

# Quantum simulations of loop quantum gravity

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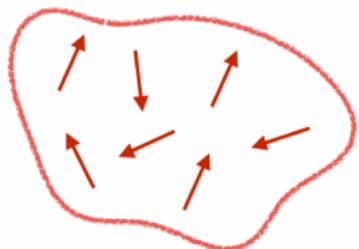


September 13, 2022

# Quantum simulations of physics

Feynman, R. P. Simulating physics with computers. Int. J. Theor. Phys. 21, 467–488 (1982)

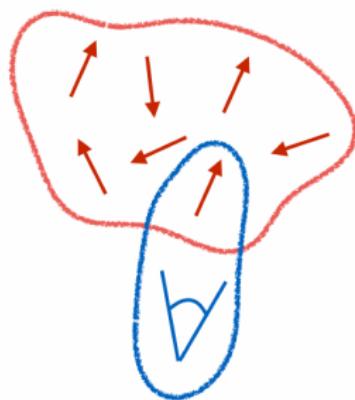
Original system at  
the Planck scale



Degrees of freedom are  
experimentally inaccessible

Exact simulation  
(e.g. using superconducting  
circuits)

Projection  
→  
Quantum structure  
of the system  
is preserved



Degrees of freedom are  
experimentally accessible

# Quantum computers and quantum systems

for one qubit

$$\dim(\mathcal{H}) = 2$$

for N qubits

$$\dim(\mathcal{H}^{\otimes N}) = 2^N$$

# Quantum computers and quantum systems

for one qubit

$$\dim(\mathcal{H}) = 2$$

for  $N$  qubits

$$\dim(\mathcal{H}^{\otimes N}) = 2^N$$

for  $N = 50$

$$\dim(\mathcal{H}^{\otimes 50}) \simeq 10^{15}$$

# Quantum computers IBM Q

IBM Q System One

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For researchers

For educators

For business

For developers

Quantum for researchers

IBM Quantum is your most passionate collaborator to advance foundational quantum computing research that will make real-world impact. Work with the best experts across experimentation, theory, and computer science and explore new possibilities in the field of quantum computing.

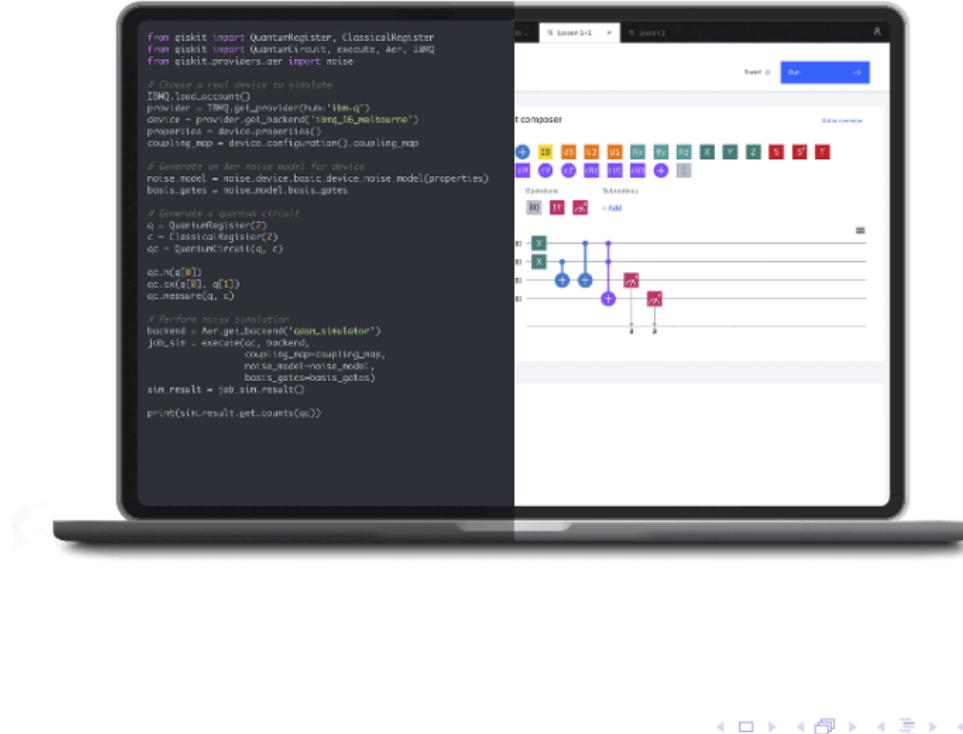
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# IBM - Qiskit library



The image shows a laptop screen with two main windows. On the left, a code editor displays a Python script using the Qiskit library. On the right, a web-based interface for Qiskit Terra shows a quantum circuit diagram.

**Code Editor Content:**

```
from qiskit import QuantumRegister, ClassicalRegister
from qiskit import QuantumCircuit, execute, Aer, IBMQ
from qiskit.providers.ibmq import noise

# Choose a real device to simulate
IBMQ.load_account()
provider = IBMQ.get_provider(hub='ibm-q')
device = provider.get_backend('ibmq_16_melbourne')
properties = device.properties()
coupling_map = device.configuration().coupling_map

# Generate an Aer noise model for device
noise_model = noise.device.basic_device_noise_model(properties)
basis_gates = noise_model.basis_gates

# Generate a quantum circuit
q = QuantumRegister(2)
c = ClassicalRegister(2)
qc = QuantumCircuit(q, c)

qc.h(q[0])
qc.cx(q[0], q[1])
qc.measure(q, c)

# Perform noisy simulation
backend = Aer.get_backend('qasm_simulator')
job_sim = execute(qc, backend,
                  coupling_map=coupling_map,
                  properties=properties,
                  basis_gates=basis_gates)
sim_result = job_sim.result()

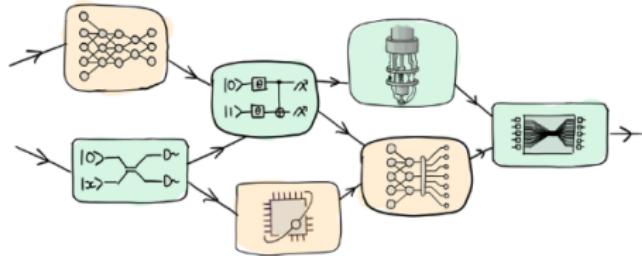
print(sim_result.get_counts(qc))
```

**Qiskit Terra Interface:**

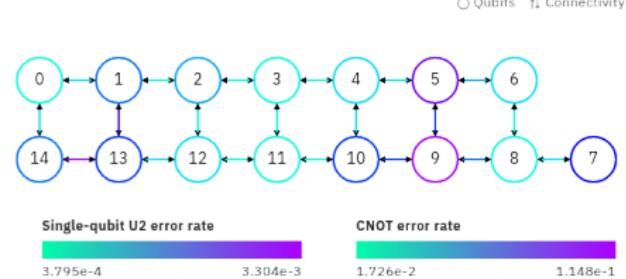
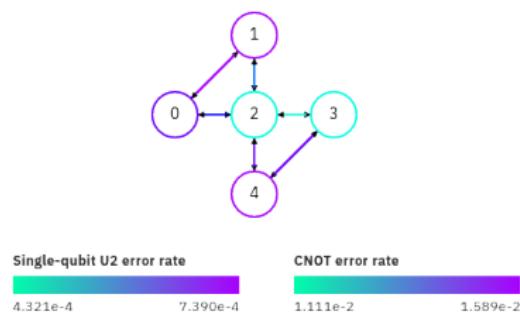
The interface includes a top navigation bar with tabs for Home, Circuits, and Data. Below the navigation is a search bar. The main area is divided into two sections: Circuit Composer and Backend Explorer. The Circuit Composer section shows the quantum circuit with two qubits and two classical bits. The Backend Explorer section shows the available backends: QASM Simulator, Statevector Simulator, and Aer Simulator.

# Xanadu - Penny Lane library

```
● ● ●  
import pennylane as qml  
from pennylane import numpy as np  
  
# create a quantum device  
dev1 = qml.device('default.qubit', wires=1)  
  
@qml.qnode(dev1)  
def circuit(phi1, phi2):  
    # a quantum node  
    qml.RX(phi1, wires=0)  
    qml.RY(phi2, wires=0)  
    return qml.expval(qml.PauliZ(0))  
  
def cost(x, y):  
    # classical processing  
    return np.sin(np.abs(circuit(x, y))) - 1  
  
# calculate the gradient  
dcost = qml.grad(cost, argnum=[0, 1])
```



# Quantum processors



# Quantum circuits

$|0\rangle$  —

$|0\rangle$  —

$|0\rangle$  —

$|0\rangle$  —

# Quantum circuits

One-qubit gates:

$|0\rangle$  —————

$|0\rangle$  ———  $U$  ———

$|0\rangle$  —————

$|0\rangle$  —————

$$U_3 = \begin{pmatrix} \cos(\theta/2) & -e^{i\lambda} \sin(\theta/2) \\ e^{i\phi} \sin(\theta/2) & e^{i\lambda+i\phi} \cos(\theta/2), \end{pmatrix}$$

$$U_3|0\rangle = \cos(\theta/2)|0\rangle + e^{i\phi} \sin(\theta/2)|1\rangle$$

# Quantum circuits

Two-qubits gates:

$|0\rangle$  —————

$|0\rangle$  ———  $U$  ————— •

$|0\rangle$  ————— ⊕

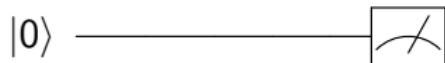
$|0\rangle$  —————

$$CX = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$CX|00\rangle = |00\rangle$   
 $CX|01\rangle = |01\rangle$   
 $CX|10\rangle = |11\rangle$   
 $CX|11\rangle = |10\rangle$

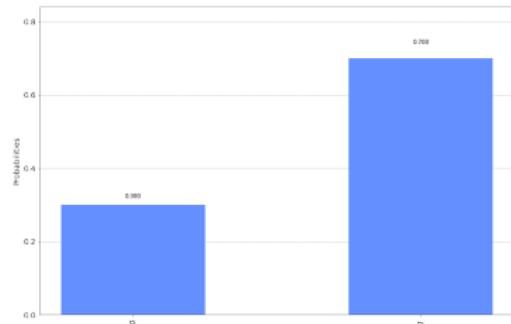
# Quantum circuits

Measurements:

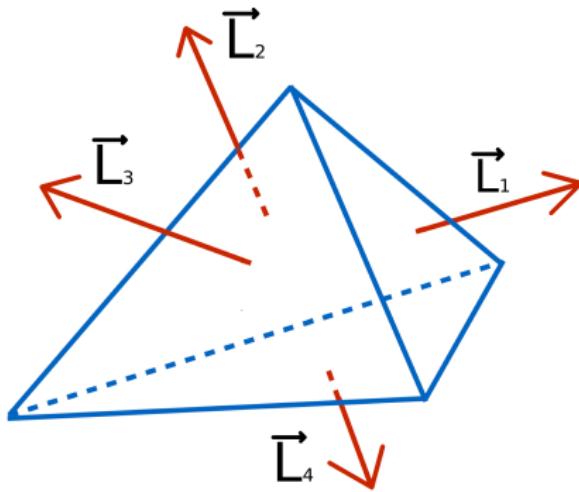


# Quantum circuits

Measurements:

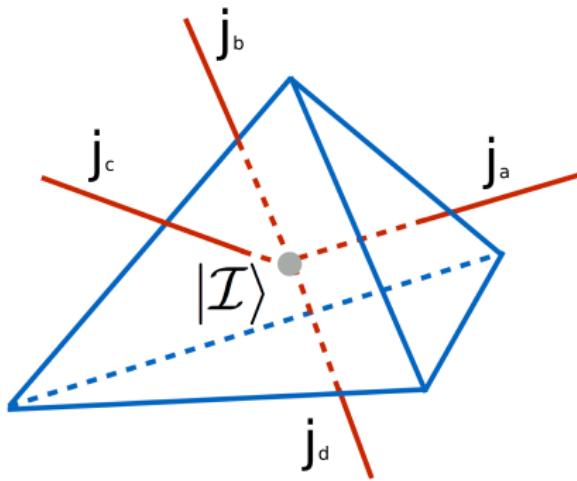


# Quantum geometry



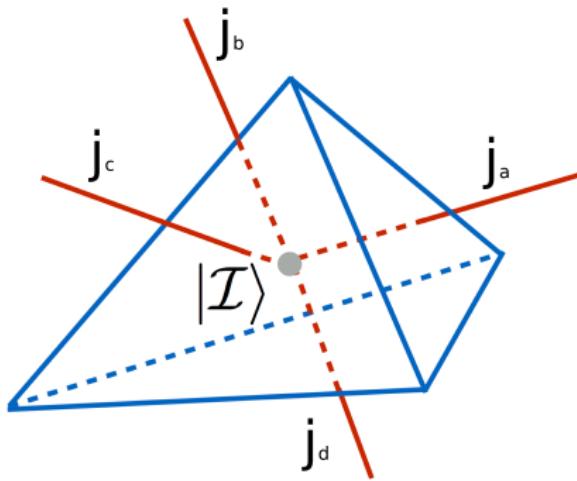
$$\vec{C} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \vec{L}_4 = 0$$

# Quantum geometry



$$[L_a^i, L_b^j] = i\delta_{ab}l_0^2\epsilon_k^{ij}L_a^k$$

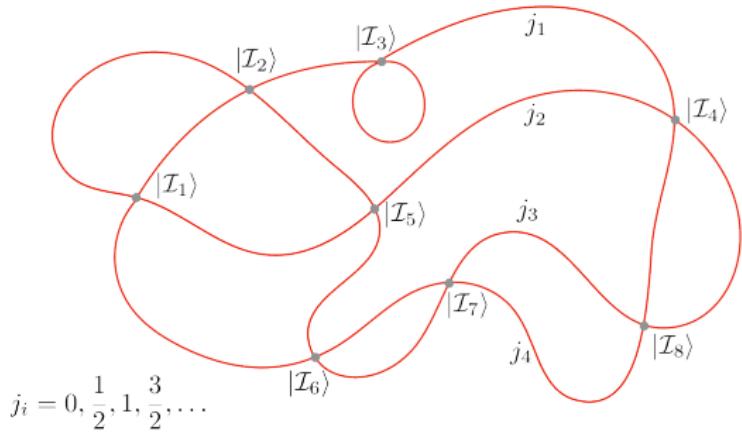
# Quantum geometry



$$[L_a^i, L_b^j] = i\delta_{ab}l_0^2\epsilon_k^{ij}L_a^k$$

$$\vec{C}|I\rangle = 0; \quad |I\rangle \in Inv_{SU(2)}(\mathcal{H}_{j_a} \otimes \mathcal{H}_{j_b} \otimes \mathcal{H}_{j_c} \otimes \mathcal{H}_{j_d})$$

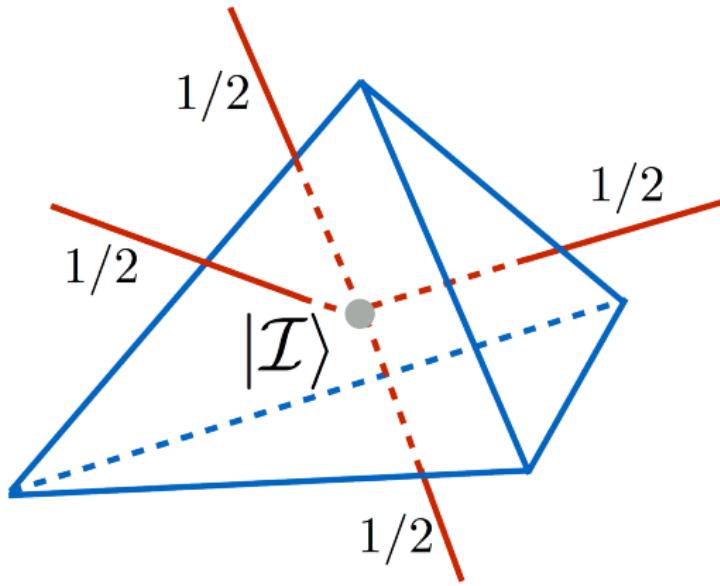
# Spin-network states



$$|\mathcal{I}_n\rangle \in \text{Inv}_{SU(2)}(\mathcal{H}_{j_a} \otimes \mathcal{H}_{j_b} \otimes \mathcal{H}_{j_c} \otimes \mathcal{H}_{j_d})$$

$$|\Gamma, j_l, \mathcal{I}_n\rangle = \bigotimes_n \mathcal{I}_n$$

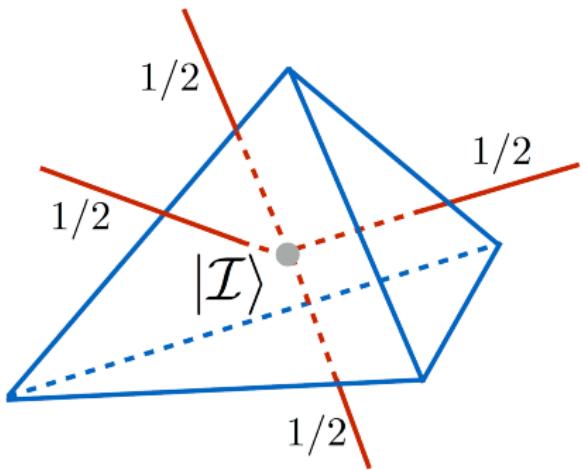
# Single node



$$|\mathcal{I}\rangle \in \text{Inv}_{SU(2)} (\mathcal{H}_{1/2} \otimes \mathcal{H}_{1/2} \otimes \mathcal{H}_{1/2} \otimes \mathcal{H}_{1/2})$$

$$|\mathcal{I}\rangle = \cos \frac{\theta}{2} |\iota_0\rangle + e^{i\phi} \sin \frac{\theta}{2} |\iota_1\rangle$$

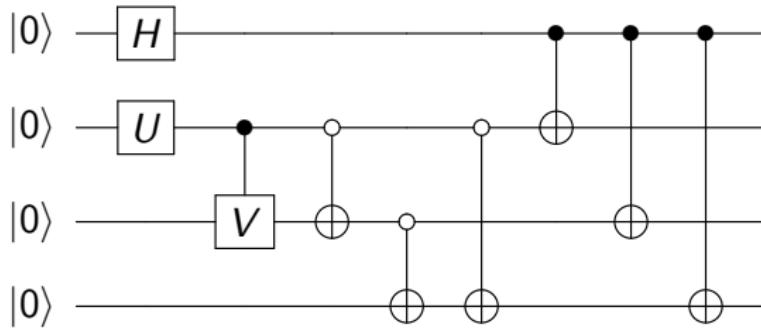
# Single node on qubits



$$\begin{aligned} |\mathcal{I}\rangle &= \cos \frac{\theta}{2} |\iota_0\rangle + e^{i\phi} \sin \frac{\theta}{2} |\iota_1\rangle = \\ &\frac{c_1}{\sqrt{2}} (|0011\rangle + |1100\rangle) + \\ &\frac{c_2}{\sqrt{2}} (|0101\rangle + |1010\rangle) + \\ &\frac{c_3}{\sqrt{2}} (|0110\rangle + |1001\rangle) \end{aligned}$$

# Single node on qubits

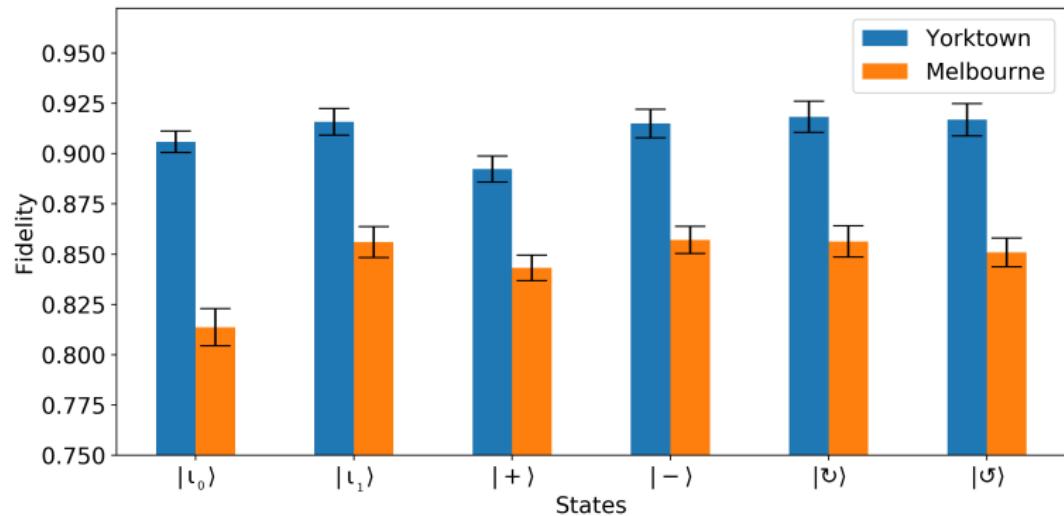
$$|\mathcal{I}\rangle = \frac{c_1}{\sqrt{2}}(|0011\rangle + |1100\rangle) + \frac{c_2}{\sqrt{2}}(|0101\rangle + |1010\rangle) + \frac{c_3}{\sqrt{2}}(|0110\rangle + |1001\rangle)$$



$$U = \begin{pmatrix} c_1 & \sqrt{|c_2|^2 + |c_3|^2} \\ -\sqrt{|c_2|^2 + |c_3|^2} & c_1^* \end{pmatrix}$$

