

Diffraction in hadronic collisions

With focus on the ep/eA at EIC

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Eberly College of Science

Outline

Lecture 1: introduction, diffraction in QCD, HERA measurements

Lecture 2: prospects at EIC: proton tagging capabilities, reduced cross section and DPDFs, longitudinal structure function, elastic vector meson production

Focus will be on ep/eA. Not pp.

EIC White paper 1212.1701

EIC Yellow Report, 2103.05419

Armesto, Newman, Słomiński, Staśto 1901.09076, 2112.06839

Frankfurt, Guzey, Staśto, Strikman 2203.12289

Diffraction in optics



PHYSICO-MATHESIS DE LUMINE, COLORIBVS. ET IRIDE.

ALIIISQVE ADNEXIS

LIBRI DVO,

In quorum Primo afferuntur Noua Experimenta, & Rationes
ab ijs deductæ pro Substantialitate Luminis.

In Secundo autem dissoluntur Argumenta in Primo adducta,
& probabiliter sustineri posse docetur Sententia,
Peripatetica de Accidentalitate Luminis.

QVA OCCASIONE

De hâtenus incognita Luminis Diffusioni, de Reflexione, Refractione, ac Diffractione Modis & Causis, de Visione, deque Speciebus Intentionalibus Visionis & Auditionis, ac de Substantiali Magnæ essentia omnia corpora penetrante, non pauca sicut digna præferantur,
& specialia etiam argumenta impugnantur Atomsilla.

AUCTORE

P. FRANCISCO MARIA GRIMALDO
SOCIETATIS IESV.

OPVS POSTHVMM.



BONONIÆ. MDCCLXV.

Ex Typographia Hæredis Victoris Benardi. Superincus permissa.
Impugni Hieronymi Ieroni Ediditque Ieronimus.



Francesco Maria Grimaldi

1618-1663

Jesuit priest from Bologna

'Light propagates and diffuses not only directly, refractively and reflectively, but also, somehow, in a fourth manner, that is DIFFRACTIVELY.'

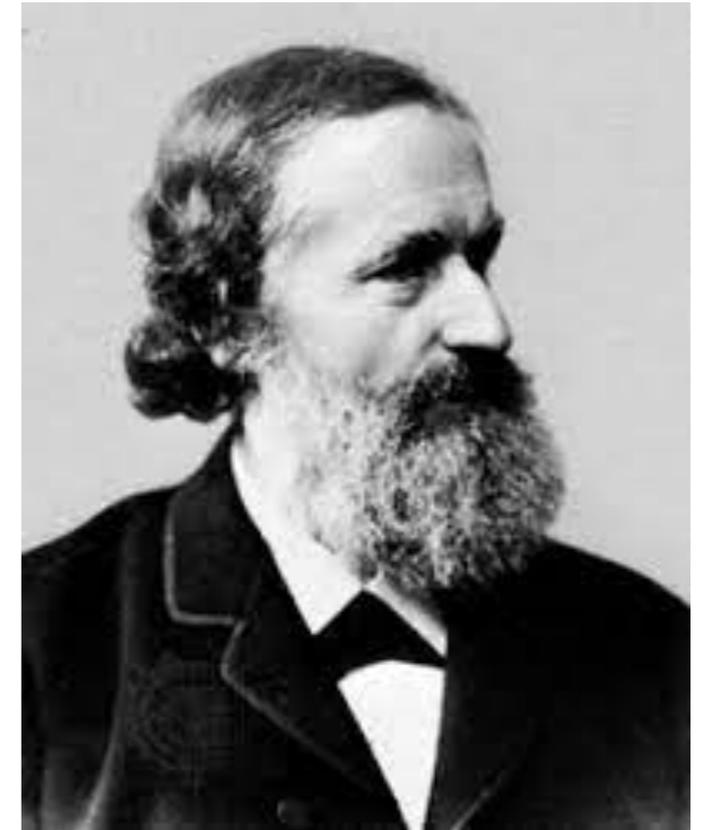
Theory of diffraction



Christiaan Huygens
1629-1695



Augustin Fresnel
1788-1827



Gustav Kirchhoff
1824-1887

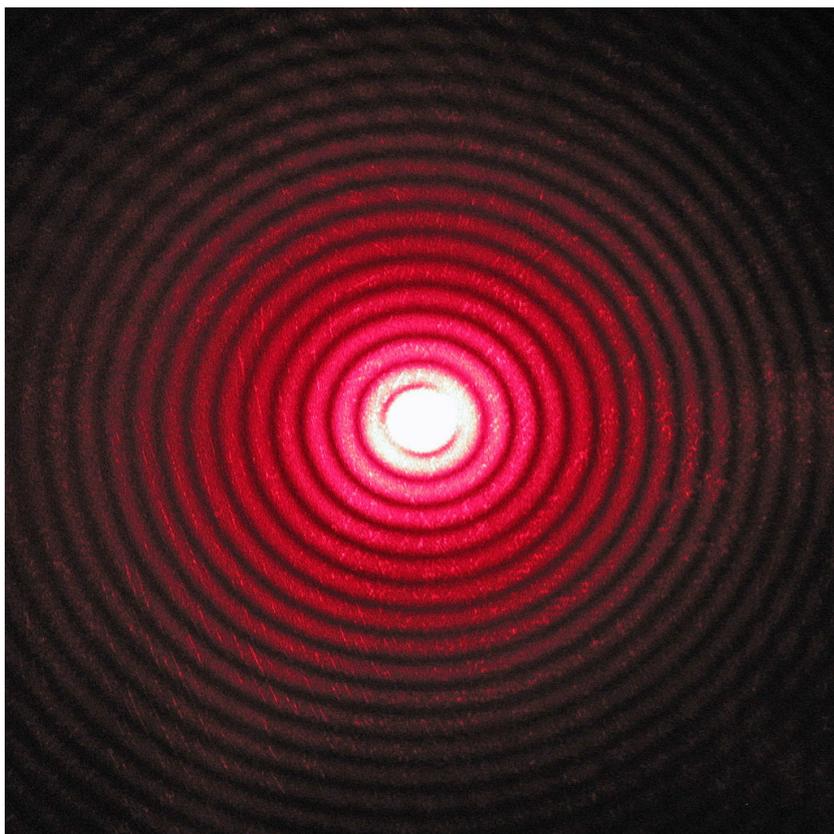
Geometrical optics: applicable in the limit when the wavelength is infinitely small

Diffraction phenomena: deviation from geometrical optics due to finite wavelength

Diffraction

Diffraction : occurs when a wave (for example light) encounters an obstacle or an opening. Most pronounced when the dimensions of obstacle/opening are comparable to wavelength

Laser light passing through a circular aperture



Source: Wikipedia
Author: Wisky

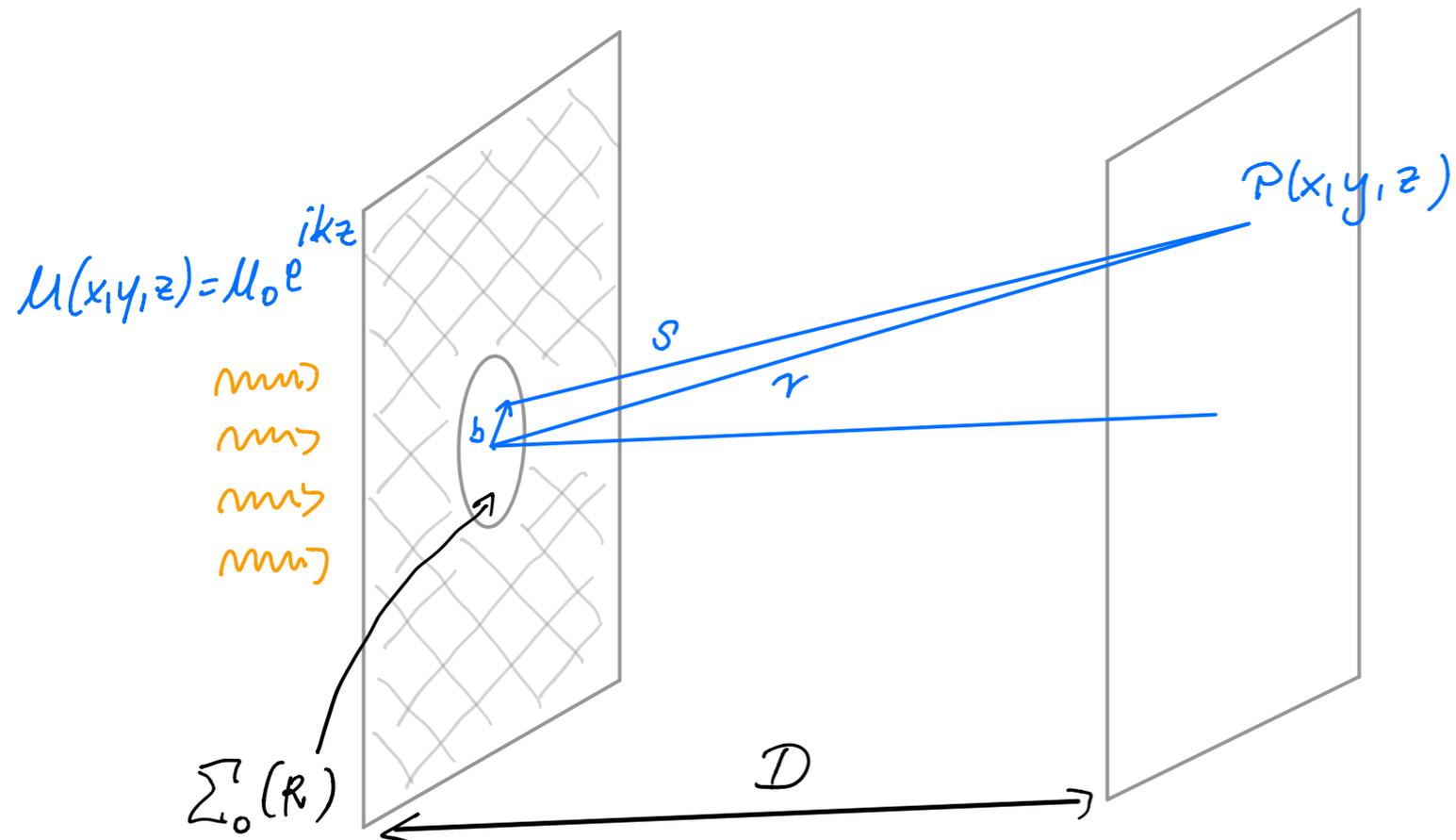
Water waves passing through small entrance



