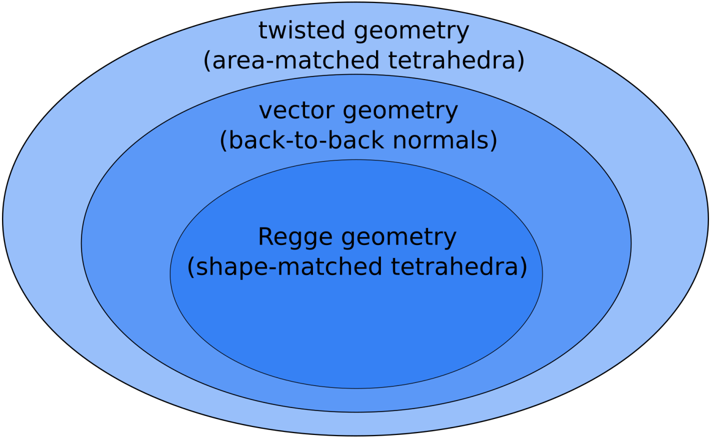
$$U = \begin{pmatrix} c_1 & \sqrt{|c_2|^2 + |c_3|^2} \\ -\sqrt{|c_2|^2 + |c_3|^2} & c_1^* \end{pmatrix} \qquad V = \begin{pmatrix} -\frac{c_2}{\sqrt{|c_2|^2+|c_3|^2}} & \frac{c_3^*}{\sqrt{|c_2|^2+|c_3|^2}} \\ -\frac{c_3}{\sqrt{|c_2|^2+|c_3|^2}} & -\frac{c_2^*}{\sqrt{|c_2|^2+|c_3|^2}} \end{pmatrix}$$

Simulation on quantum processor



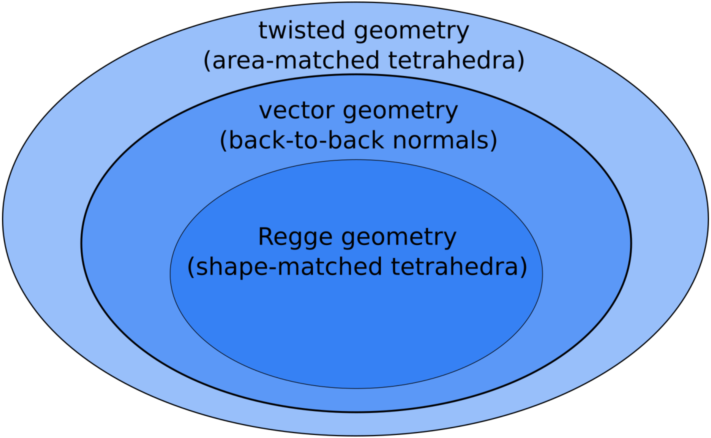
G. Cz, Jakub Mielczarek, "Quantum simulations of a qubit of space" Phys. Rev. D 103, 046001 (2021)

Gluing tetrahedrons



Spin-network basis states are un-entangled $|\Gamma, j_l, \mathcal{I}_n\rangle = \bigotimes_n \mathcal{I}_n$

Gluing tetrahedrons



Spin-network basis states are un-entangled $|\Gamma, j_l, \mathcal{I}_n\rangle = \bigotimes_n \mathcal{I}_n$

Gluing tetrahedrons

squeezed states

$$|\mathcal{B}, \lambda_l\rangle = \left(1 - |\lambda_l|^2\right) \sum_j \sqrt{2j+1}\lambda_l^{2j} |\mathcal{B}, j\rangle \quad (1)$$

maximally entangled state of spin j

$$|\mathcal{B}, j\rangle = \frac{1}{\sqrt{2j+1}} \sum_{m=-j}^{j} (-1)^{j-m} |j, m\rangle_s |j, -m\rangle_t$$

Gluing tetrahedrons

squeezed states

$$|\mathcal{B}, \lambda_l\rangle = \left(1 - |\lambda_l|^2\right) \sum_j \sqrt{2j+1} \lambda_l^{2j} |\mathcal{B}, j\rangle \tag{1}$$

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$$|\mathcal{B}, j\rangle = \frac{1}{\sqrt{2j+1}} \sum_{m=-j}^{j} (-1)^{j-m} |j, m\rangle_s |j, -m\rangle_t$$

projection on spin-network basis states

$$P_\Gamma = \sum_{j_l, \mathcal{I}_n} |\Gamma, j_l, \mathcal{I}_n\rangle \langle \Gamma, j_l, \mathcal{I}_n|$$

$$|\Gamma, \mathcal{B}, \lambda_l\rangle = P_\Gamma \bigotimes_l |\mathcal{B}, \lambda_l\rangle$$