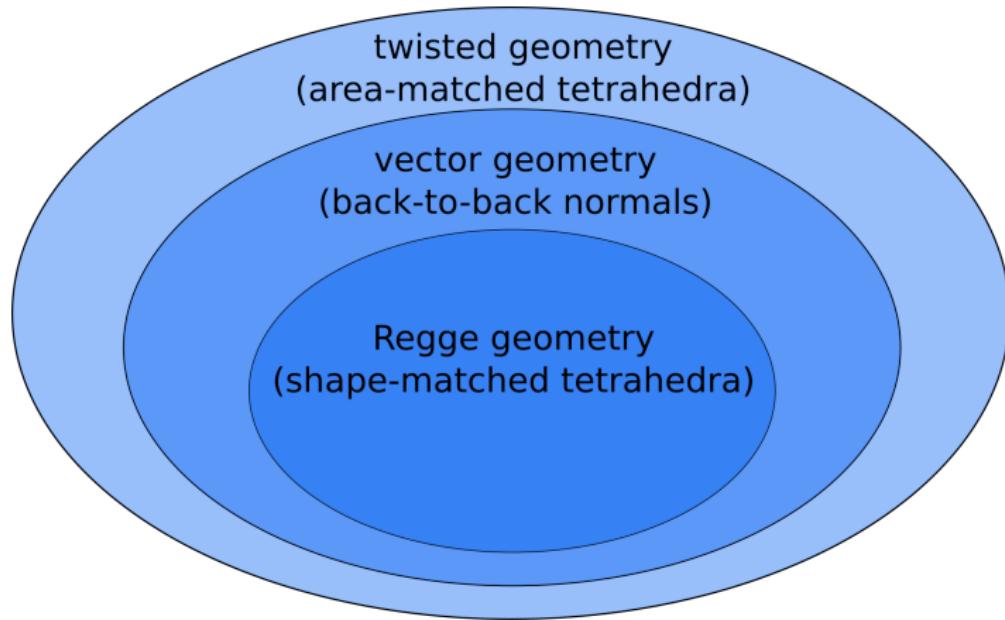
$$V = \begin{pmatrix} -\frac{c_2}{\sqrt{|c_2|^2 + |c_3|^2}} & \frac{c_3^*}{\sqrt{|c_2|^2 + |c_3|^2}} \\ -\frac{c_3}{\sqrt{|c_2|^2 + |c_3|^2}} & -\frac{c_2^*}{\sqrt{|c_2|^2 + |c_3|^2}} \end{pmatrix}$$

# Simulation on quantum processor



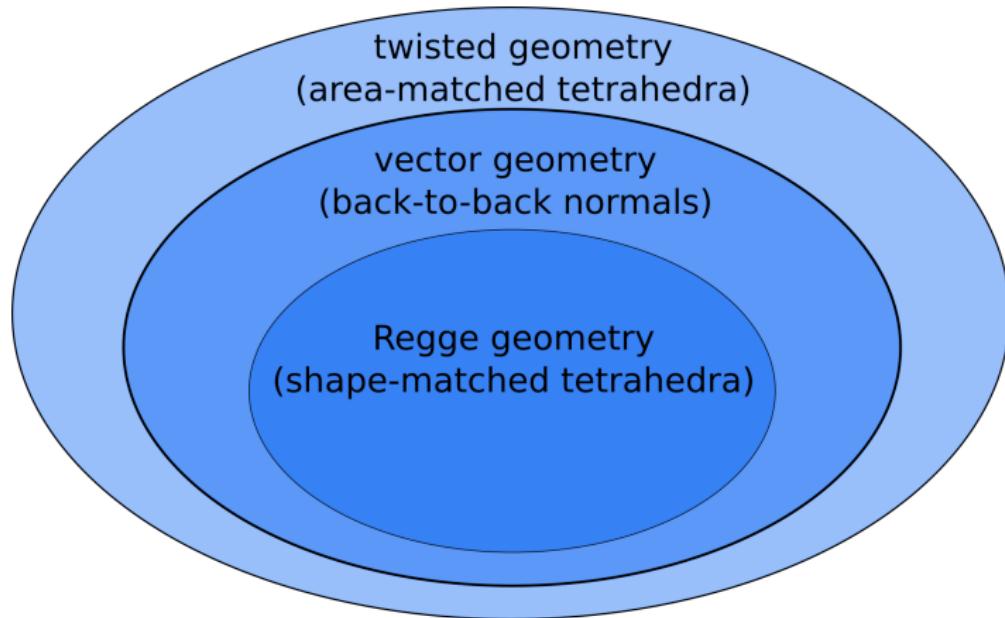
G. Cz, Jakub Mielczarek, "Quantum simulations of a qubit of space"  
Phys. Rev. D 103, 046001 (2021)

# Gluing tetrahedrons



Spin-network basis states are un-entangled  $|\Gamma, j_l, \mathcal{I}_n\rangle = \bigotimes_n \mathcal{I}_n$

# Gluing tetrahedrons



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# Gluing tetrahedrons

squeezed states

$$|\mathcal{B}, \lambda_I\rangle = \left(1 - |\lambda_I|^2\right) \sum_j \sqrt{2j+1} \lambda_I^{2j} |\mathcal{B}, j\rangle \quad (1)$$

maximally entangled state of spin  $j$

$$|\mathcal{B}, j\rangle = \frac{1}{\sqrt{2j+1}} \sum_{m=-j}^j (-1)^{j-m} |j, m\rangle_s |j, -m\rangle_t$$

# Gluing tetrahedrons

squeezed states

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$$|\mathcal{B}, j\rangle = \frac{1}{\sqrt{2j+1}} \sum_{m=-j}^j (-1)^{j-m} |j, m\rangle_s |j, -m\rangle_t$$

projection on spin-network basis states

$$P_\Gamma = \sum_{j_I, \mathcal{I}_n} |\Gamma, j_I, \mathcal{I}_n\rangle \langle \Gamma, j_I, \mathcal{I}_n|$$

$$|\Gamma, \mathcal{B}, \lambda_I\rangle = P_\Gamma \bigotimes_I |\mathcal{B}, \lambda_I\rangle$$

# Gluing tetrahedrons

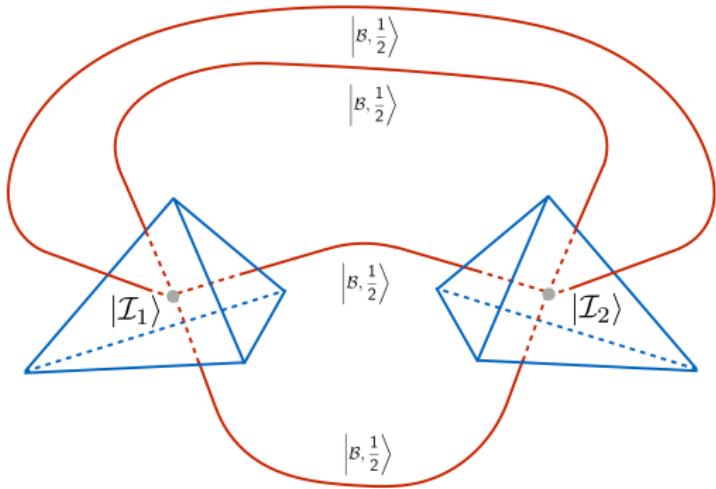
for  $j = \frac{1}{2}$ :

$$|\mathcal{B}, j\rangle = \frac{1}{\sqrt{2}} \left( \left| \frac{1}{2}, \frac{1}{2} \right\rangle \left| \frac{1}{2}, -\frac{1}{2} \right\rangle - \left| \frac{1}{2}, -\frac{1}{2} \right\rangle \left| \frac{1}{2}, \frac{1}{2} \right\rangle \right)$$

in qubit notations

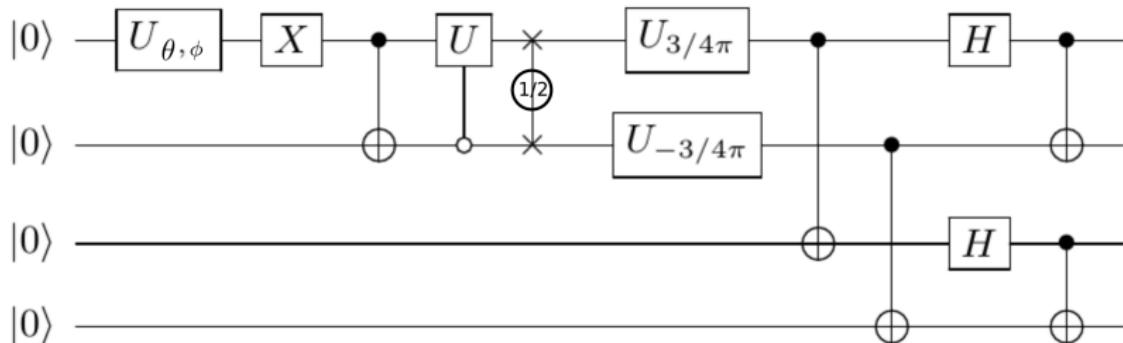
$$\left| \mathcal{B}, \frac{1}{2} \right\rangle = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle)$$

# Dipole



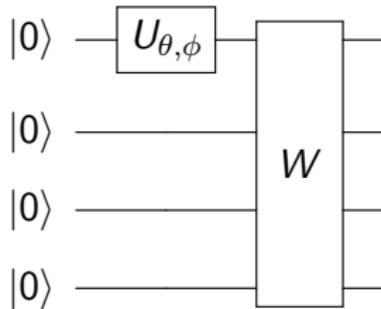
$$\begin{aligned}\left| \Gamma_2, \mathcal{B}, \frac{1}{2} \right\rangle &= P_\Gamma \bigotimes_I \left| \mathcal{B}, \frac{1}{2} \right\rangle = \sum_{k,l} \iota_{(k)}^{m_1 m_2 m_3 m_4} \iota_{(l)} m_1 m_2 m_3 m_4 \left| \iota_k \iota_l \right\rangle \\ &= \frac{1}{\sqrt{2}} (\left| \iota_0 \iota_0 \right\rangle + \left| \iota_1 \iota_1 \right\rangle)\end{aligned}$$

# New circuit for node



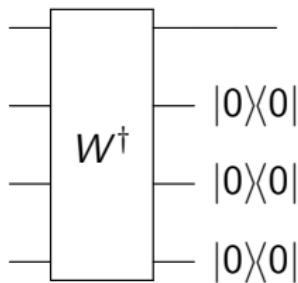
$$|\mathcal{I}\rangle = \cos \frac{\theta}{2} |\psi_0\rangle + e^{i\phi} \sin \frac{\theta}{2} |\psi_1\rangle = \frac{c_1}{\sqrt{2}} (|0011\rangle + |1100\rangle) + \frac{c_2}{\sqrt{2}} (|0101\rangle + |1010\rangle) + \frac{c_3}{\sqrt{2}} (|0110\rangle + |1001\rangle)$$

# New circuit for node



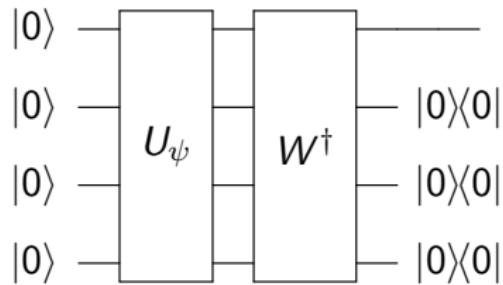
$$|\mathcal{I}\rangle = \cos \frac{\theta}{2} |\iota_0\rangle + e^{i\phi} \sin \frac{\theta}{2} |\iota_1\rangle = \frac{c_1}{\sqrt{2}} (|0011\rangle + |1100\rangle) + \frac{c_2}{\sqrt{2}} (|0101\rangle + |1010\rangle) + \frac{c_3}{\sqrt{2}} (|0110\rangle + |1001\rangle)$$

# Projector



Projection operator on intertwiner subspace, expressed in one-qubit representation.