Source: Wikipedia
Author: Verbcatcher

In quantum theory: hadronic and nuclear **diffractive** scattering

Kirchhoff theory



$$P \equiv (x, y, z)$$

$$k = 2\pi / \lambda$$

Wave number

U Amplitude

$$\phi(x, y, z, t) = U(x, y, z)e^{-i\omega t}$$

$$(\nabla^2 + k^2)U = 0$$

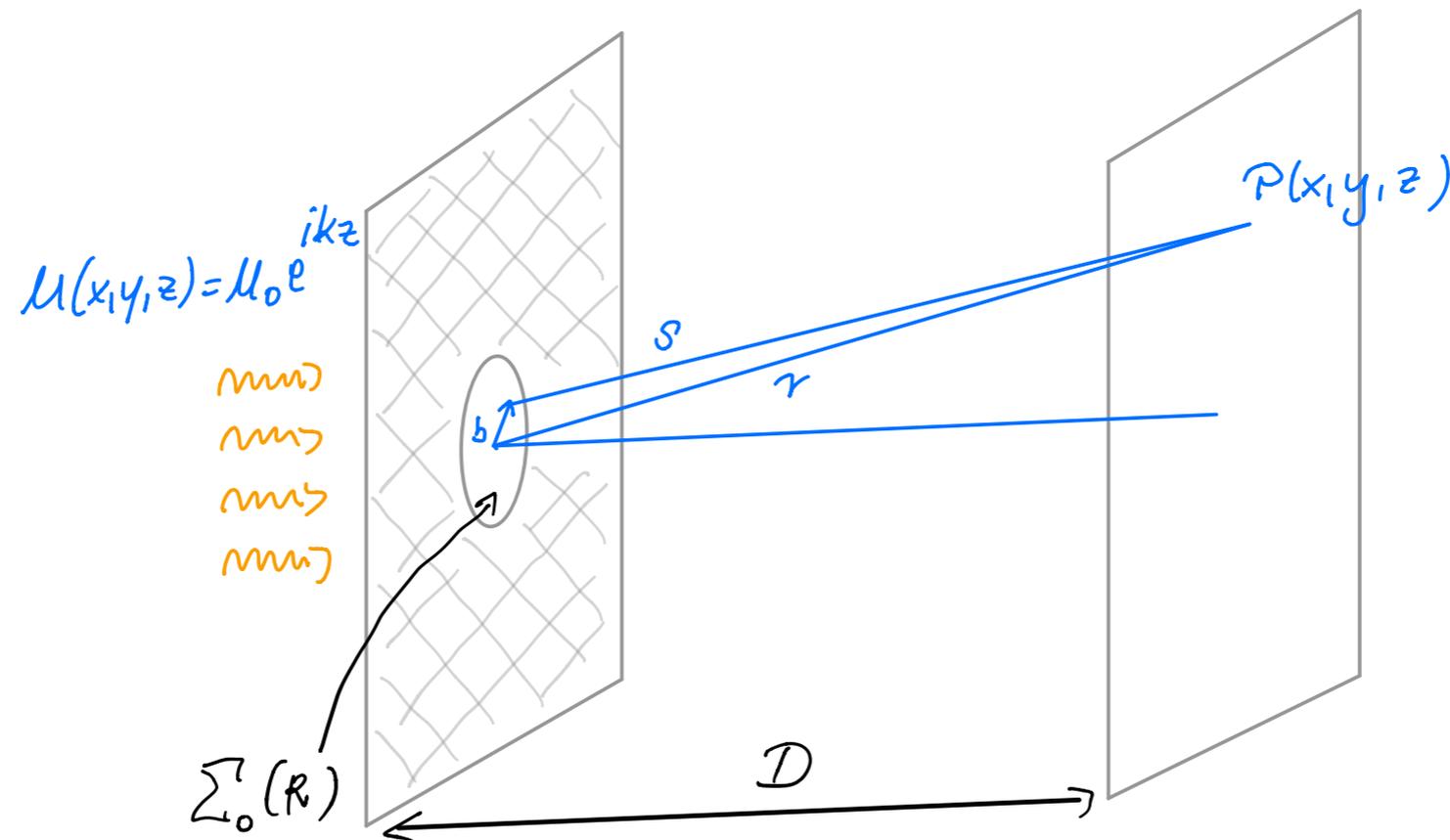
Helmholtz equation

Short wave length limit ($kR > 1$)

$$U(x, y, z) = -\frac{ik}{2\pi} U_0 \int_{\Sigma_0} d^2\mathbf{b} \frac{e^{iks}}{s}$$

Fresnel-Kirchhoff
integral

Kirchhoff theory



Geometrical optics:

$$kR^2 / D \gg 1$$

Fresnel diffraction:

$$kR^2 / D \approx 1$$

Near field

Fraunhofer diffraction:

$$kR^2 / D \ll 1$$

Far field

Relevant for hadronic physics

Fraunhofer diffraction

For the hole in the screen:

$$U(x, y, z) = -\frac{ik}{2\pi} U_0 \frac{e^{ikr}}{r} \int d^2\mathbf{b} \Gamma(\mathbf{b}) e^{-i\mathbf{q}\mathbf{b}}$$

$$\mathbf{q} \approx \mathbf{k}' - \mathbf{k}$$

Momentum transfer (2 dimensional vector)

\mathbf{k} Incoming wave vector

\mathbf{k}' Outgoing wave vector

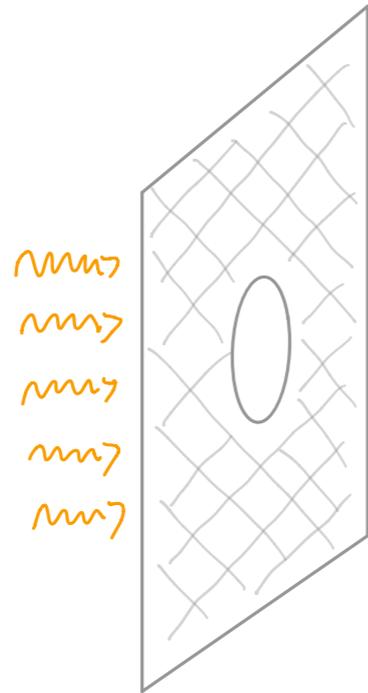
$\Gamma(\mathbf{b})$ Profile function

For the hole:

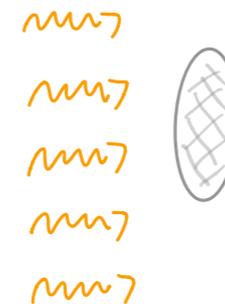
$$\Gamma(\mathbf{b}) = \begin{cases} 1, & \text{on } \Sigma_0 \\ 0, & \text{outside } \Sigma_0 \end{cases}$$

Diffraction off an obstacle

Babinet's principle:



S1 Screen with a hole



S2 Obstacle with the same size and shape as a hole

Waves diffracted by a hole S1 and obstacle S2 must combine to reconstruct the incident wave front.

Diffraction patterns away from incident direction are the same for screen with hole and complementary obstacle.

Diffraction off an obstacle



$$U(x, y, z) = U_{\text{inc}} + U_{\text{scat}}$$

$$U(x, y, z) = U_0 \left(e^{ikz} + f(\mathbf{q}) \frac{e^{ikr}}{r} \right)$$

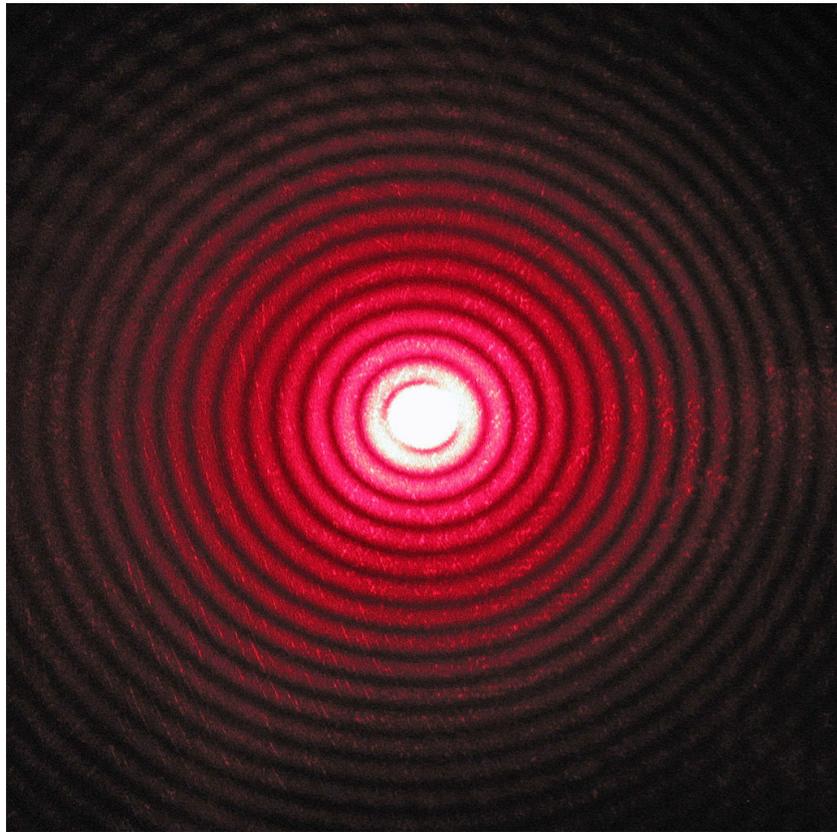
Scattering amplitude

$$f(\mathbf{q}) = \frac{ik}{2\pi} \int d^2\mathbf{b} \Gamma(\mathbf{b}) e^{-i\mathbf{q}\cdot\mathbf{b}}$$

$$\Gamma(\mathbf{b}) = \frac{1}{2\pi ik} \int d^2\mathbf{q} f(\mathbf{q}) e^{i\mathbf{q}\cdot\mathbf{b}}$$

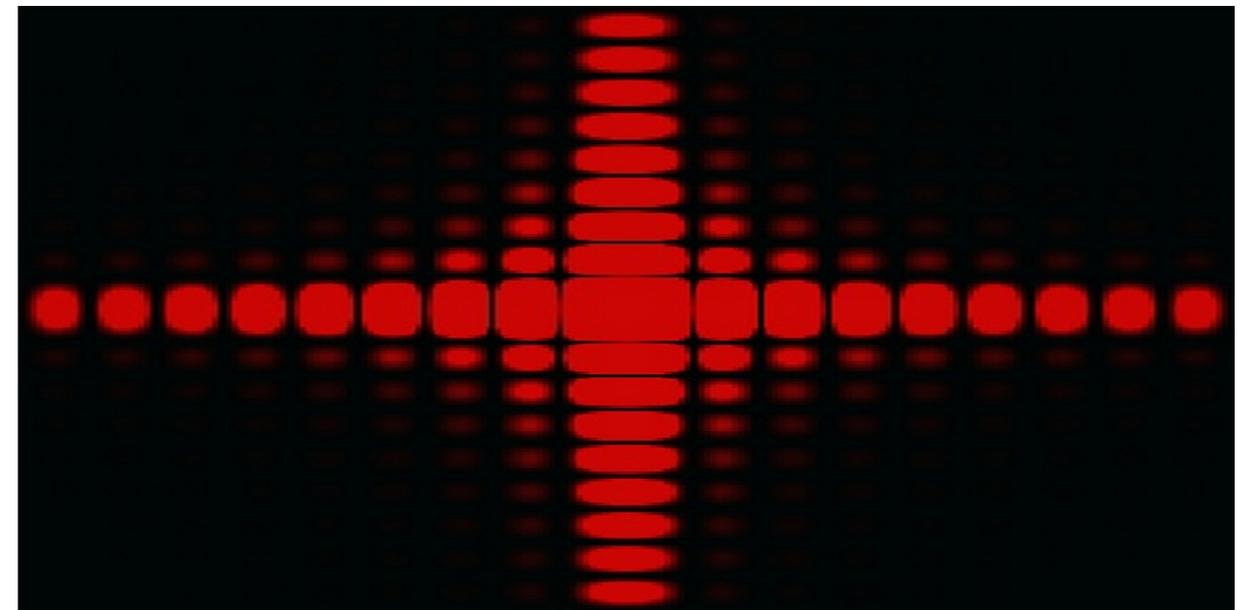
Scattering amplitude is Fourier transform of the profile function.
Profile function is inverse Fourier transform of the scattering amplitude.

Diffraction patterns



Source: Wikipedia
Author: Wisky

Circular aperture



Source: Wikipedia
Author: Epzcaw

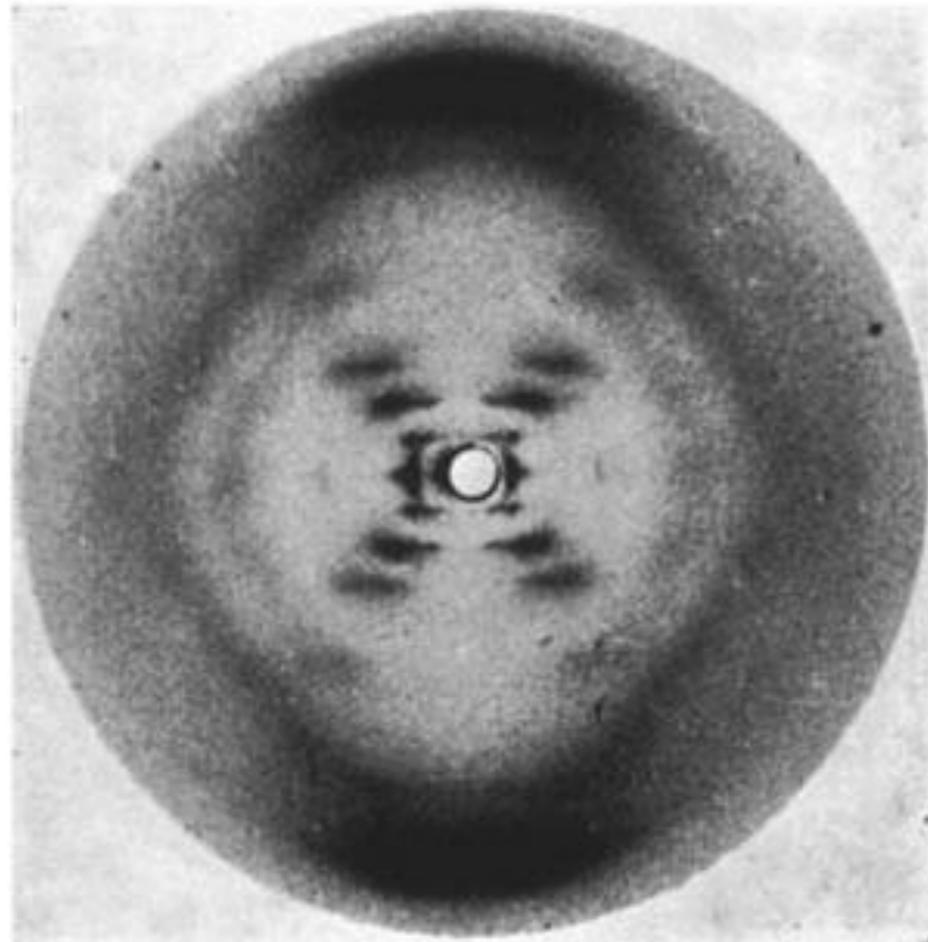
Rectangular aperture

The diffraction pattern (far away from obstacle) is a Fourier transform of the apertured field.