Gluing tetrahedrons

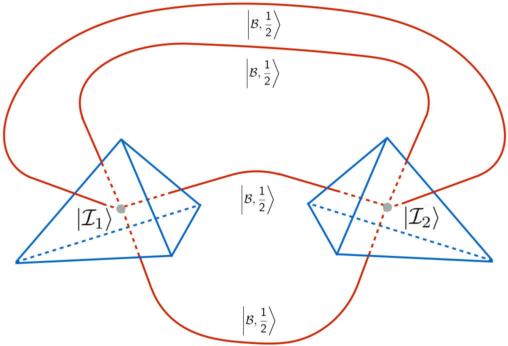
for $j = \frac{1}{2}$:

$$|\mathcal{B}, j\rangle = \frac{1}{\sqrt{2}} \left(\left| \frac{1}{2}, \frac{1}{2} \right\rangle \left| \frac{1}{2}, -\frac{1}{2} \right\rangle - \left| \frac{1}{2}, -\frac{1}{2} \right\rangle \left| \frac{1}{2}, \frac{1}{2} \right\rangle \right)$$

in qubit notations

$$\left| \mathcal{B}, \frac{1}{2} \right\rangle = \frac{1}{\sqrt{2}} \left(|01\rangle - |10\rangle \right)$$

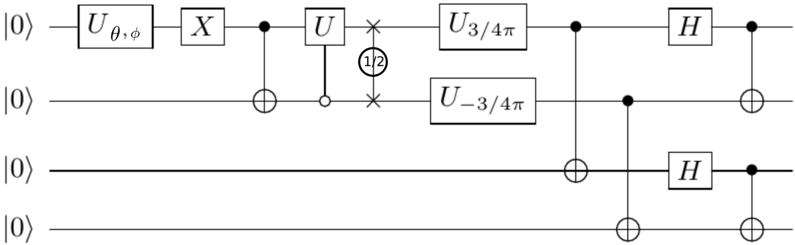
Dipole



$$\left|\Gamma_2,\mathcal{B},\frac{1}{2}\right\rangle = P_\Gamma \bigotimes_l \left|\mathcal{B},\frac{1}{2}\right\rangle = \sum_{k,l} \iota_{(k)}^{m_1 m_2 m_3 m_4}\, \iota_{(l) m_1 m_2 m_3 m_4} \left|\iota_k \iota_l\right\rangle$$

$$= \frac{1}{\sqrt{2}}\left(|\iota_0\iota_0\rangle + |\iota_1\iota_1\rangle\right)$$

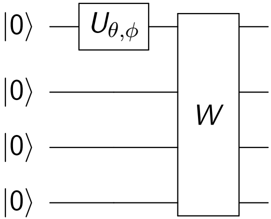
New circuit for node



$$|\mathcal{I}\rangle = \cos\frac{\theta}{2}|\iota_0\rangle + e^{i\phi}\sin\frac{\theta}{2}|\iota_1\rangle =$$

$$\frac{c_1}{\sqrt{2}}(|0011\rangle + |1100\rangle) + \frac{c_2}{\sqrt{2}}(|0101\rangle + |1010\rangle) + \frac{c_3}{\sqrt{2}}(|0110\rangle + |1001\rangle)$$

New circuit for node



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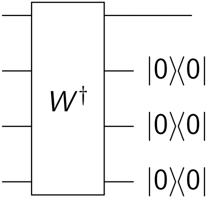
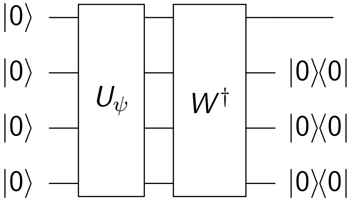
Projector



Projection operator on intertwiner subspace, expressed in one-qubit representation.

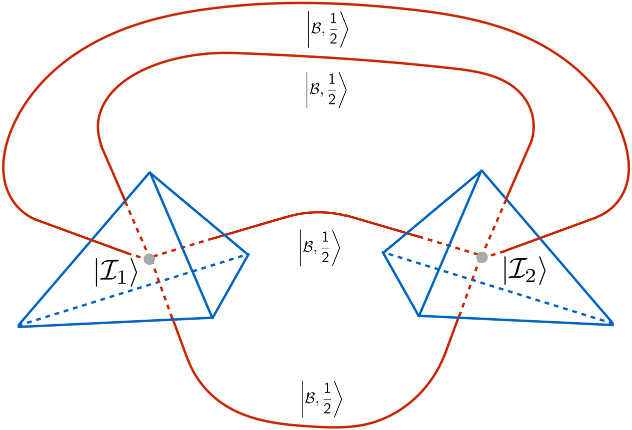
Projector



Projection of state $|\psi\rangle$ on intertwiner subspace, expressed in one-qubit representation.