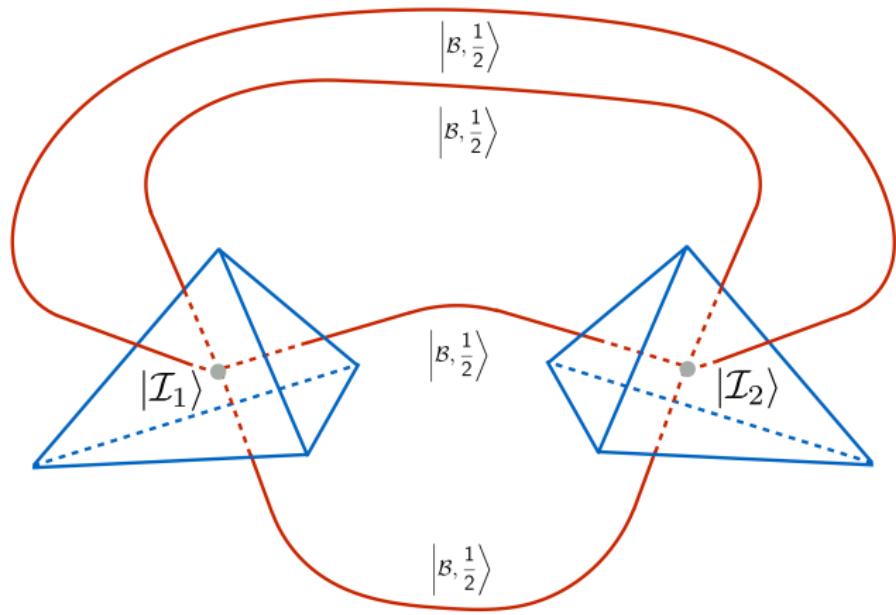
# Projector



Projection of state  $|\psi\rangle$  on intertwiner subspace, expressed in one-qubit representation.

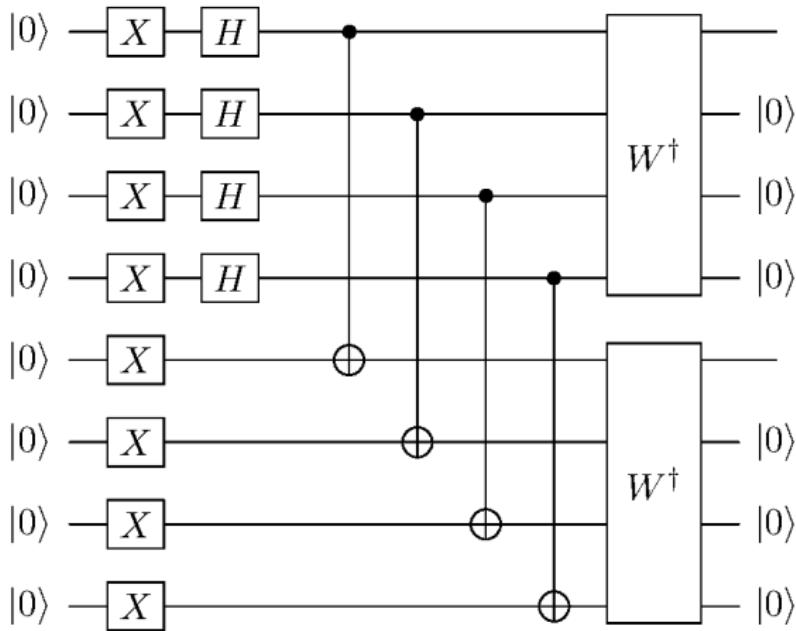
$$\left( \sum_k |\iota_k\rangle \langle \iota_k| \right) |\psi\rangle$$

# Dipole



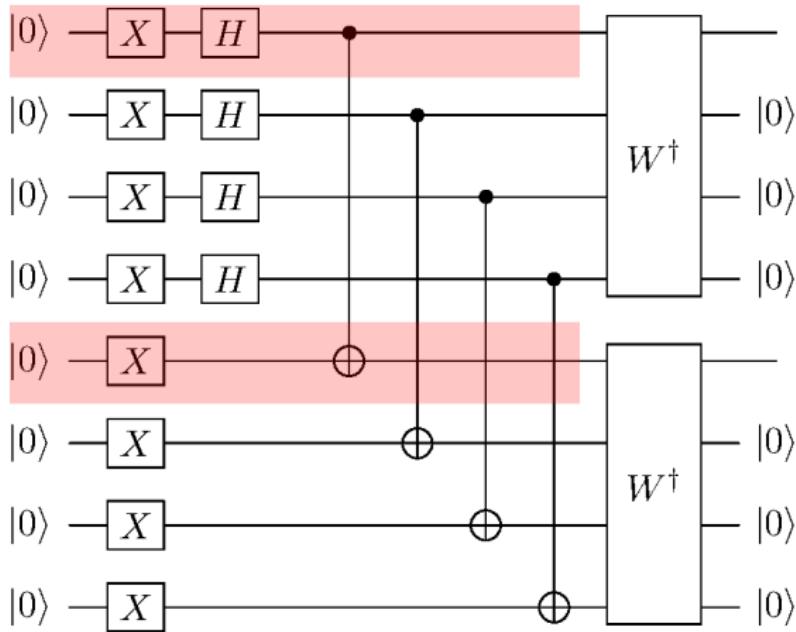
$$\left| \Gamma_2, \mathcal{B}, \frac{1}{2} \right\rangle = \frac{1}{\sqrt{2}} (|\iota_0 \iota_0 \rangle + |\iota_1 \iota_1 \rangle)$$

# Dipole



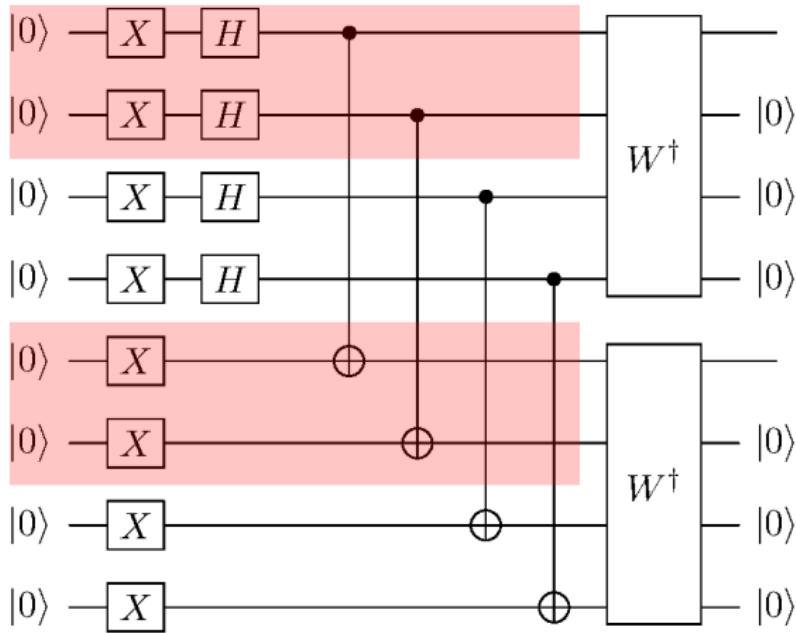
$$\left| \Gamma_2, \mathcal{B}, \frac{1}{2} \right\rangle = \frac{1}{\sqrt{2}} (|\iota_0 \iota_0 \rangle + |\iota_1 \iota_1 \rangle)$$

# Dipole



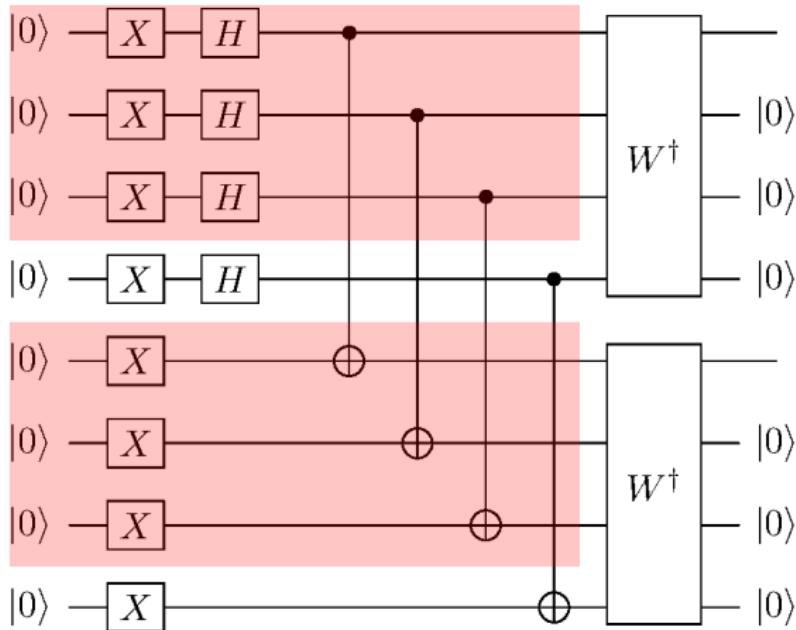
$$\left| \Gamma_2, \mathcal{B}, \frac{1}{2} \right\rangle = \frac{1}{\sqrt{2}} (|\iota_0 \iota_0 \rangle + |\iota_1 \iota_1 \rangle)$$