Diffraction pattern

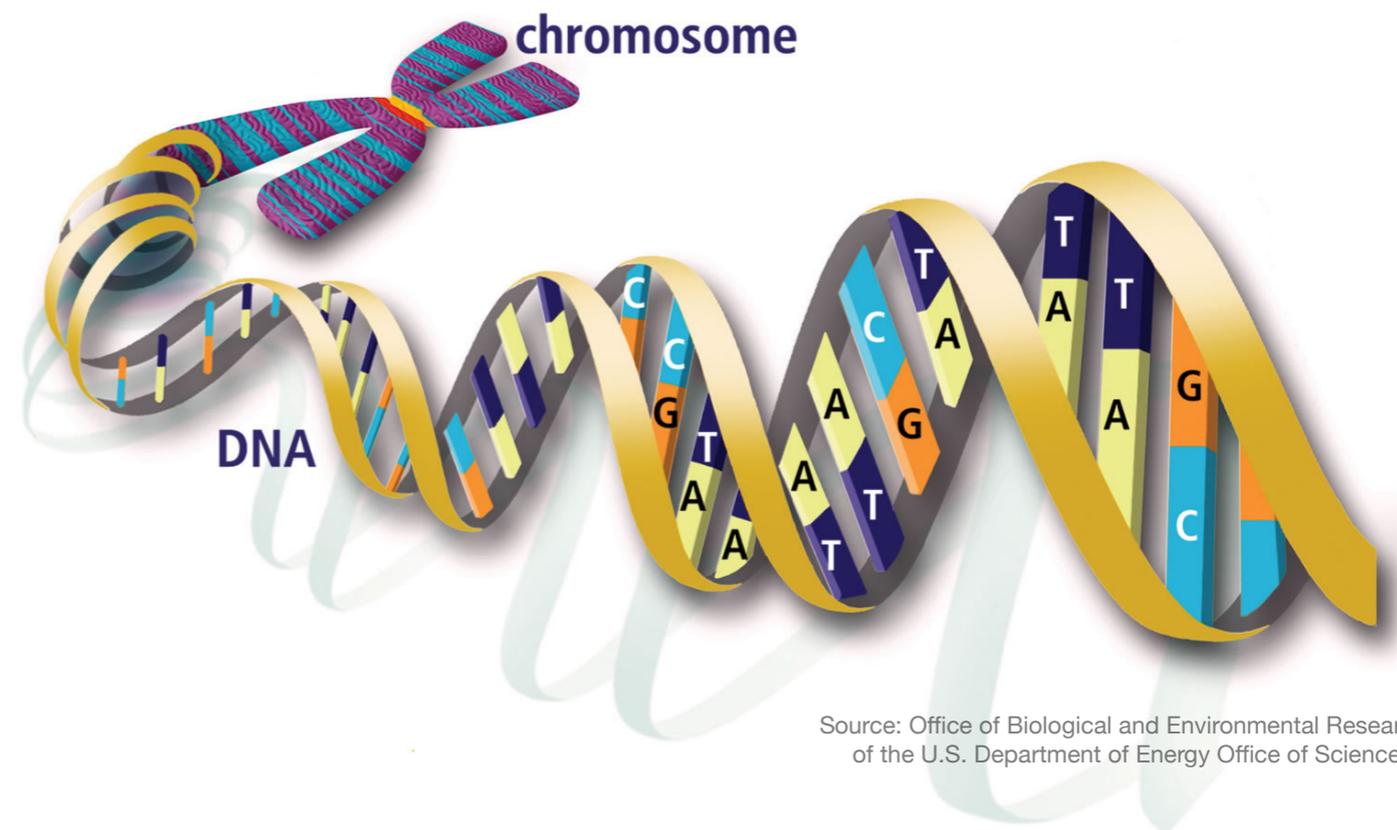
Photo 51

Gosling-Franklin



Source: Wikipedia

Watson-Crick



Diffraction can provide very detailed information about the structure of an object.
The object cannot be destroyed in this process.

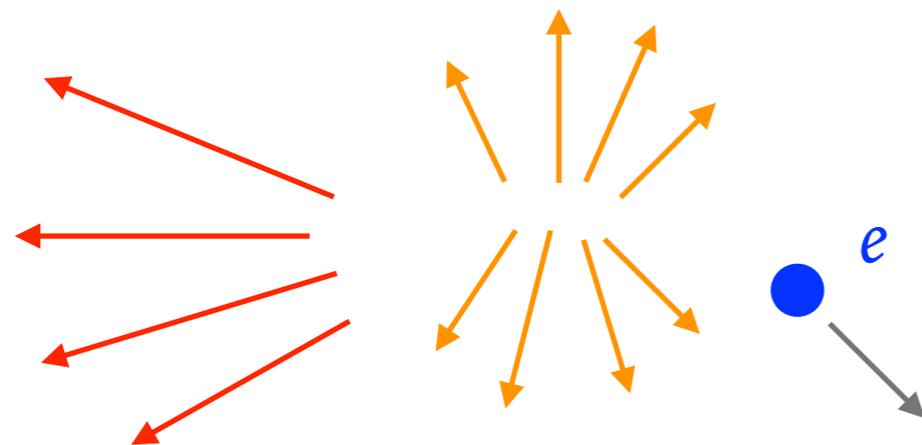
Diffraction in hadronic / nuclear physics

In quantum physics: propagation and interaction of particles as an absorption of the various components of their wavefunction

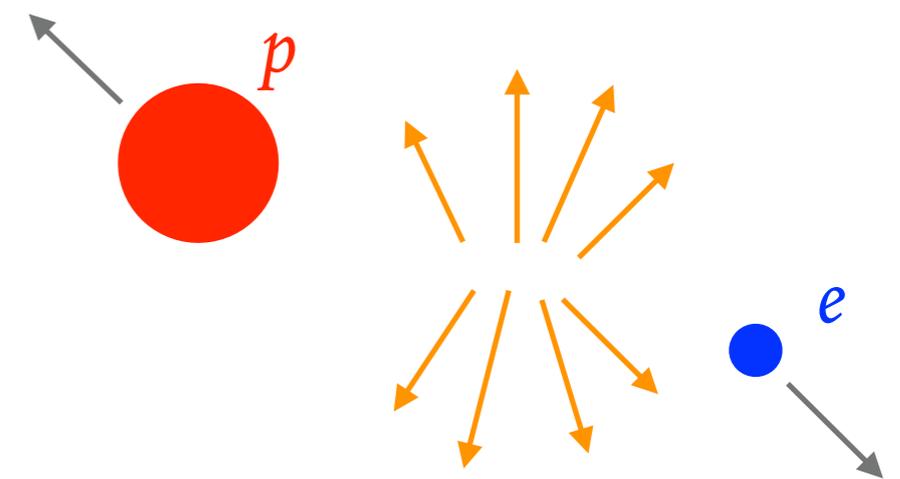
Electron - hadron(nucleus) scattering (like at EIC)



10% processes at HERA were diffractive



Proton is fragmented (inelastic process)



Target (proton) is intact
Activity in central region

Scattering at ep collider HERA

HERA: (1992-2007)

27.5 GeV electrons/positrons

820/920 GeV protons

318 GeV CoM energy

Lumi: $10^{31} \text{ cm}^{-2} \text{ s}^{-1}$

Electrons, positrons and protons

Physics:

Structure functions

Parton density functions

Established growth of gluon with decreasing Bjorken x

Measurement of coupling constant

Diffraction

Jets, heavy quarks

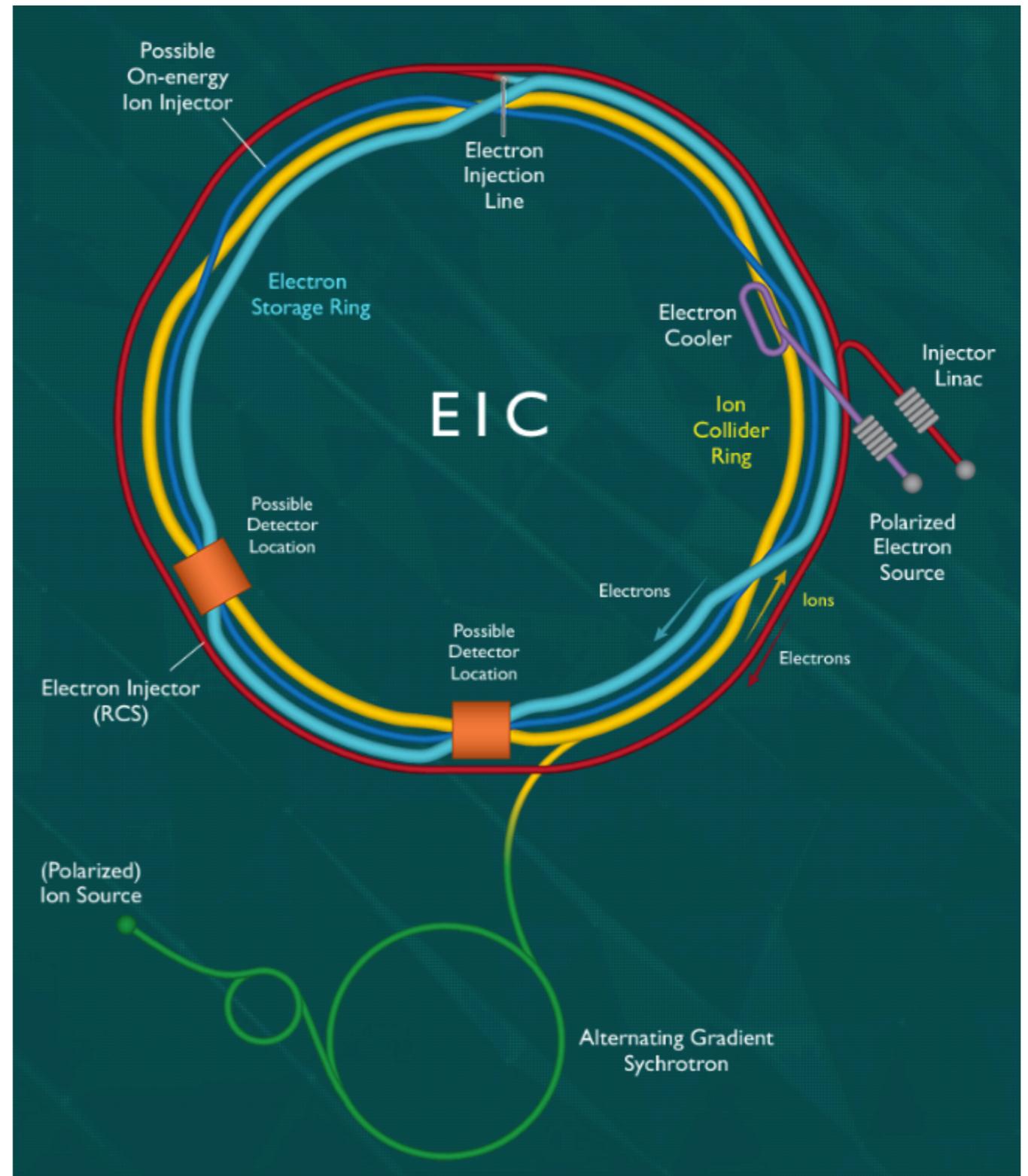
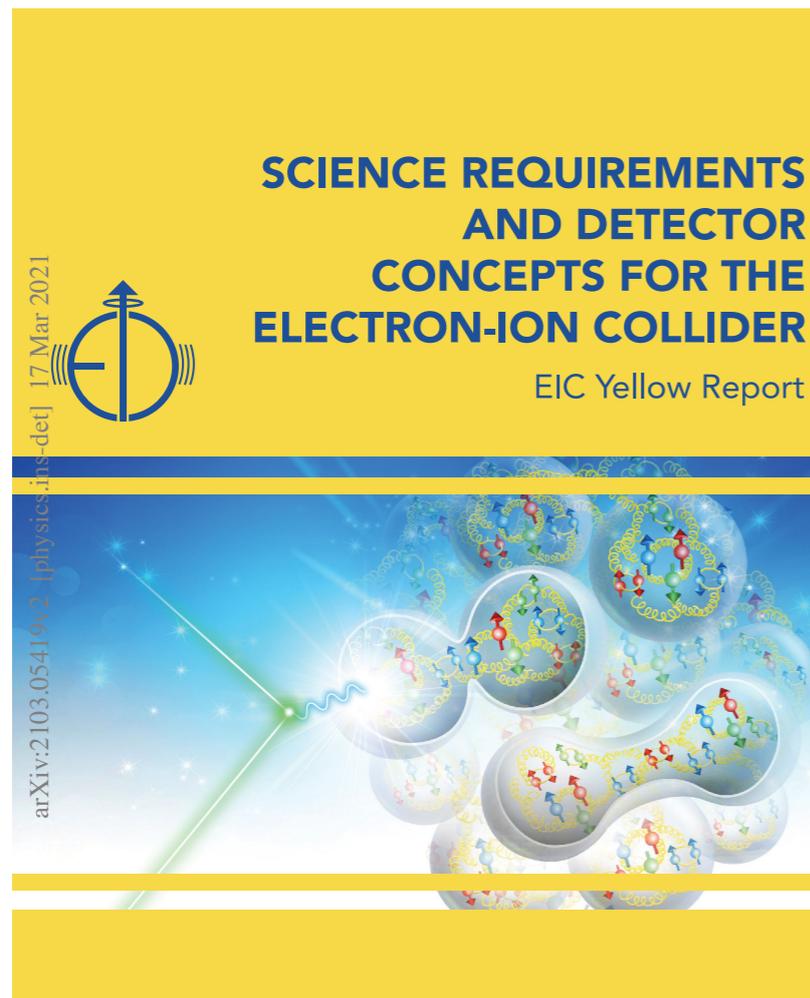
BSM searches