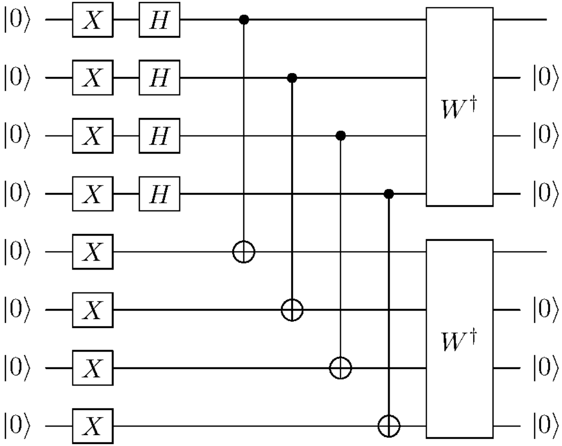
$$\left(\sum_k |\iota_k\rangle \langle \iota_k|\right) |\psi\rangle$$

Dipole



$$\left|\Gamma_2, \mathcal{B}, \frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}\left(|\iota_0\iota_0\rangle + |\iota_1\iota_1\rangle\right)$$

Dipole



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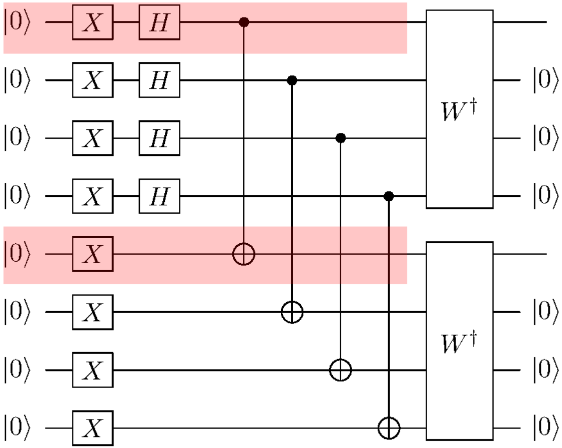
Dipole



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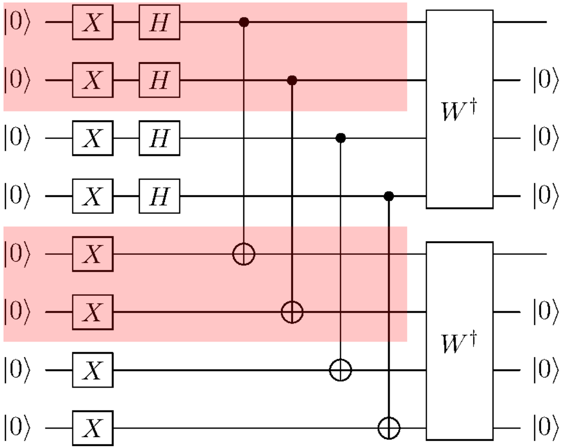
Dipole



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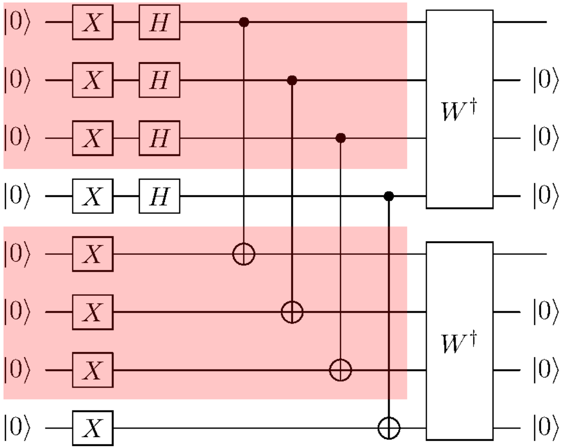
Dipole



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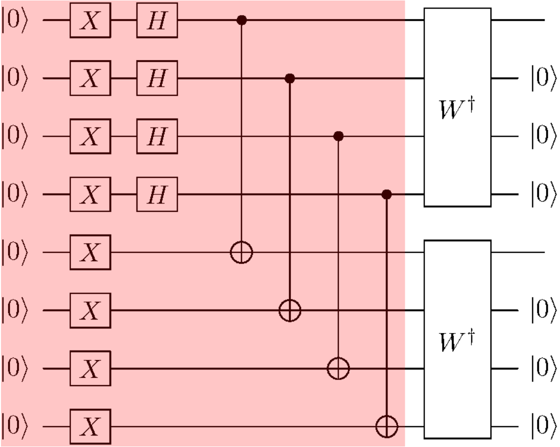
Dipole