# Dipole



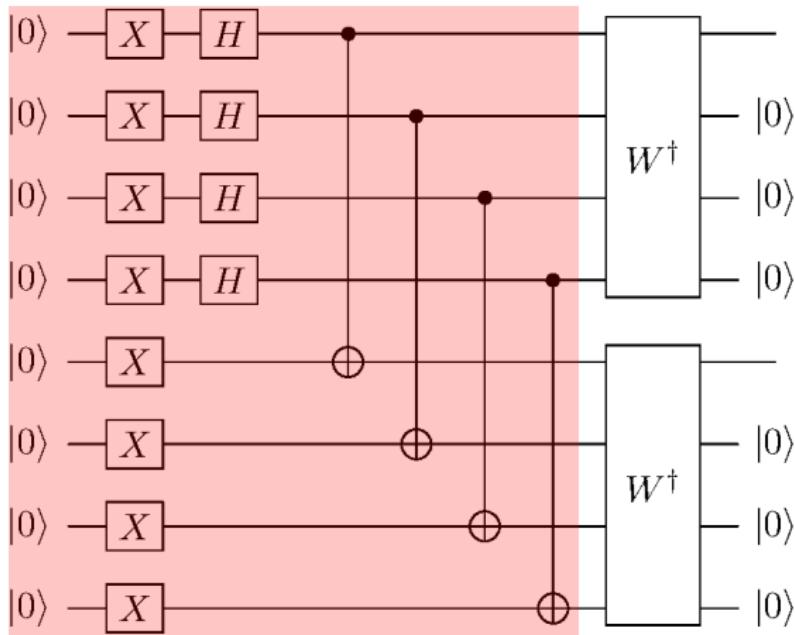
$$\left| \Gamma_2, \mathcal{B}, \frac{1}{2} \right\rangle = \frac{1}{\sqrt{2}} (|\iota_0 \iota_0 \rangle + |\iota_1 \iota_1 \rangle)$$

# Dipole



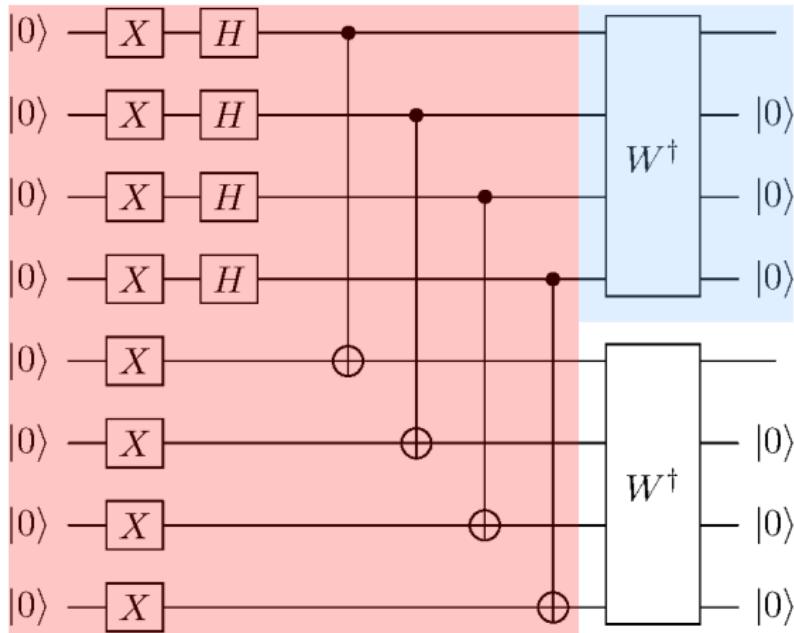
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# Dipole



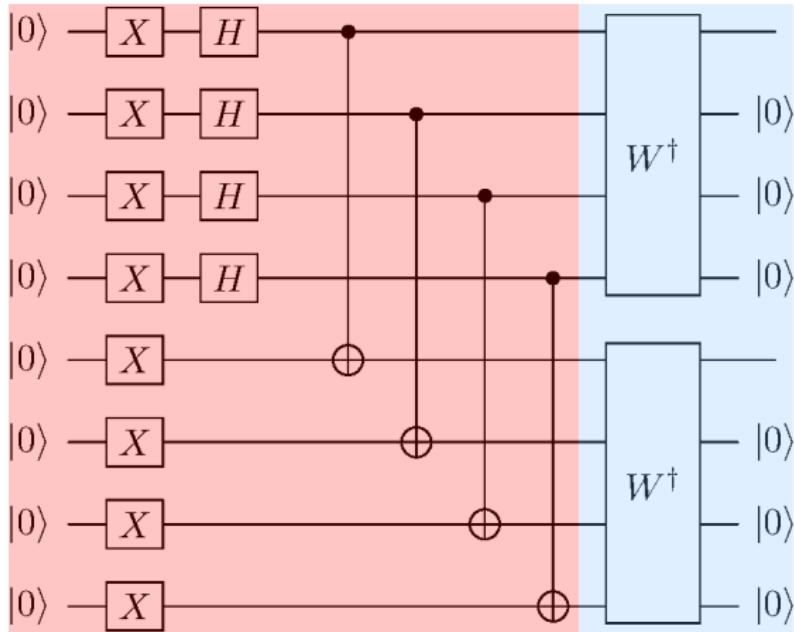
$$\left| \Gamma_2, \mathcal{B}, \frac{1}{2} \right\rangle = \frac{1}{\sqrt{2}} (|\iota_0 \iota_0 \rangle + |\iota_1 \iota_1 \rangle)$$

# Dipole



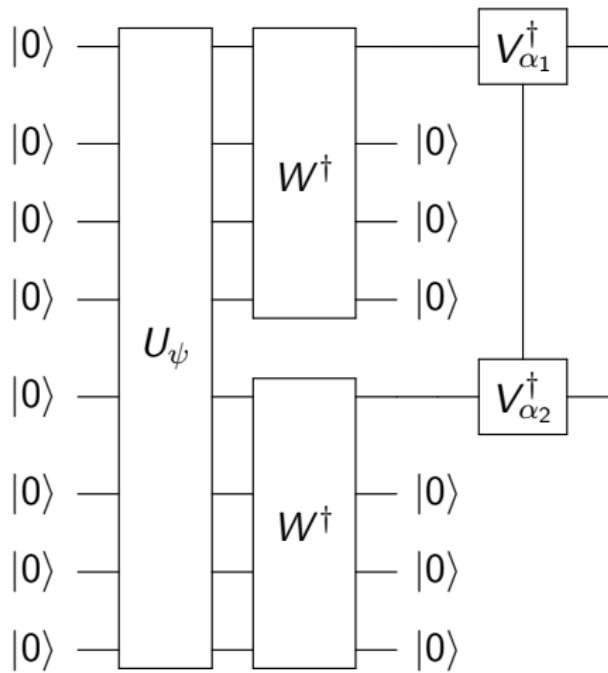
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# Dipole



$$\left| \Gamma_2, \mathcal{B}, \frac{1}{2} \right\rangle = \frac{1}{\sqrt{2}} (|\iota_0 \iota_0 \rangle + |\iota_1 \iota_1 \rangle)$$

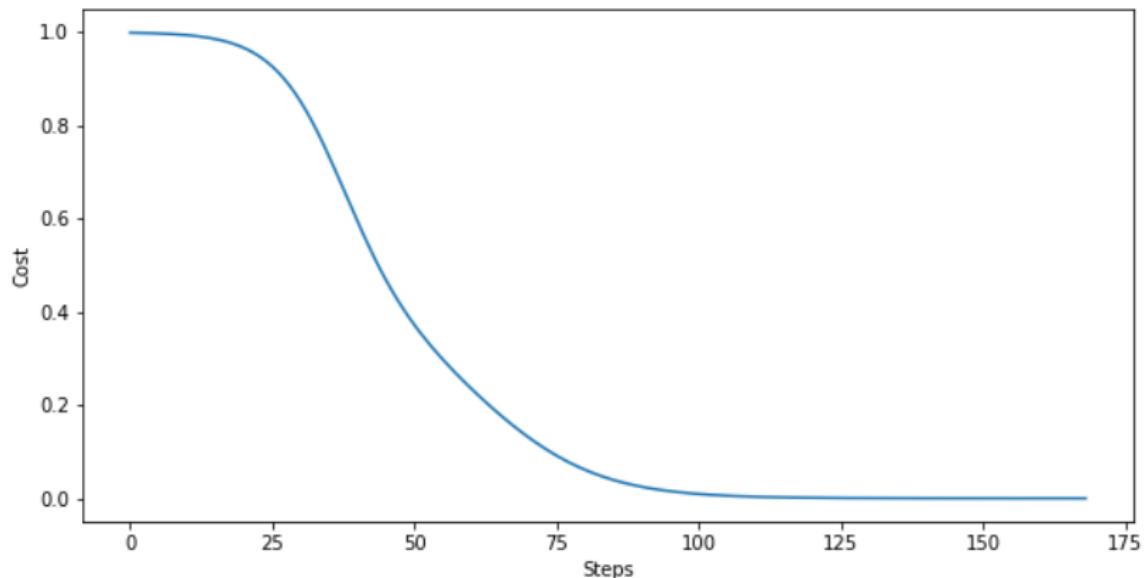
# Dipole



# Dipole

## Variational algorithm

$$cost(\alpha) = 1 - Prob(q_0q_4 = 00)$$

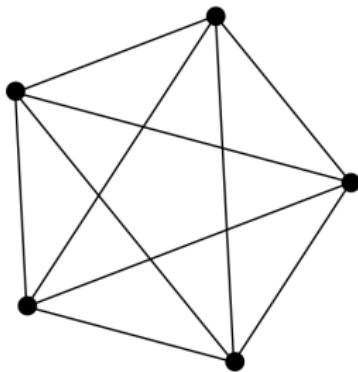


# Pentagram

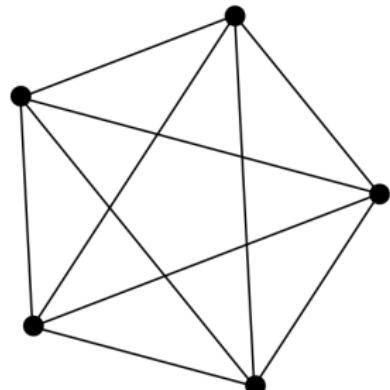
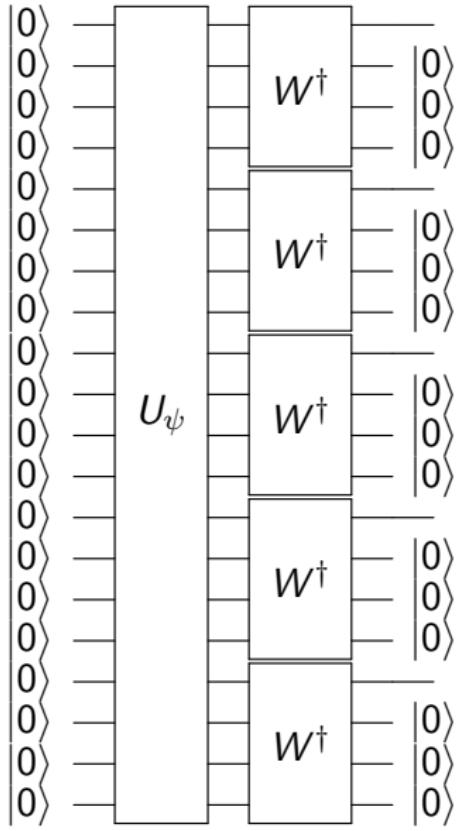
$$P |\psi\rangle = \sum_{\iota_{k_i}} \overline{\{15j\}} |\iota_{k_1} \iota_{k_2} \iota_{k_3} \iota_{k_4} \iota_{k_5}\rangle$$