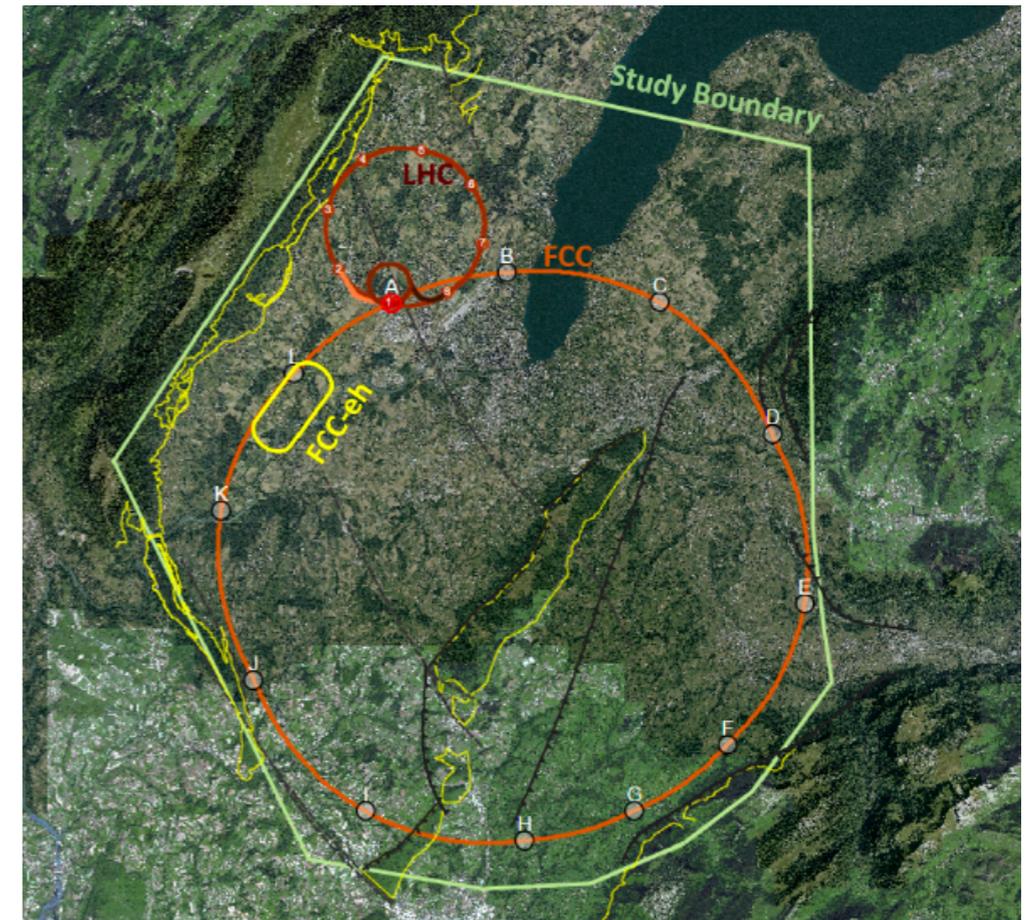
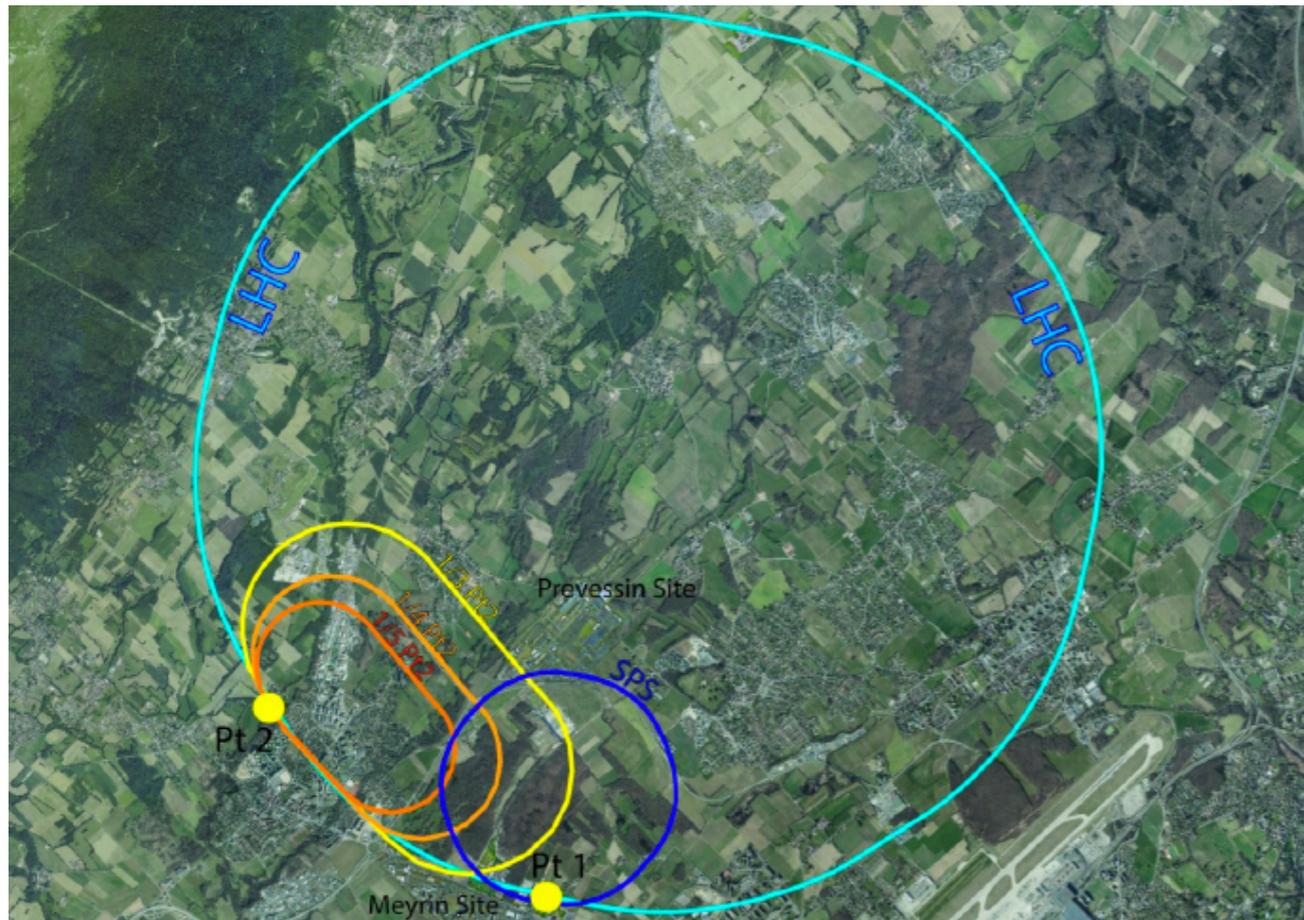
Low luminosity/limited statistics, no nuclei, no polarized beams

Future DIS machines EIC

EIC: 5-20 GeV electrons
20-140 GeV CoM energy
Lumi: $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$
Polarized e,p,d, ^3He
Wide range of nuclei



Future DIS machines LHeC, FCC-eh at CERN

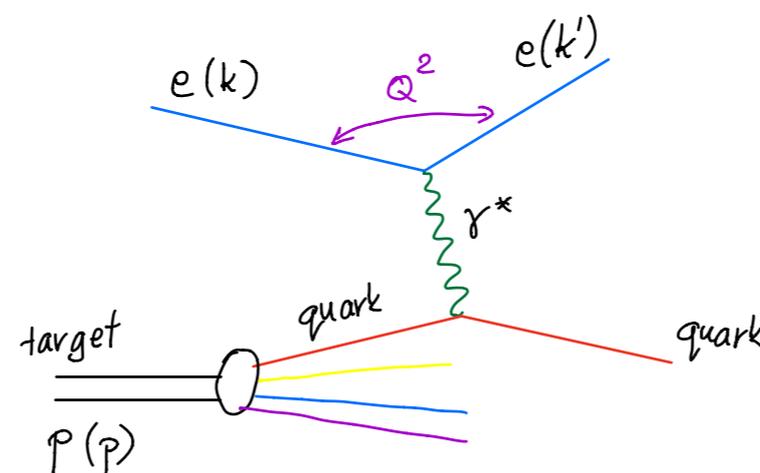
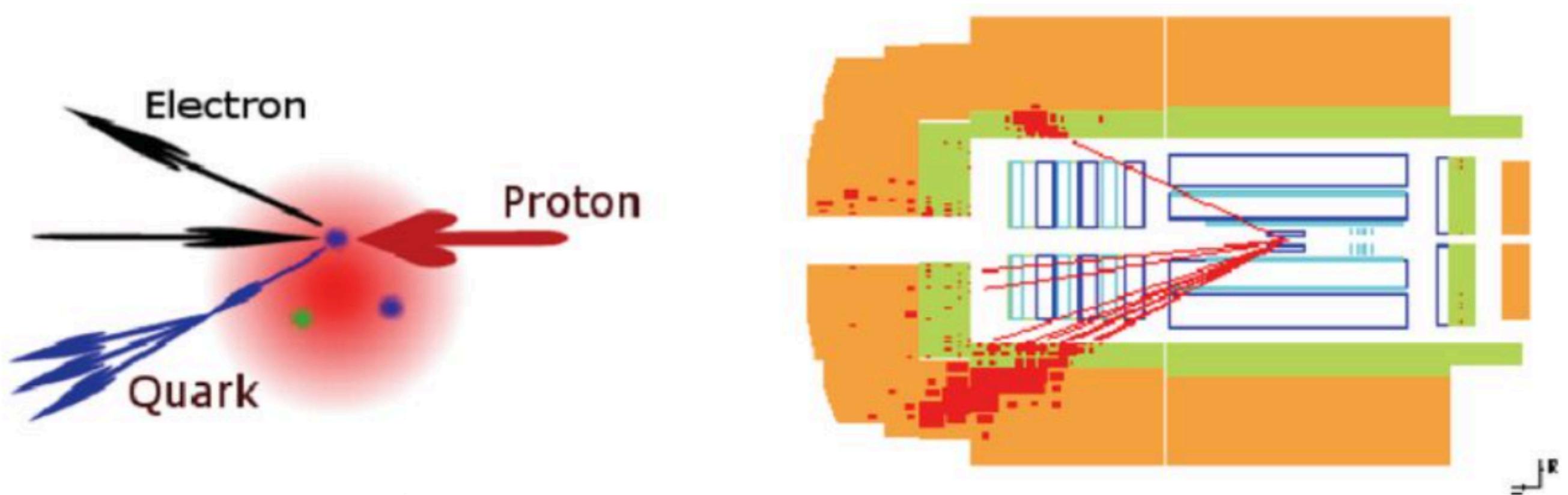


LHeC: 60(50) GeV electrons x
LHC protons and ions
1.3 TeV CoM energy for ep
812 GeV CoM for ePb
Lumi: $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$
Simultaneous running with ATLAS
and CMS in HL-LHC period