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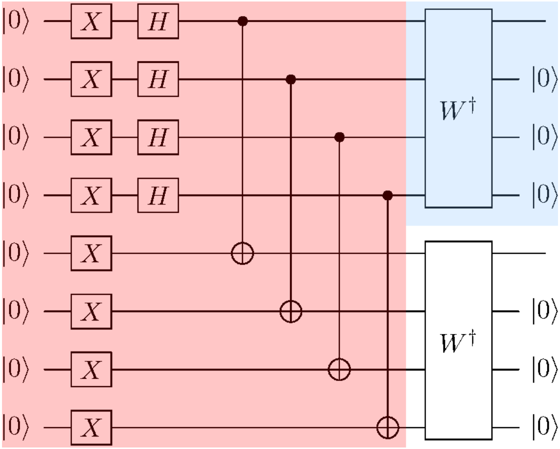
Dipole



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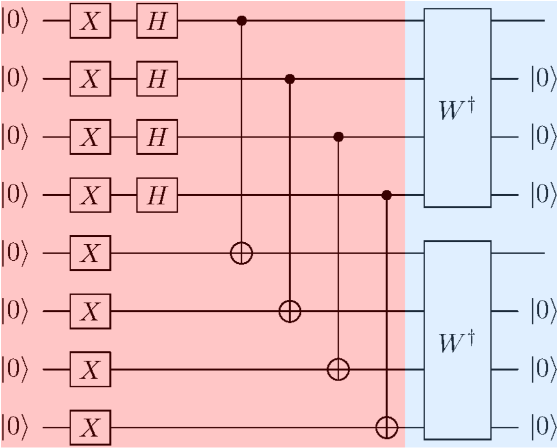
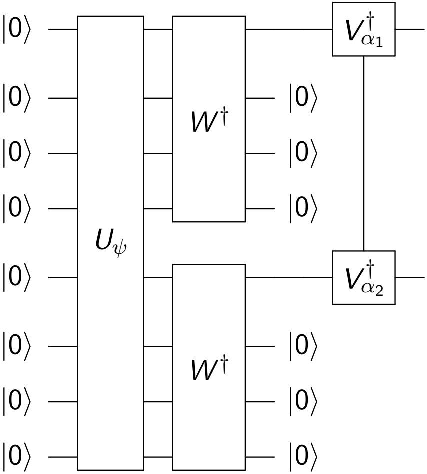
Dipole



$$\left|\Gamma_2, \mathcal{B}, \frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}\left(|\iota_0\iota_0\rangle + |\iota_1\iota_1\rangle\right)$$

Dipole



$|0\rangle$
$|0\rangle$
$|0\rangle$
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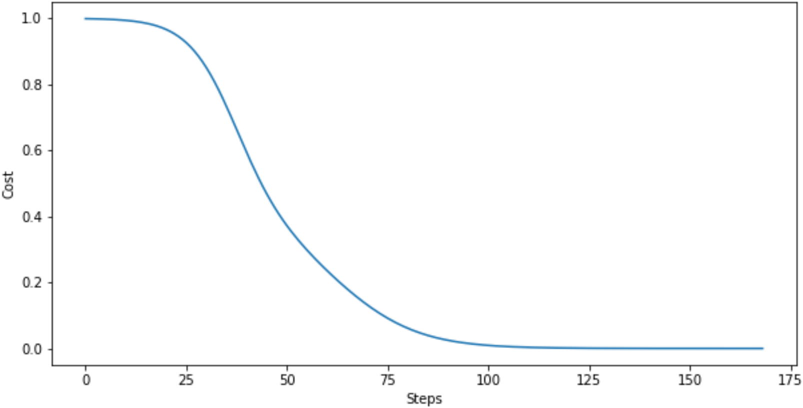
$|0\rangle$
$|0\rangle$
U_ψ
$W^\dagger$
$W^\dagger$
$|0\rangle$
$|0\rangle$
$|0\rangle$
$|0\rangle$
$|0\rangle$
$|0\rangle$
$V^\dagger_{\alpha_1}$
$V^\dagger_{\alpha_2}$