where

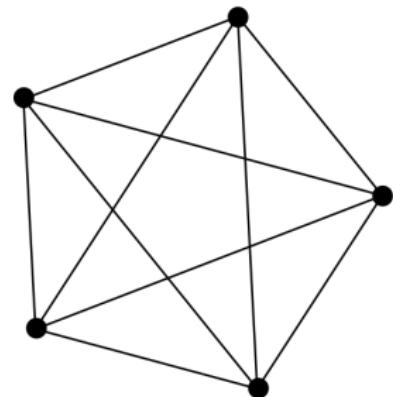
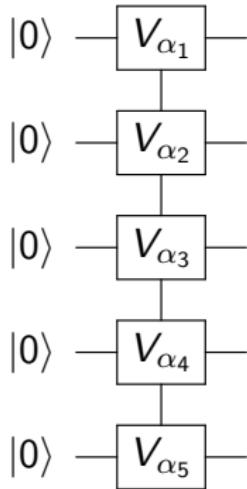
$$\begin{aligned} \{15j\} = & \iota_1^{m_{12}m_{13}m_{14}m_{15}} \iota_{2;m_{12}}^{m_{13}m_{14}m_{15}} \iota_{3;m_{12}m_{13}}^{m_{14}m_{15}} \\ & \iota_{4;m_{12}m_{13}m_{14}}^{m_{15}} \iota_{5;m_{12}m_{13}m_{14}m_{15}} \end{aligned}$$



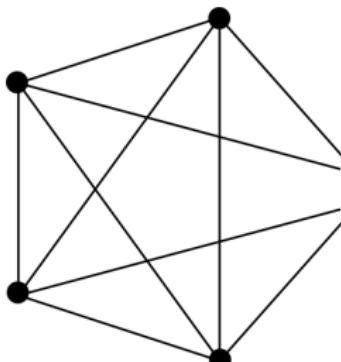
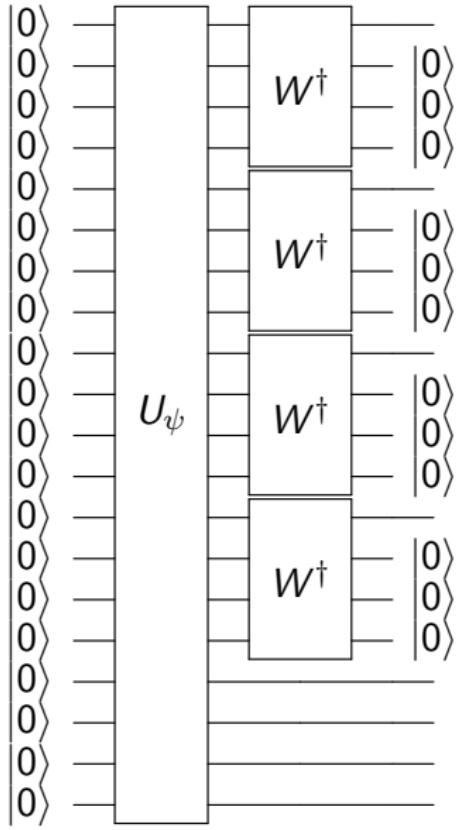
# Pentagram



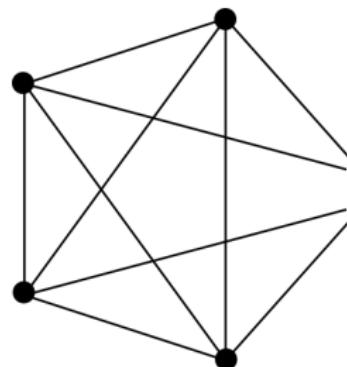
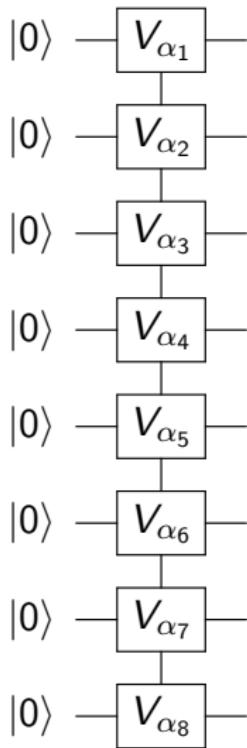
# Pentagram



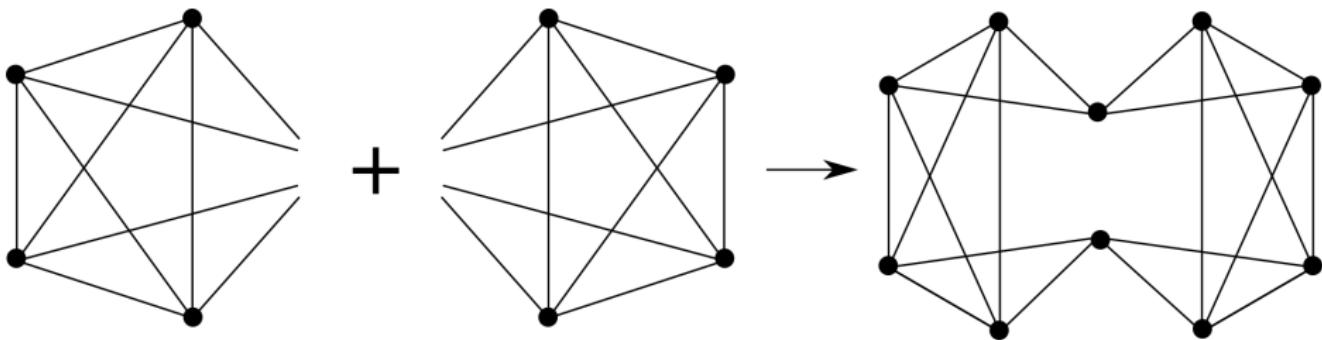
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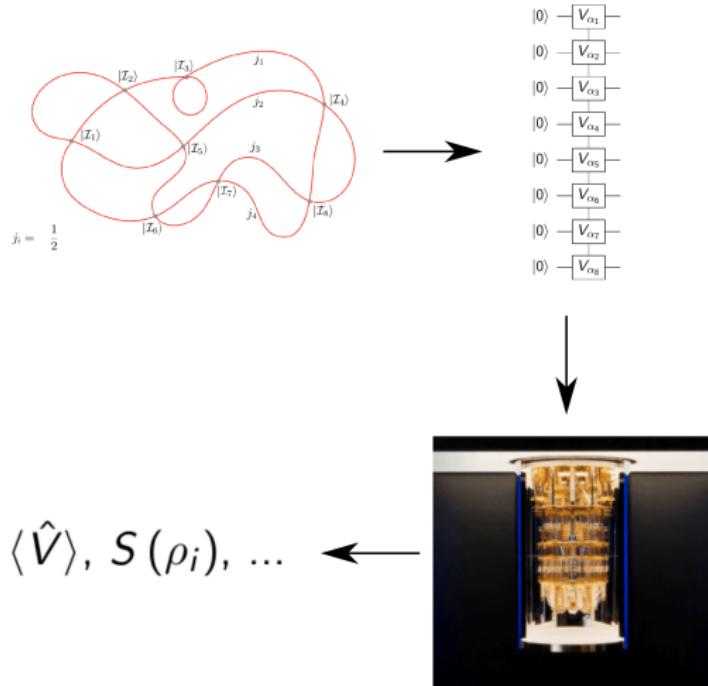


# Decagram



8 qubits + 8 qubits → 10 qubits

# Simulations of spin networks



# Mutual information

$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

$$S(\rho) = -Tr\rho \ln \rho$$

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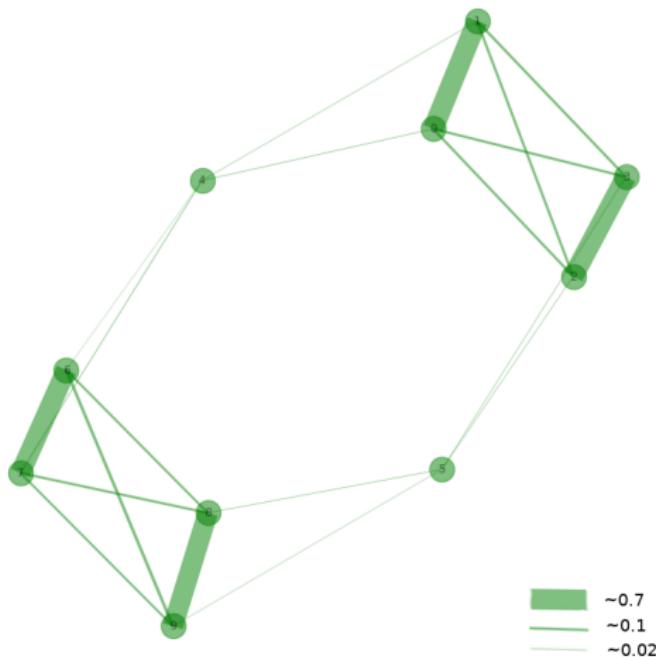
$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

$$S(\rho) = -Tr\rho \ln \rho$$

$$\mathcal{C}(O_A, O_B) = \frac{1}{\|O_A\| \|O_B\|} (\langle O_A O_B \rangle - \langle O_A \rangle \langle O_B \rangle)$$

$$\frac{1}{2}\mathcal{C}(O_A, O_B)^2 \leq I(\rho_{AB})$$

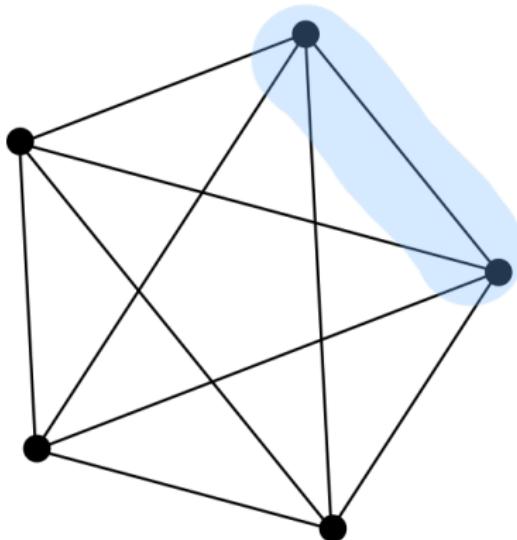
# Mutual information: Decagram



$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

# Page curve - random pentagram

Pairs of spins in random state on the links:



Entropy of subsystem:

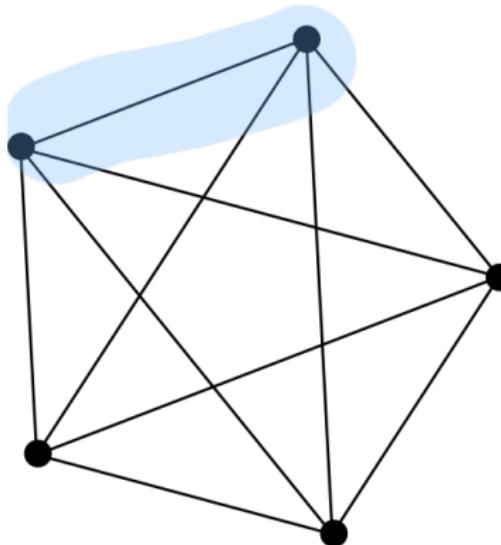
$$N_A = 2$$

$$\rho = Tr_A |\Gamma_5\rangle$$

$$S(\rho) = -Tr \rho \log \rho$$

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Entropy of subsystem:

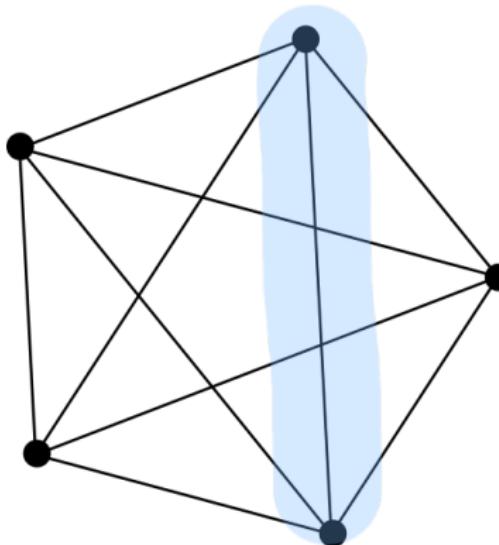
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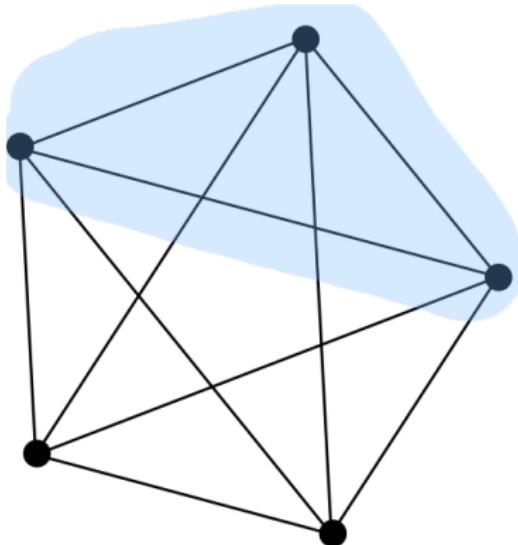
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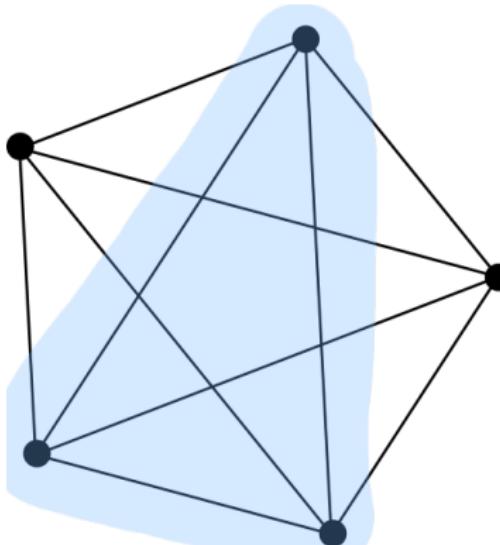
$$N_A = 3$$

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# Page curve - random pentagram

Pairs of spins in random state on the links:



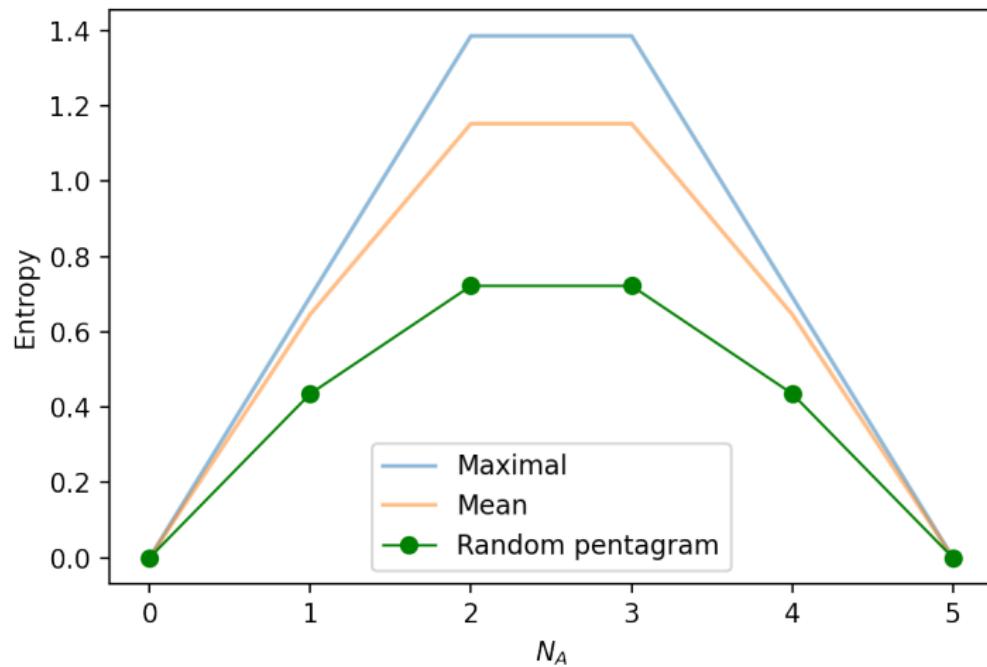
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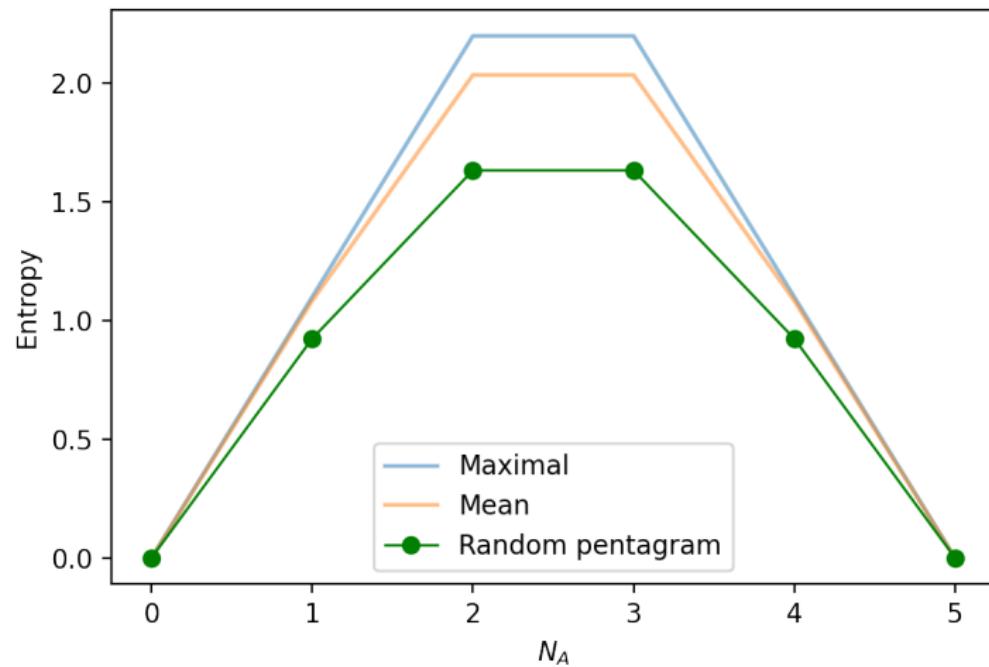
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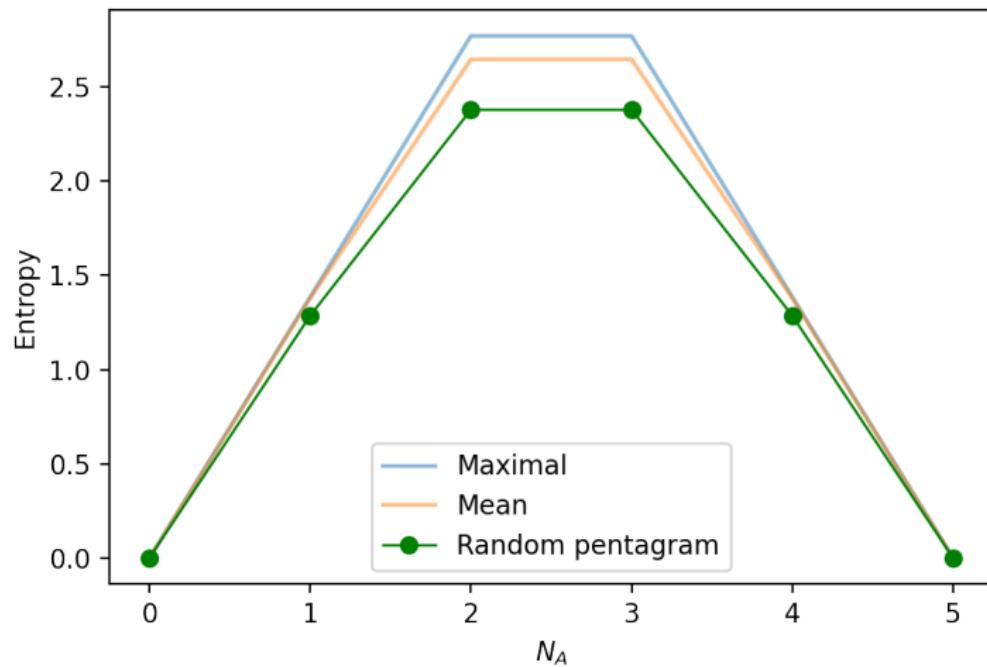
# Random pentagram - spin $\frac{1}{2}$