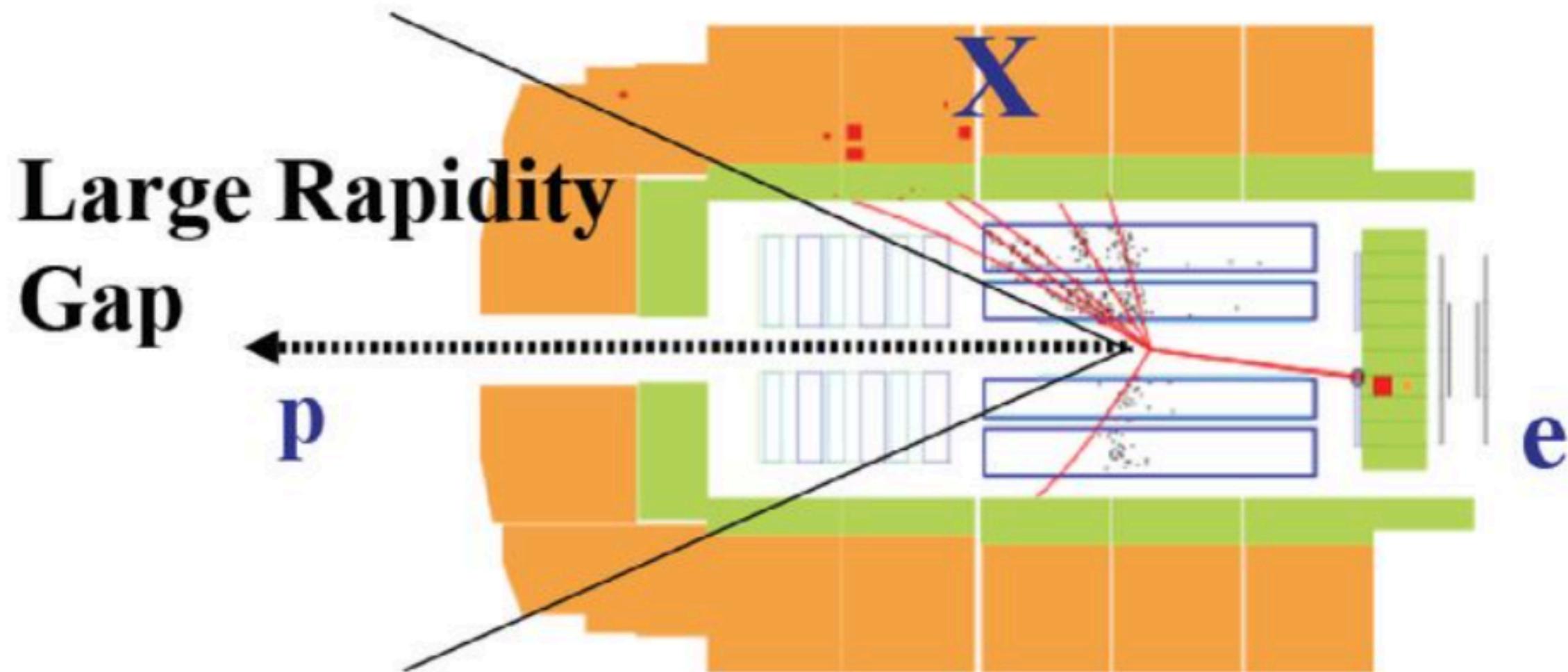
FCC-ep: 60(50) GeV
electrons x 50 TeV protons
from FCC, lead beams 19.7
TeV/per nucleon
3.5 TeV CoM energy for ep
2.2 TeV CoM for ePb
Lumi: $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$

Scattering at ep collider HERA

Non-diffractive DIS event



Diffraction at HERA



10% events at HERA were of diffractive type

Large portion of the detector void of any particle activity: **rapidity gap**

Proton stays intact despite undergoing violent collision with a 50 TeV electron (in its rest frame)

Rapidity: recap

$$p^\mu = (E, \vec{p}_T, p_z) \quad y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} \quad \frac{E}{p_z} = \tanh y$$

Under boosts in z direction rapidity transforms additively

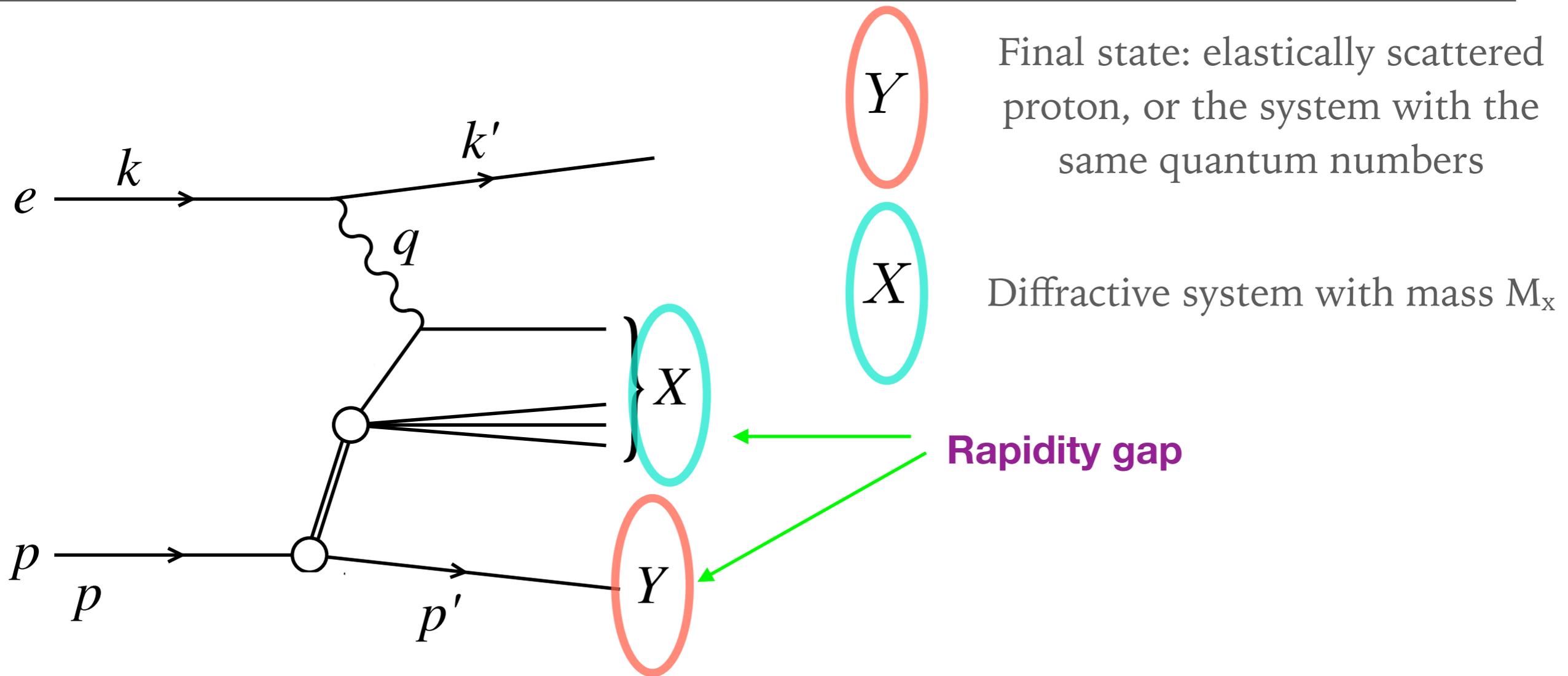
$$\begin{pmatrix} p_z \\ E \end{pmatrix} = \begin{pmatrix} \cosh \phi & \sinh \phi \\ \sinh \phi & \cosh \phi \end{pmatrix} \begin{pmatrix} p'_z \\ E' \end{pmatrix}$$

Then $y = y' + \phi$

Pseudorapidity $\eta = -\ln \tan \frac{\theta}{2}$ Angle between 3-momentum and z-axis

When $m \ll |\vec{p}_T|$ then $y \rightarrow \eta$

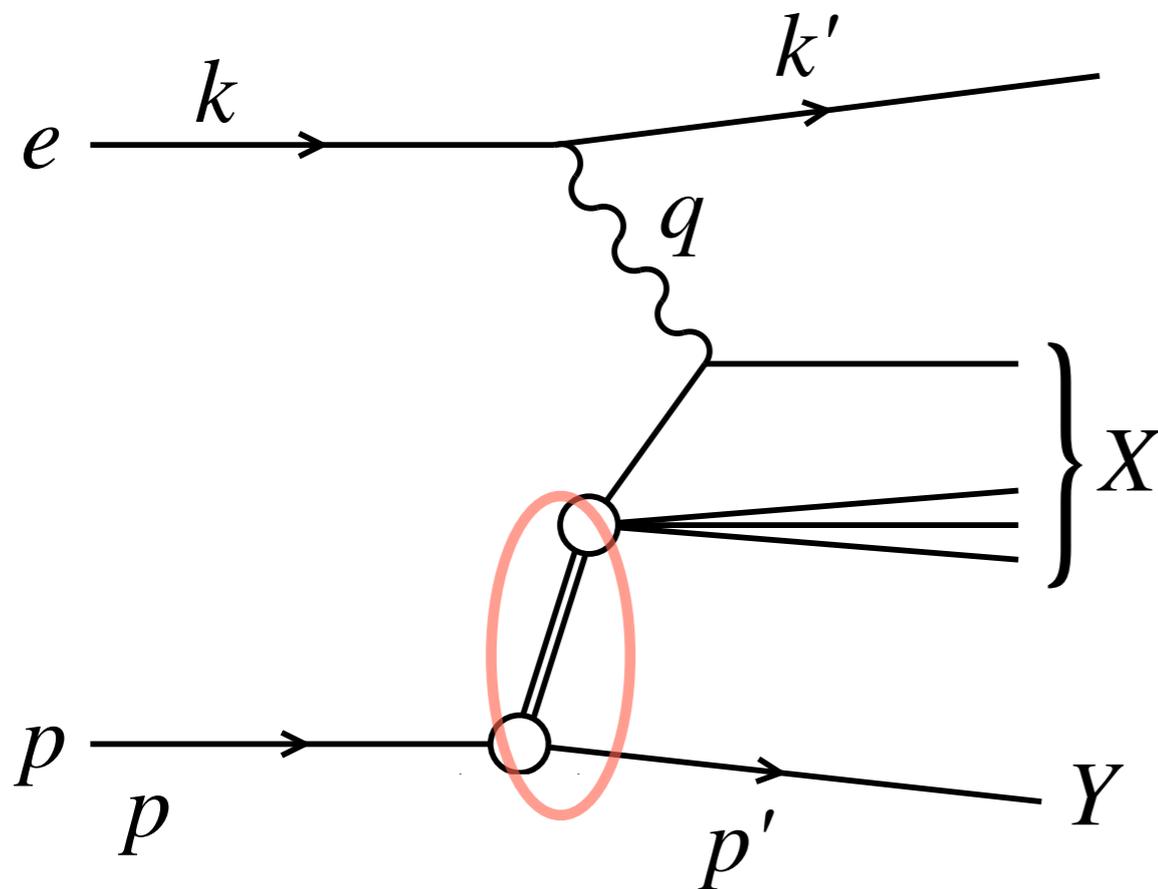
Diffraction in electron - proton(nucleus)



In order for the rapidity gap to exist it needs to be mediated by the **colorless** exchange

Diffraction: a reaction characterized by a **rapidity** gap in the final state

Diffraction and the Pomeron



Diffraction: a reaction characterized by a large rapidity gap in the final state

In order for the rapidity gap to exist it needs to be mediated by the **colorless diffractive exchange**

But what is this **diffractive exchange** ?

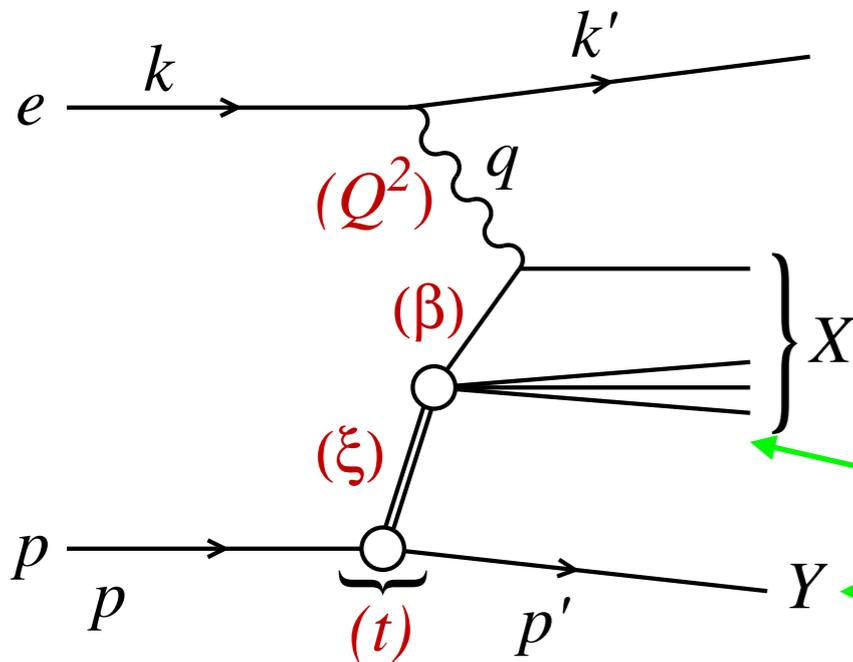
Usually referred to as the **Pomeron**.
Quantum numbers of the vacuum

Modeled as a composite system of gluons and/or quarks.