Dipole

Variational algorithm

$$cost(\alpha) = 1 - Prob(q_0 q_4 = 00)$$

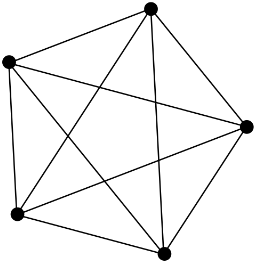


Pentagram

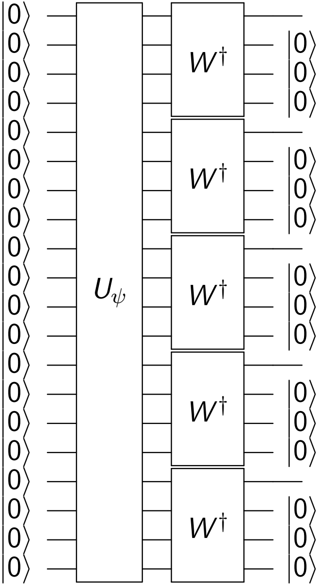
$$P\,|\psi\rangle = \sum_{\iota_{k_i}} \overline{\{15j\}}\,|\iota_{k_1}\iota_{k_2}\iota_{k_3}\iota_{k_4}\iota_{k_5}\rangle$$

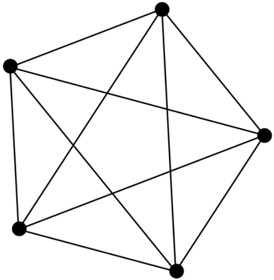
where

$$\{15j\} = \iota_1^{m_{12}m_{13}m_{14}m_{15}}\,\iota_{2;m_{12}}{}^{m_{13}m_{14}m_{15}}\,\iota_{3;m_{12}m_{13}}{}^{m_{14}m_{15}}\,\iota_{4;m_{12}m_{13}m_{14}}{}^{m_{15}}\,\iota_{5;m_{12}m_{13}m_{14}m_{15}}$$

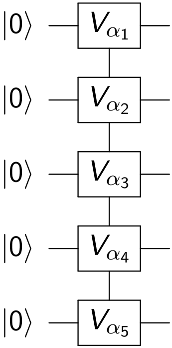


Pentagram

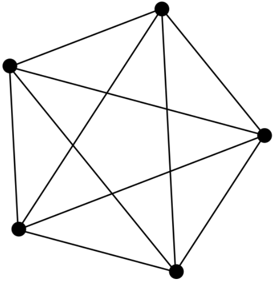




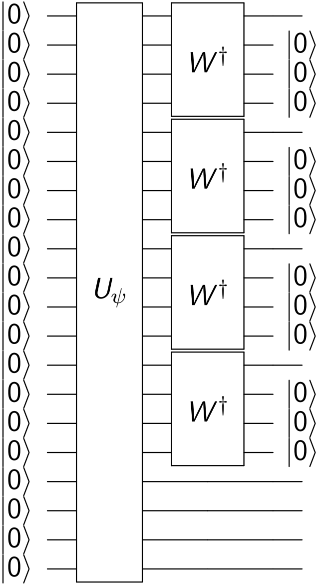
Pentagram

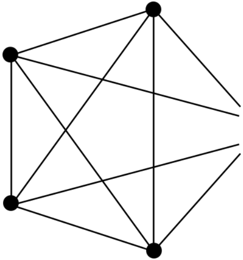


$|0\rangle$ V_{α_1}
$|0\rangle$ V_{α_2}
$|0\rangle$ V_{α_3}
$|0\rangle$ V_{α_4}
$|0\rangle$ V_{α_5}

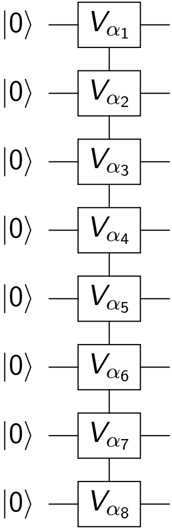


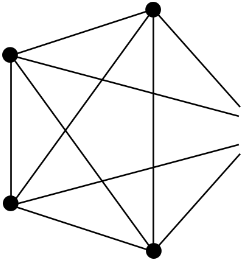
Pentagram



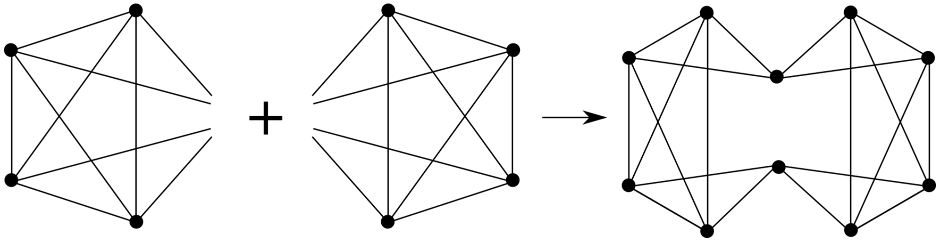


Pentagram

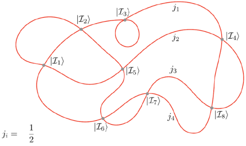

$|0\rangle$
V_{α_1}
$|0\rangle$
V_{α_2}
$|0\rangle$
V_{α_3}
$|0\rangle$
V_{α_4}
$|0\rangle$
V_{α_5}
$|0\rangle$
V_{α_6}
$|0\rangle$
V_{α_7}
$|0\rangle$
V_{α_8}