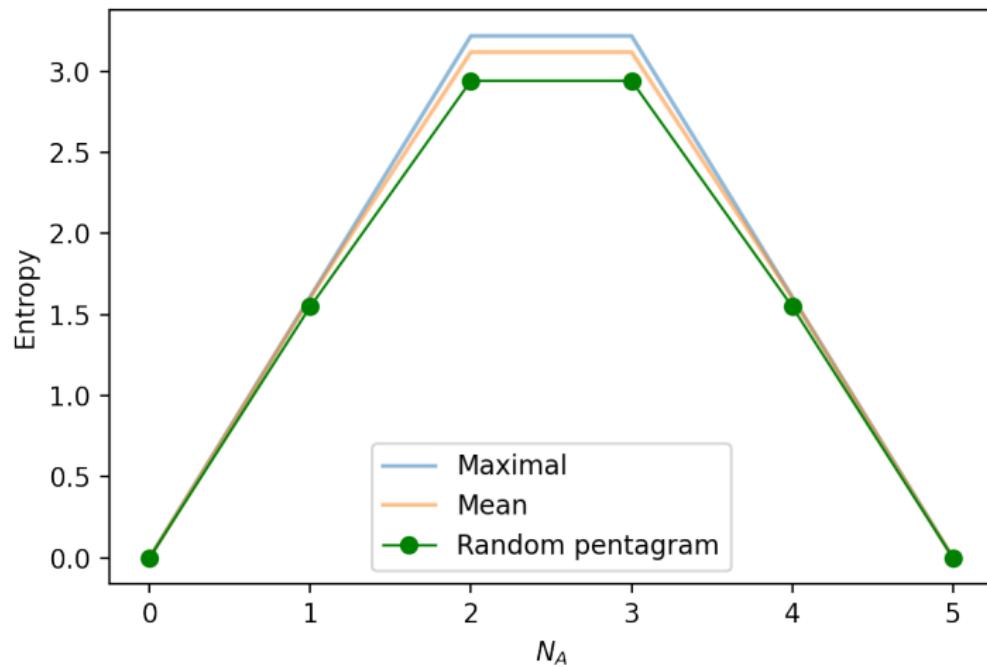
# Random pentagram - spin 1



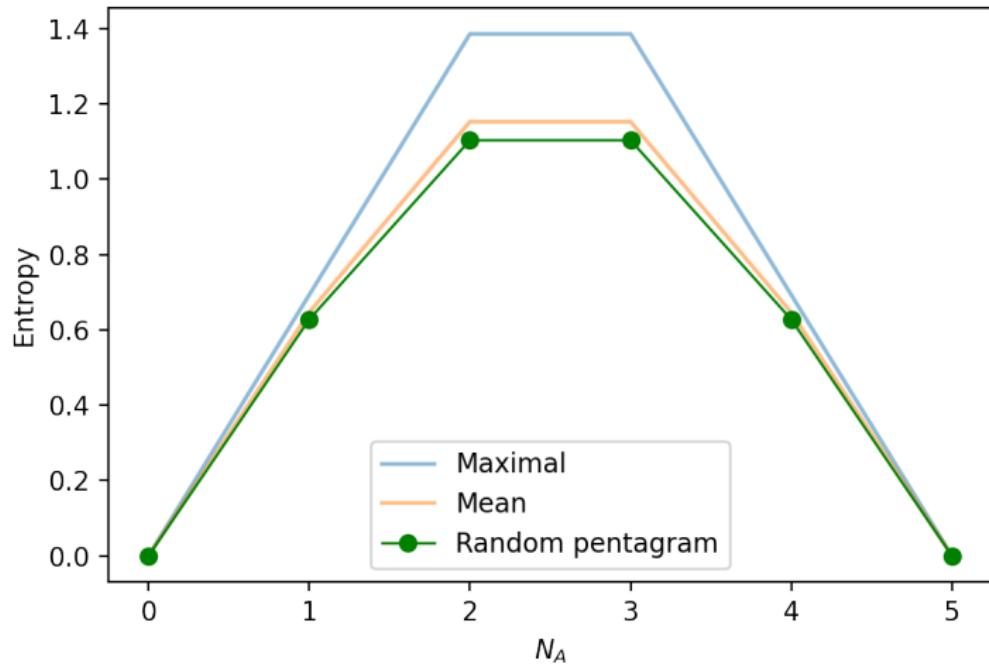
# Random pentagram - spin $\frac{3}{2}$



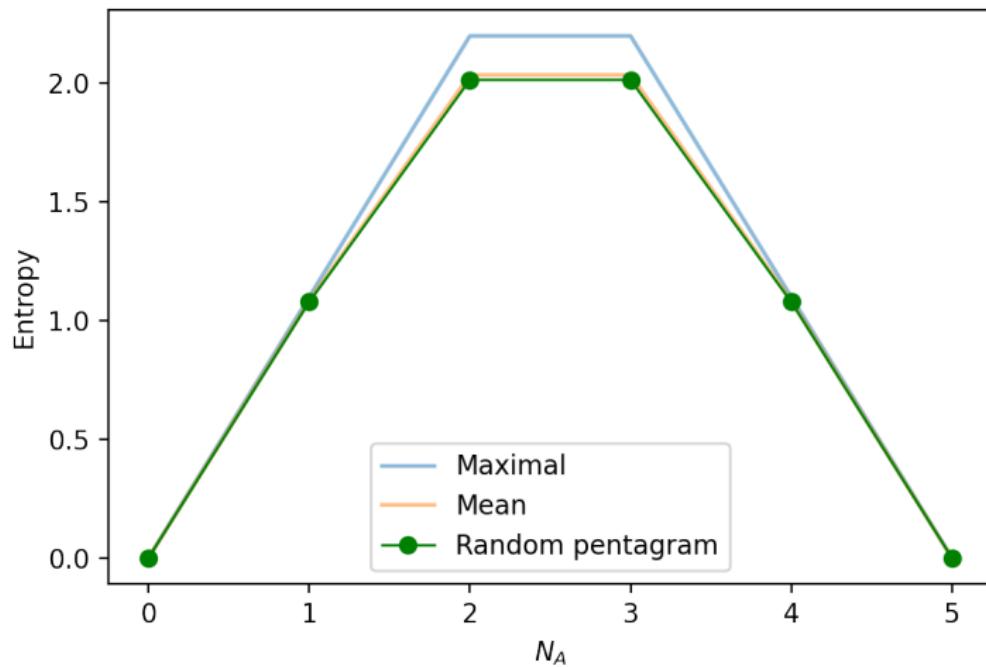
# Random pentagram - spin 2



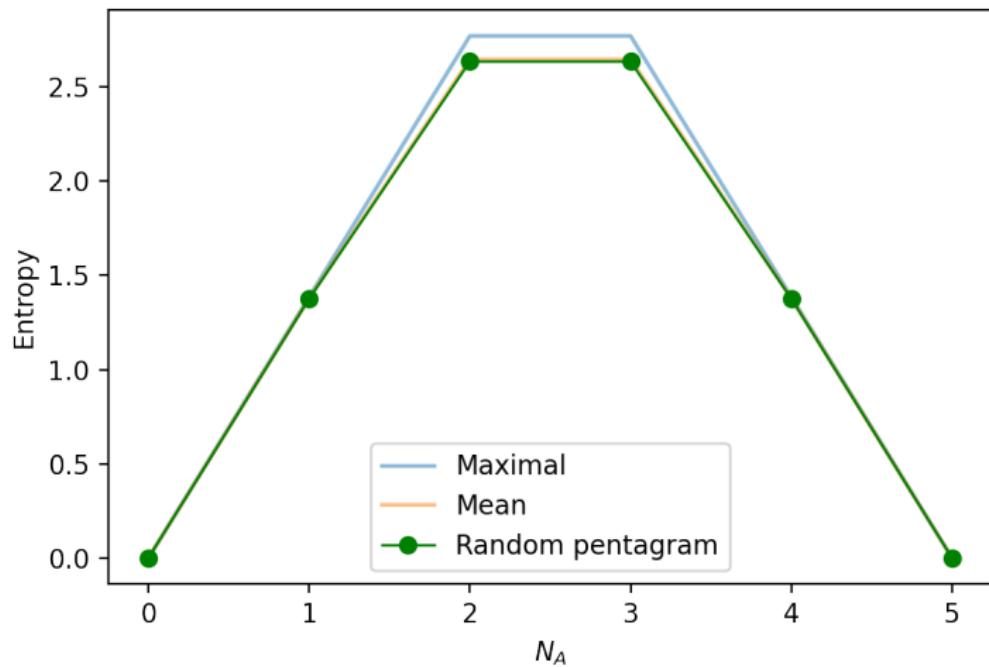
# Random pentagram - maximally entangled links - spin $\frac{1}{2}$



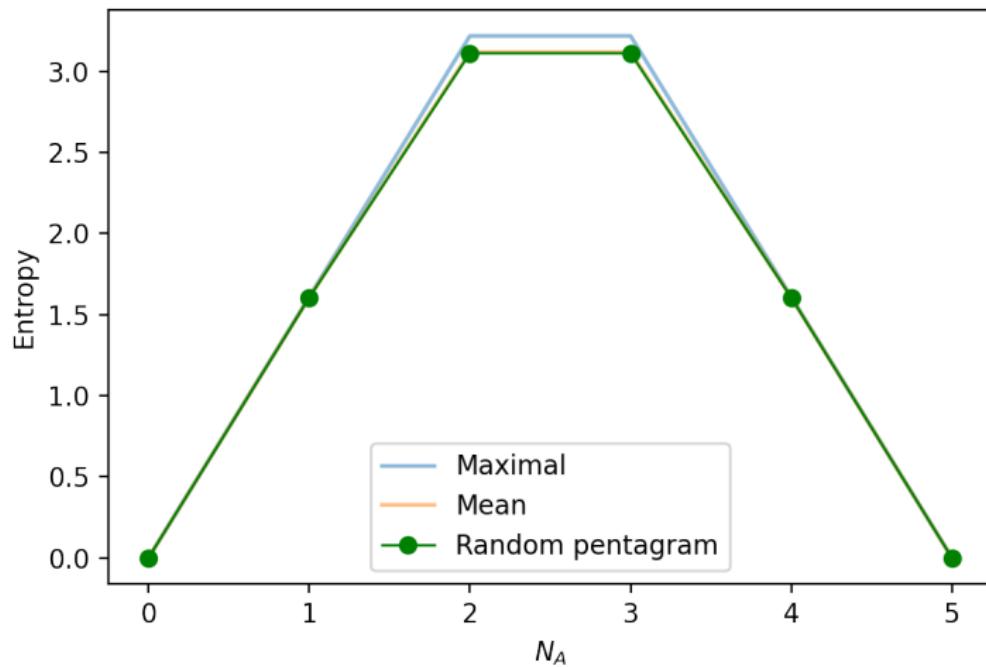
# Random pentagram - maximally entangled links - spin 1



# Random pentagram - maximally entangled links - spin $\frac{3}{2}$



# Random pentagram - maximally entangled links - spin 2



# Summary

- we can prepare a broad class of spin-network states on a quantum register
- measurements can be performed both on simulator and real quantum processor
- a number of required qubits is not much greater than the number of nodes
- entropies of subsystems and Page curve can be extracted
- for low spins, typical entropy of a subsystem in random pentagram is smaller than in random 5-qubit quantum system
- entropy of subsystems in pentagram grows up to typical entropy of a 5-qubit quantum system if maximally entangled links are used

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Thank you for your attention