Studying diffractive processes can shed light onto properties of this intriguing object.

Diffractive kinematics in DIS

Standard DIS variables:



electron-proton
cms energy squared:

$$s = (k + p)^2$$

photon-proton
cms energy squared:

$$W^2 = (q + p)^2$$

inelasticity

$$y = \frac{p \cdot q}{p \cdot k}$$

Bjorken x

$$x = \frac{-q^2}{2p \cdot q}$$

(minus) photon virtuality

$$Q^2 = -q^2$$

Rapidity gap

Target is scattered elastically:
elastic scattering

It can also dissociate into a
state Y with the same quantum
numbers, but still separated
from the rest of particles

Diffractive DIS variables:

$$\xi \equiv x_{IP} = \frac{Q^2 + M_X^2 - t}{Q^2 + W^2}$$

$$\beta = \frac{Q^2}{Q^2 + M_X^2 - t}$$

$$t = (p - p')^2$$

momentum fraction of the
Pomeron w.r.t hadron

momentum fraction of parton
w.r.t Pomeron

4-momentum transfer squared

$$x = \xi \beta$$

Reduced cross section, structure functions

Recall the **reduced cross section** in inclusive DIS:

$$\frac{d^2\sigma_{NC}}{dx dQ^2} = \frac{2\pi\alpha_{em}^2}{xQ^4} Y_+ \sigma_r(x, Q^2)$$

Dimensions:

$$Y_+ = 1 + (1 - y)^2$$

σ_r Dimensionless

Reduced cross section depends on two **structure functions**:

$$\sigma_r(x, Q^2) = F_2(x, Q^2) - \frac{y^2}{Y_+} F_L(x, Q^2)$$

$$F_T = F_2 - F_L \quad \text{transverse structure function}$$

$$F_L \quad \text{longitudinal structure function}$$

Diffractive cross section, structure functions

Diffractive cross section depends on 4 variables (ξ, β, Q^2, t) :

$$\frac{d^4 \sigma^D}{d\xi d\beta dQ^2 dt} = \frac{2\pi \alpha_{\text{em}}^2}{\beta Q^4} Y_+ \sigma_r^{\text{D}(4)}(\xi, \beta, Q^2, t)$$

$$Y_+ = 1 + (1 - y)^2$$

Reduced cross section depends on two structure functions:

$$\sigma_r^{\text{D}(4)}(\xi, \beta, Q^2, t) = F_2^{\text{D}(4)}(\xi, \beta, Q^2, t) - \frac{y^2}{Y_+} F_L^{\text{D}(4)}(\xi, \beta, Q^2, t)$$

Upon integration over t :

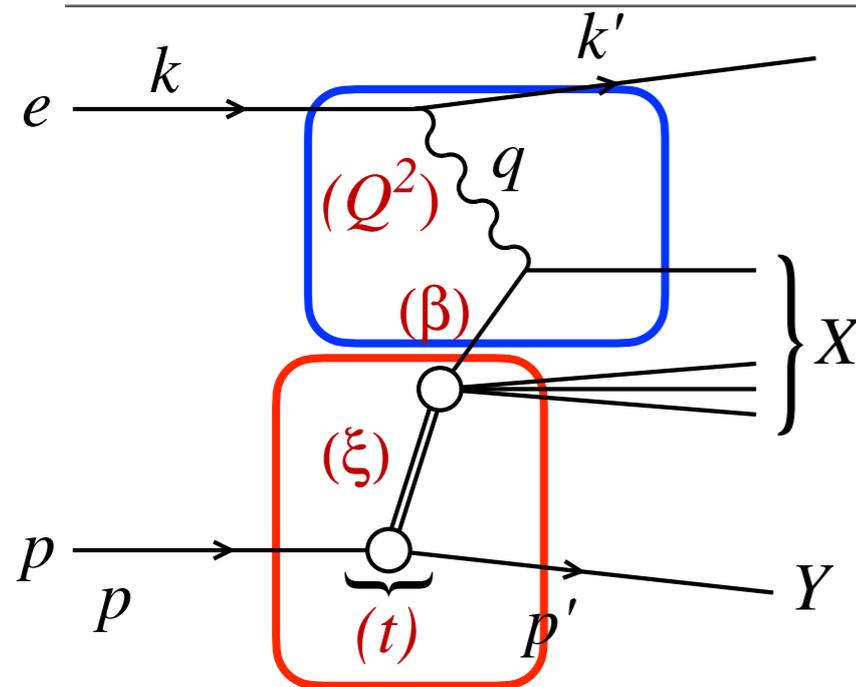
$$F_{2,L}^{\text{D}(3)}(\xi, \beta, Q^2) = \int_{-\infty}^0 dt F_{2,L}^{\text{D}(4)}(\xi, \beta, Q^2, t)$$

Dimensions:

$$[\sigma_r^{\text{D}(4)}] = \text{GeV}^{-2}$$

$$\sigma_r^{\text{D}(3)} \quad \text{Dimensionless}$$

Collinear factorization for diffraction



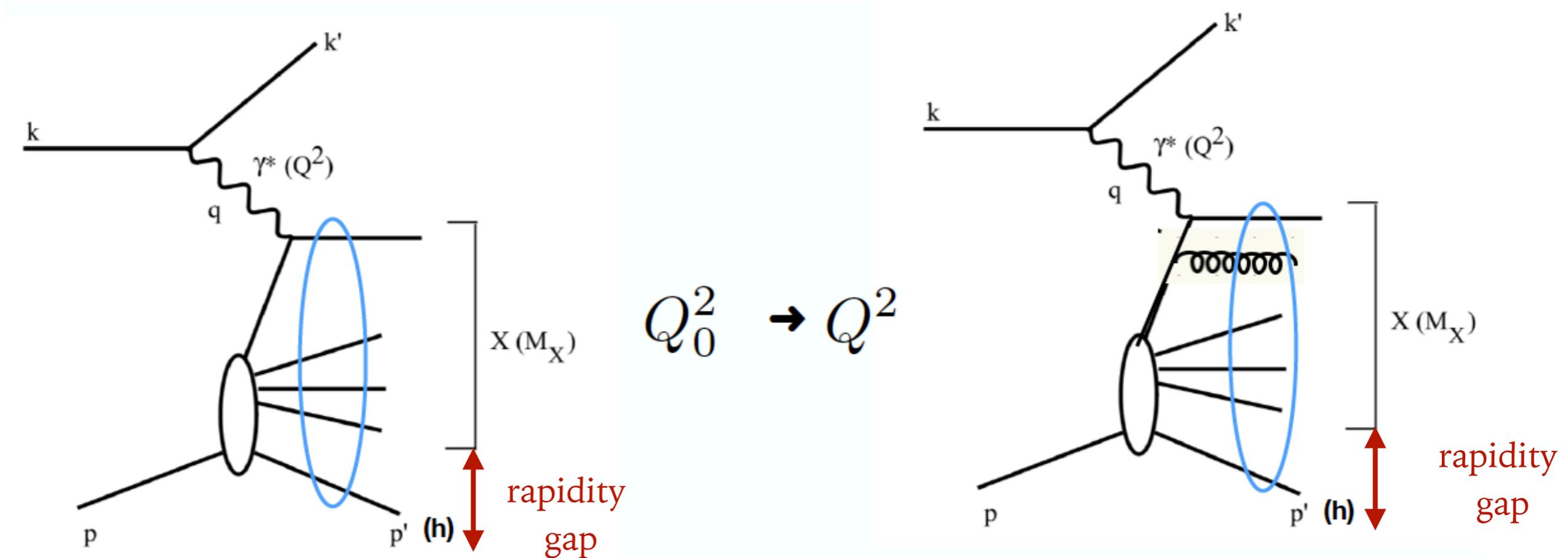
Collins

Collinear factorization in diffractive DIS

$$F_{2/L}^{D(4)}(\beta, \xi, Q^2, t) = \sum_i \int_{\beta}^1 \frac{dz}{z} C_{2/L,i} \left(\frac{\beta}{z}, Q^2 \right) f_i^D(z, \xi, Q^2, t)$$

- Diffractive cross section can be factorized into the convolution of the perturbatively calculable **partonic cross sections** and **diffractive parton distributions** (DPDFs).
- **Partonic cross sections** are the same as for the inclusive DIS.
- The DPDFs represent (at least in LO) the **probability distributions** for partons i in the proton under the constraint that the proton is scattered into the system Y with a specified 4-momentum.
- **Factorization** should be valid for sufficiently(?) large Q^2 (and fixed t and ξ).

Factorization for diffraction



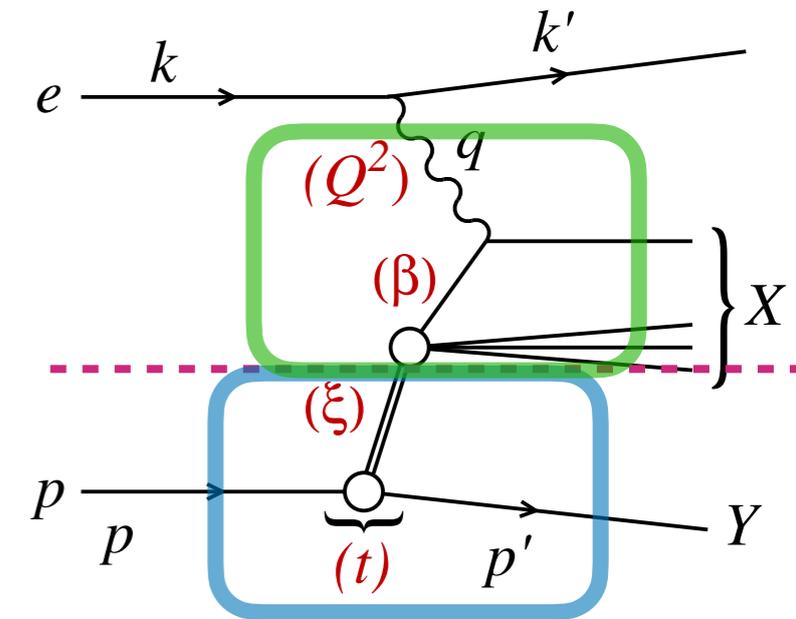
By changing the scale from Q_0^2 to Q^2 additional gluon is emitted in the diffractive system X
 Soft gluons forming the system h are not able to resolve the qg system since it is localized at a distance smaller than $1/Q_0$

Model for diffractive structure functions

Regge factorization with **Pomeron** terms works for small $\xi < 0.01$

At higher ξ additional exchanges '**Reggeons**' need to be included

$$f_i^{D(4)}(z, \xi, Q^2, t) = \underbrace{f_{\mathbb{P}}^p(\xi, t)}_{\text{Pomeron}} \underbrace{f_i^{\mathbb{P}}(z, Q^2)}_{\text{Reggeon}} + \underbrace{f_{\mathbb{R}}^p(\xi, t)}_{\text{Reggeon}} \underbrace{f_i^{\mathbb{R}}(z, Q^2)}_{\text{Reggeon}}$$



Regge type flux:

$$f_{\mathbb{P},\mathbb{R}}^p(\xi, t) = A_{\mathbb{P},\mathbb{R}} \frac{e^{B_{\mathbb{P},\mathbb{R}}t}}{\xi^{2\alpha_{\mathbb{P},\mathbb{R}}(t)-1}}$$

Trajectory:

$$\alpha_{\mathbb{P},\mathbb{R}}(t) = \alpha_{\mathbb{P},\mathbb{R}}(0) + \alpha'_{\mathbb{P},\mathbb{R}} t.$$

For t-integrated case

$$f_i^{D(3)}(z, \xi, Q^2) = \phi_{\mathbb{P}}^p(\xi) f_i^{\mathbb{P}}(z, Q^2) + \phi_{\mathbb{R}}^p(\xi) f_i^{\mathbb{R}}(z, Q^2)$$

Integrated flux:

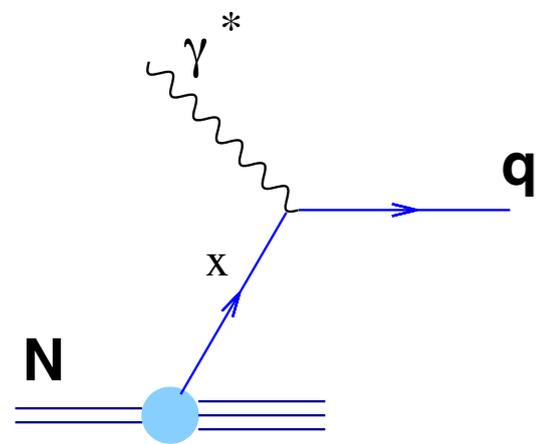
$$\phi_{\mathbb{P},\mathbb{R}}^p(\xi) = \int dt f_{\mathbb{P},\mathbb{R}}^p(\xi, t)$$