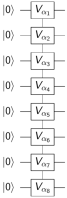


Decagram



8 qubits + 8 qubits → 10 qubits





$$\langle \hat{V} \rangle,\ S(\rho_i),\ \ldots$$

Mutual information

$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

$$S(\rho) = -Tr\rho \ln \rho$$

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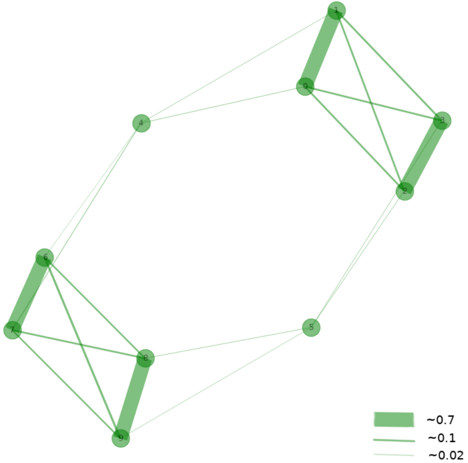
$$\mathcal{C}(O_A, O_B) = \frac{1}{||O_A||\,||O_B||}\left(\langle O_A O_B\rangle - \langle O_A\rangle\langle O_B\rangle\right)$$

$$\frac{1}{2}\mathcal{C}(O_A, O_B)^2 \leq I(\rho_{AB})$$

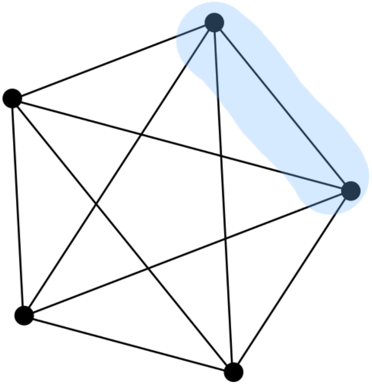
Mutual information: Decagram



$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

Page curve - random pentagram

Pairs of spins in random state on the links:



Entropy of subsystem:

$$N_A = 2$$

$$\rho = Tr_A|\Gamma_5\rangle$$

$$S(\rho) = -Tr\rho\log\rho$$

Page curve - random pentagram

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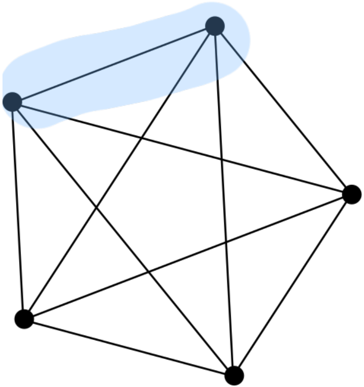

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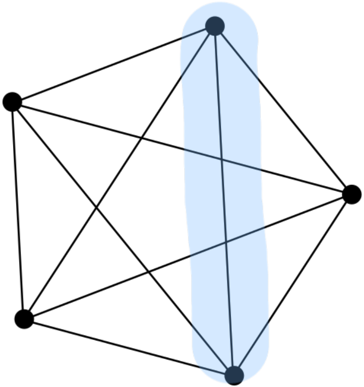

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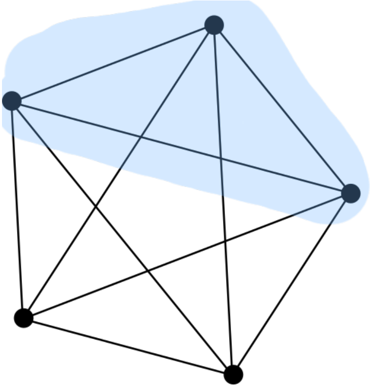

Entropy of subsystem:

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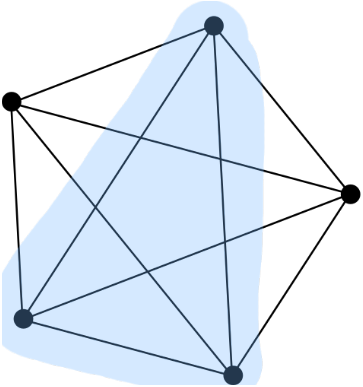
Page curve - random pentagram

Pairs of spins in random state on the links:



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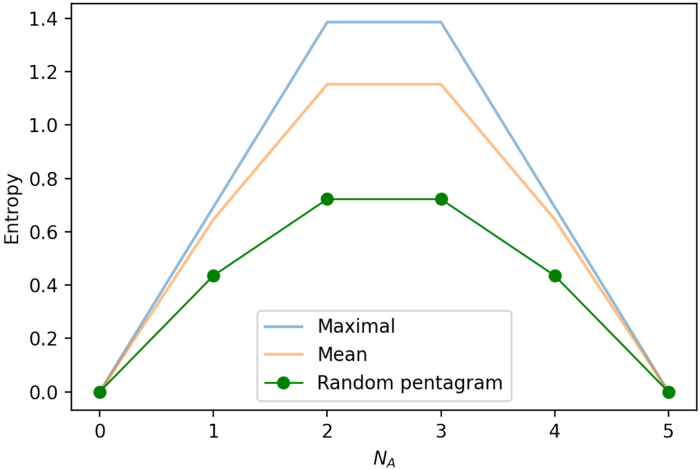
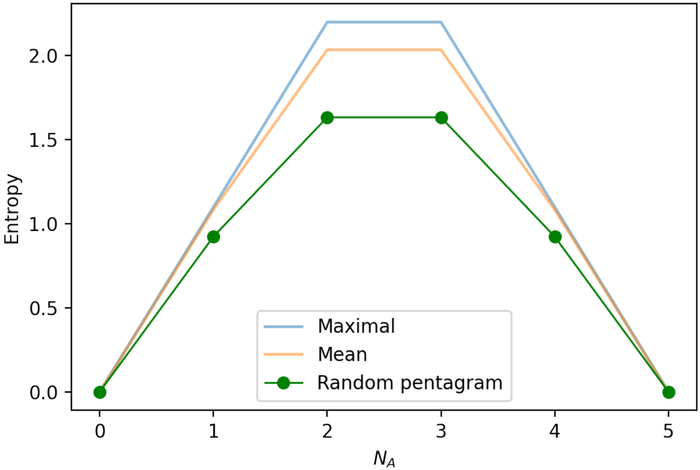
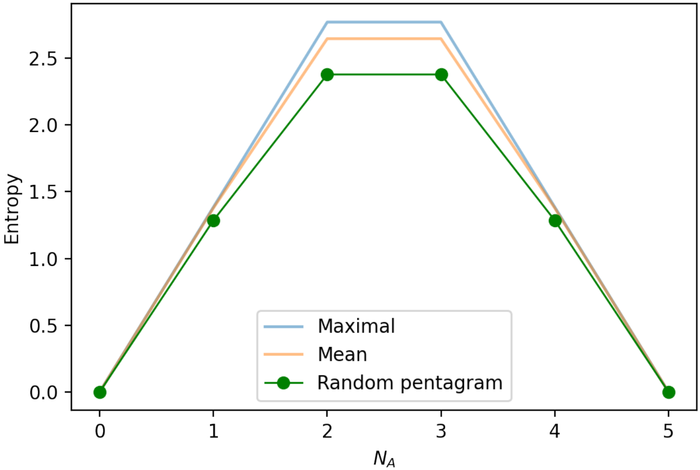
Random pentagram - spin $\frac{1}{2}$



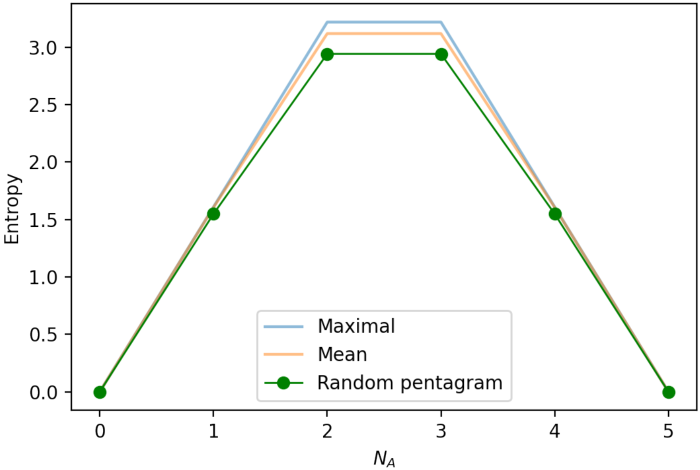
Random pentagram - spin 1



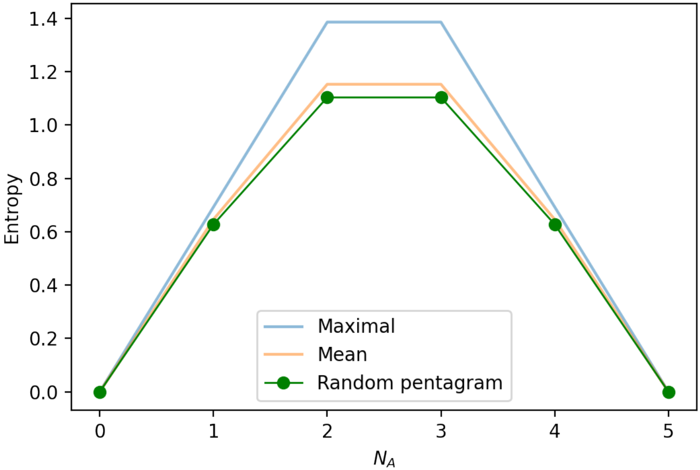
Random pentagram - spin $\frac{3}{2}$



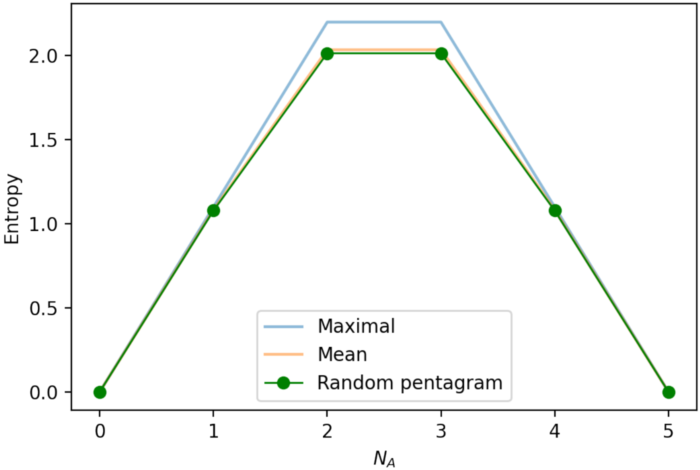
Random pentagram - spin 2



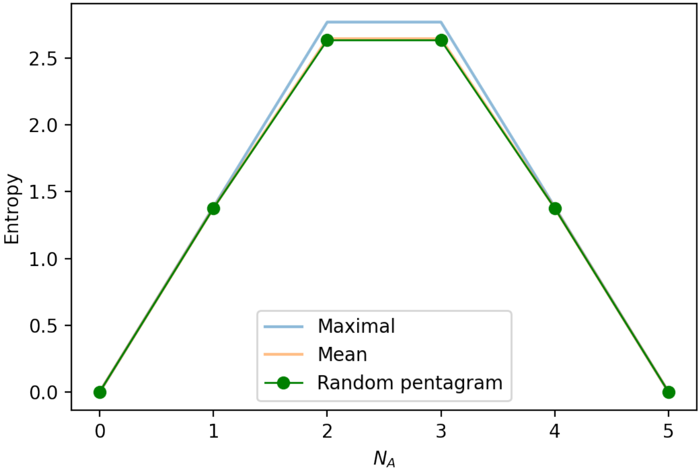
Random pentagram - maximally entangled links - spin $\frac{1}{2}$



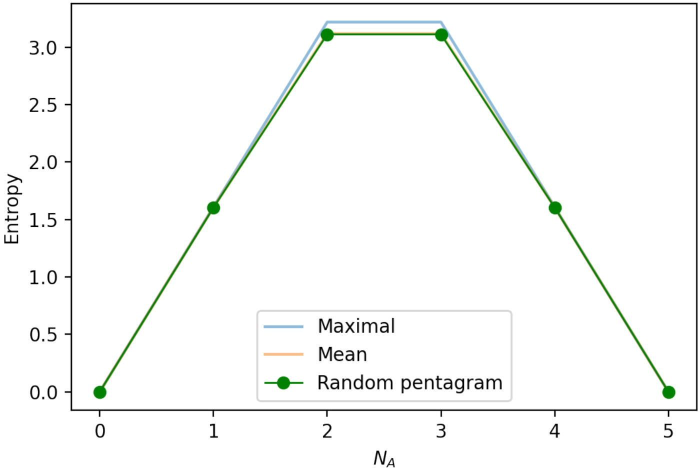
Random pentagram - maximally entangled links - spin 1



Random pentagram - maximally entangled links - spin $\frac{3}{2}$



Random pentagram - maximally entangled links - spin 2



Summary

- we can prepare a broad class of spin-network states on a quantum register
- measurements can be performed both on simulator and real quantum processor
- a number of required qubits is not much greater than the number of nodes
- entropies of subsystems and Page curve can be extracted
- for low spins, typical entropy of a subsytem in random pentagram is smaller than in random 5-qubit quantum system
- entropy of subsystems in pentagram grows up to typical entropy of a 5-qubit quantum system if maximally entangled links are used

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Thank you for your attention