Pomeron PDFs obtained via NLO DGLAP evolution starting at initial scale $\mu_0^2 = 1.8 \text{ GeV}^2$

$$z f_i(z, \mu_0^2) = A_i z^{B_i} (1-z)^{C_i} \quad i=q,g$$

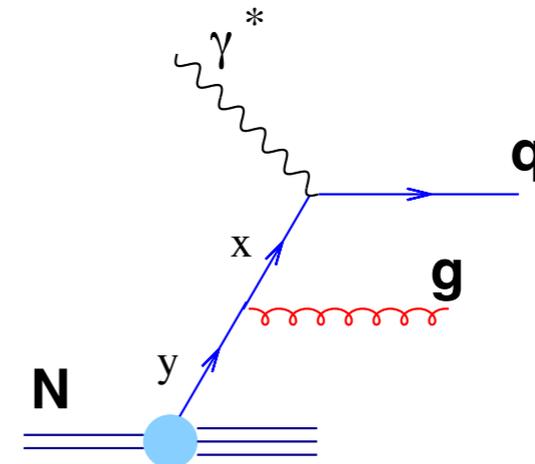
DGLAP evolution equations: recap

$$t = \ln Q^2 / Q_0^2$$



$$q(x, t)$$

$$t + \delta t$$



$$q(x, t) + \delta q(x, t)$$

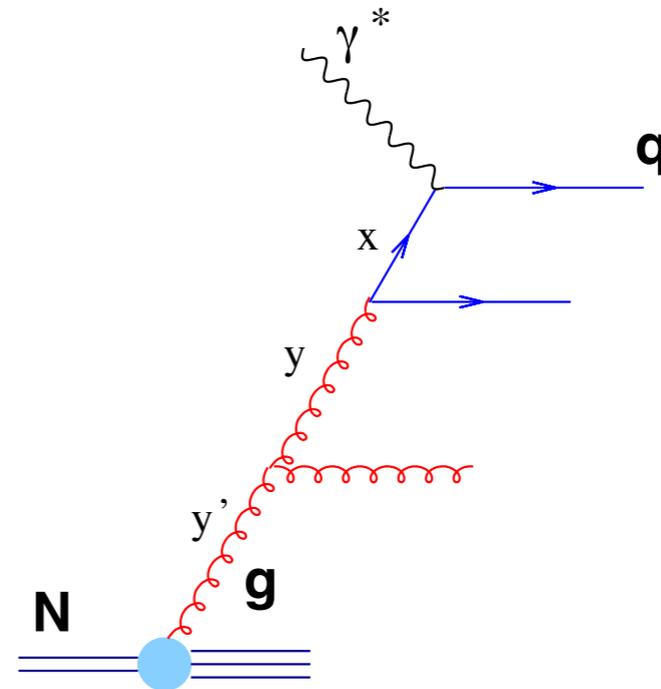
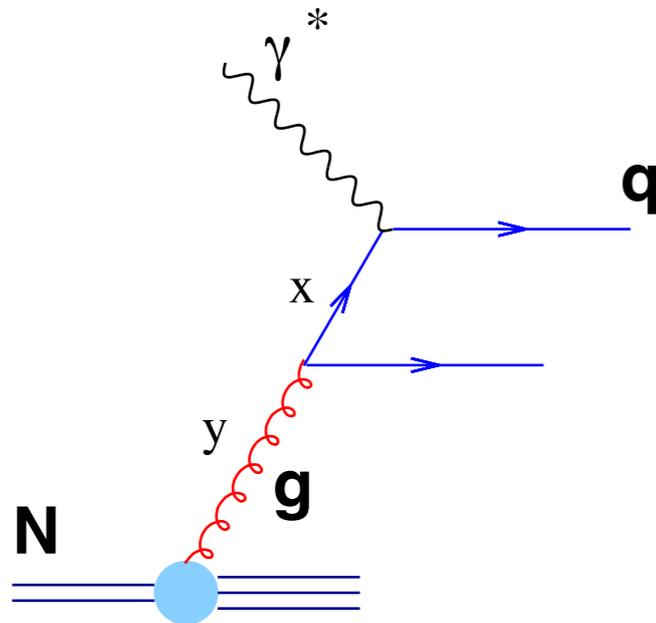
quarks with fraction $x = \frac{Q^2}{s}$ of target momentum

DGLAP evolution equations: recap

Apart from $q(x,t)$ one has to include $g(x,t)$

$$t = \ln Q^2 / Q_0^2$$

$$t + \delta t$$

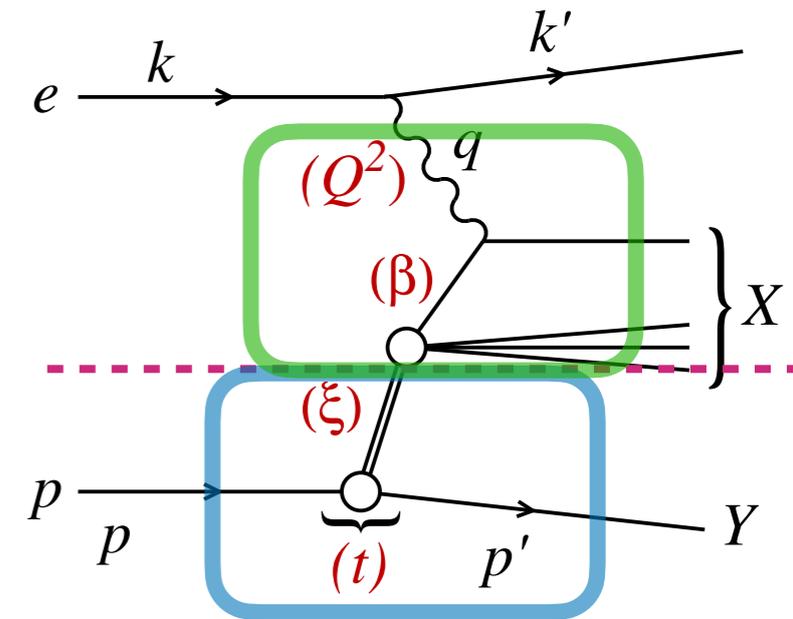


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Regge type flux:

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Trajectory:

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For t-integrated case

$$f_i^{D(3)}(z, \xi, Q^2) = \phi_{\mathbb{P}}^p(\xi) f_i^{\mathbb{P}}(z, Q^2) + \phi_{\mathbb{R}}^p(\xi) f_i^{\mathbb{R}}(z, Q^2)$$

Integrated flux:

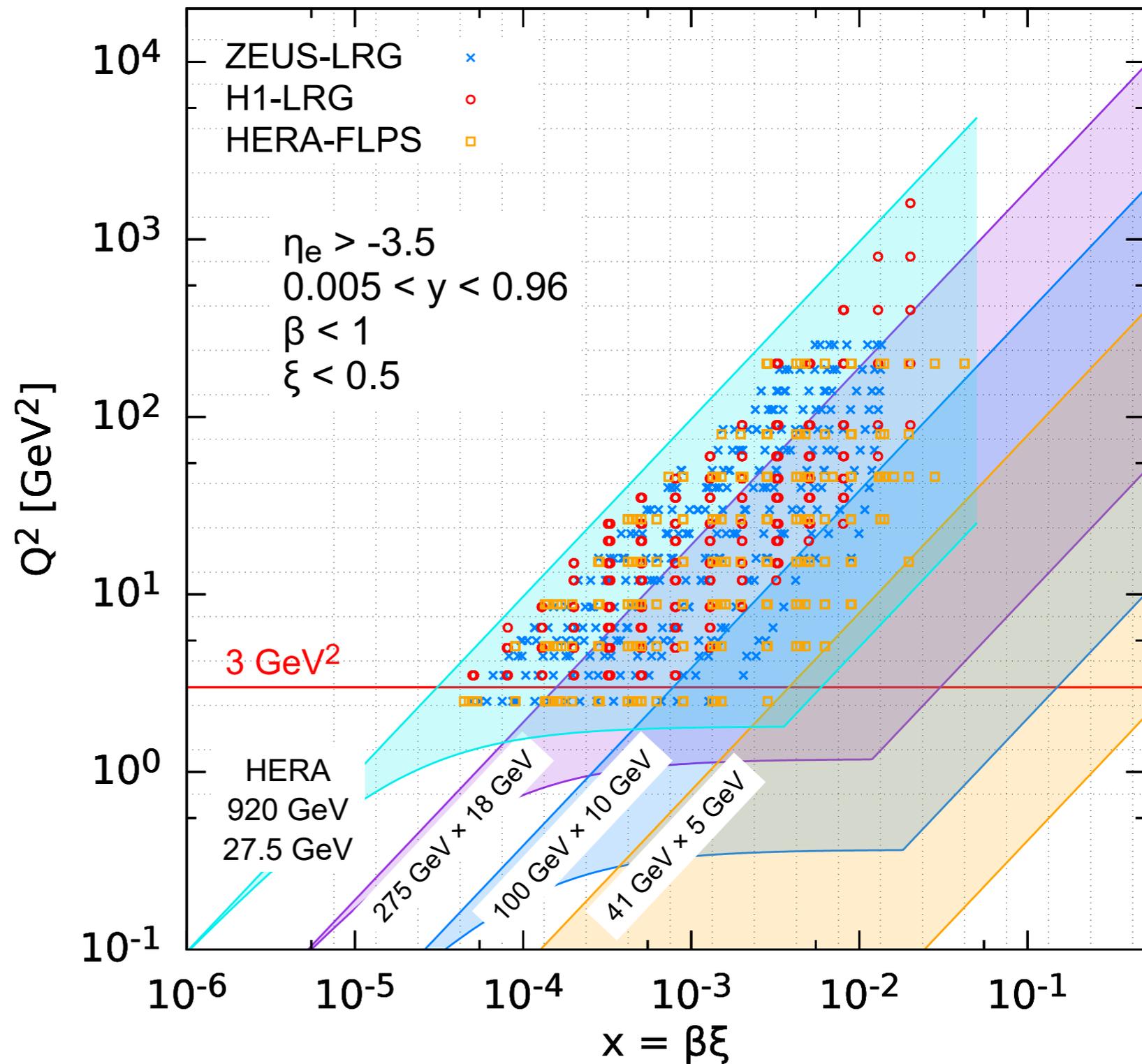
$$\phi_{\mathbb{P},\mathbb{R}}^p(\xi) = \int dt f_{\mathbb{P},\mathbb{R}}^p(\xi, t)$$

Pomeron PDFs obtained via NLO DGLAP evolution starting at initial scale $\mu_0^2 = 1.8 \text{ GeV}^2$

$$z f_i(z, \mu_0^2) = A_i z^{B_i} (1-z)^{C_i} \quad i=q,g$$

Phase space (x,Q²) HERA-EIC

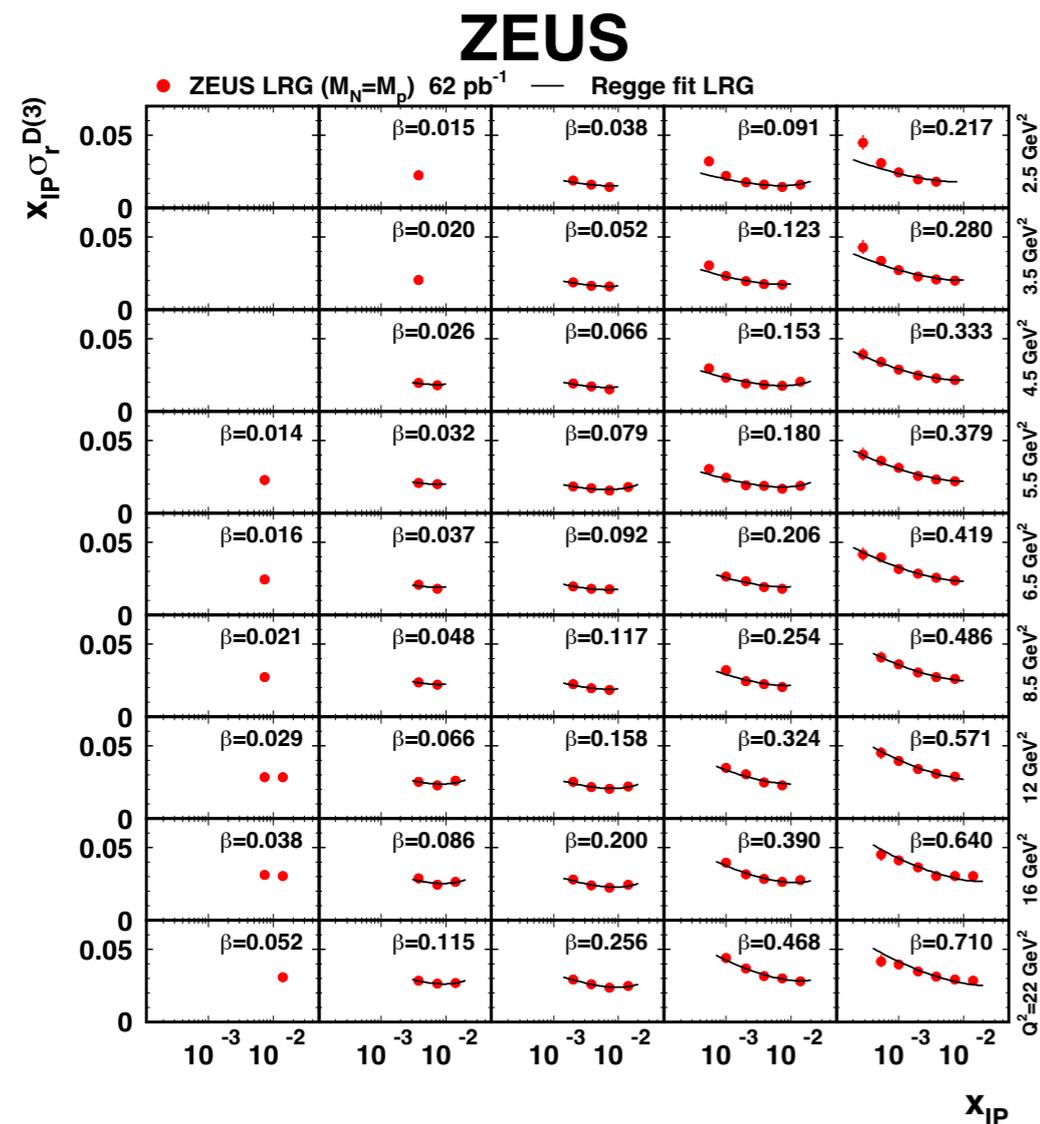
EIC 3 scenarios - HERA



Measurement methods: LRG vs LP

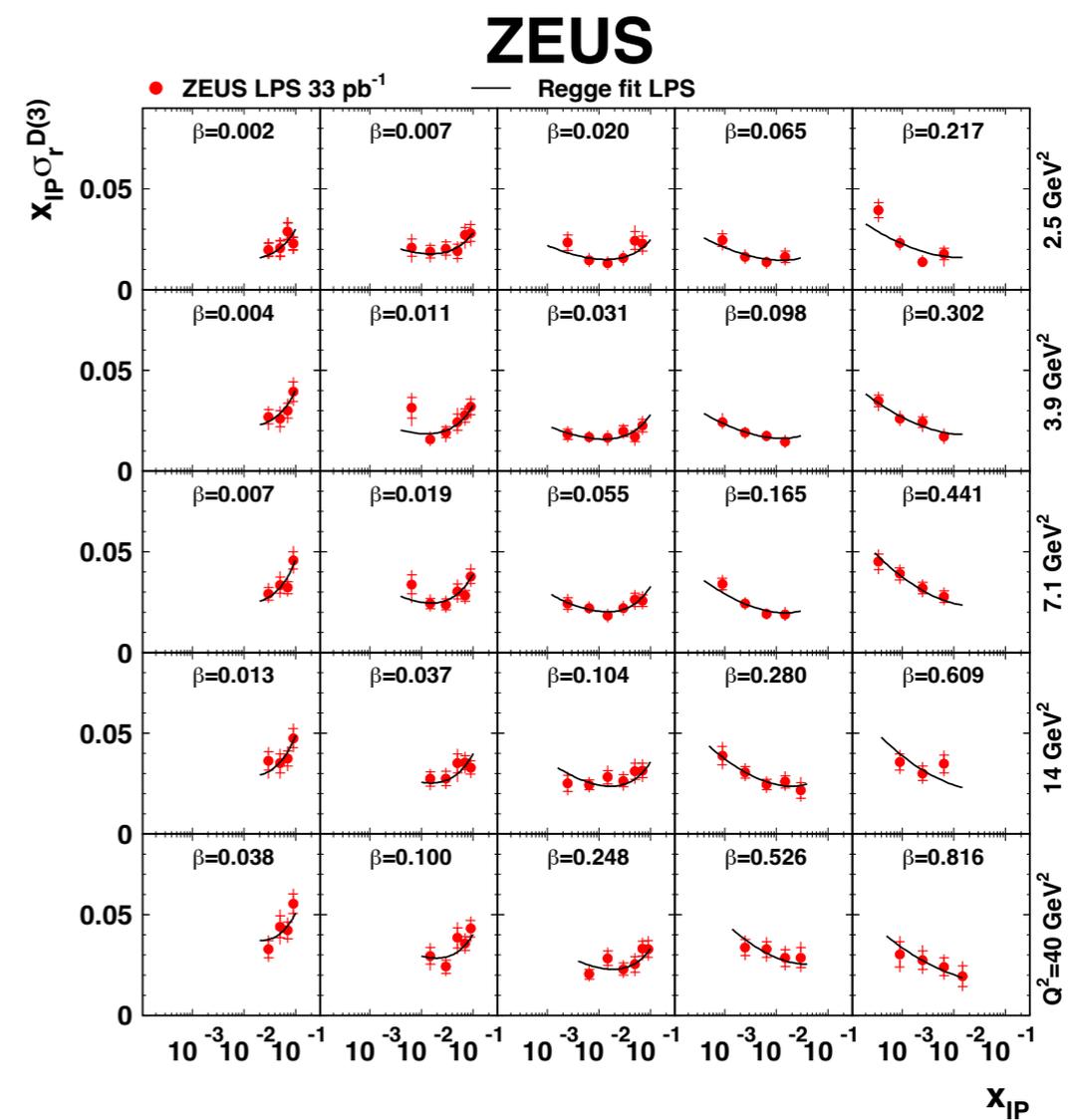
Large Rapidity Gap method:

request a large rapidity gap (ex. ZEUS 2009 $\xi < 0.02$)



Proton Tagging (Leading Proton) method:

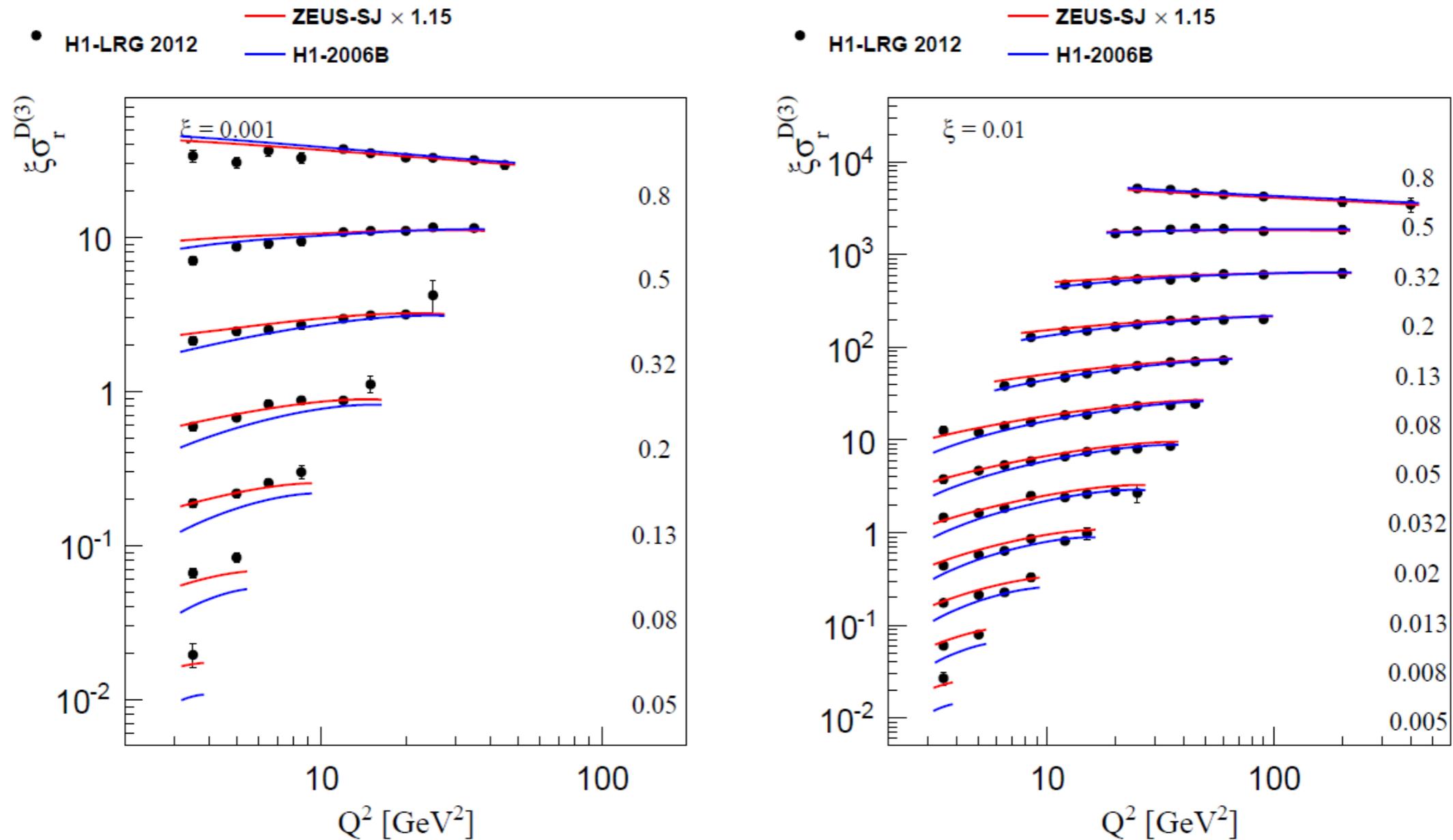
detection of a leading proton (ex. Leading Proton Spectrometer in ZEUS, Forward Proton Spectrometer in H1, can go to higher $\xi < 0.1$)



Diffractive fits

$$\xi = x_{IP}$$

Example of the DGLAP fit to the diffractive data



Comparison of H1-2006B and ZEUS-SJ fits to the H1-LRG 2012 data
 ZEUS-SJ fit seems to better describe the data in the low β region

Fit examples to diffractive data at HERA

$$f_i^{\text{D}(4)}(z, \xi, Q^2, t) = f_{\text{IP}}^p(\xi, t) f_i^{\text{IP}}(z, Q^2) + f_{\text{IR}}^p(\xi, t) f_i^{\text{IR}}(z, Q^2)$$

$$f_{\text{IP},\text{IR}}^p(\xi, t) = A_{\text{IP},\text{IR}} \frac{e^{B_{\text{IP},\text{IR}}t}}{\xi^{2\alpha_{\text{IP},\text{IR}}(t)-1}}$$

$$z f_i(z, \mu_0^2) = A_i z^{B_i} (1-z)^{C_i} \quad i=q,g$$

$$\alpha_{\text{IP},\text{IR}}(t) = \alpha_{\text{IP},\text{IR}}(0) + \alpha'_{\text{IP},\text{IR}} t.$$

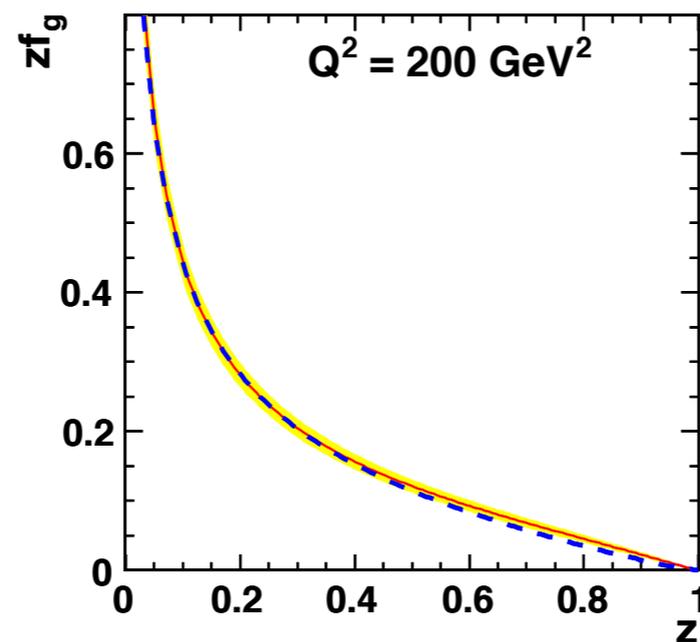
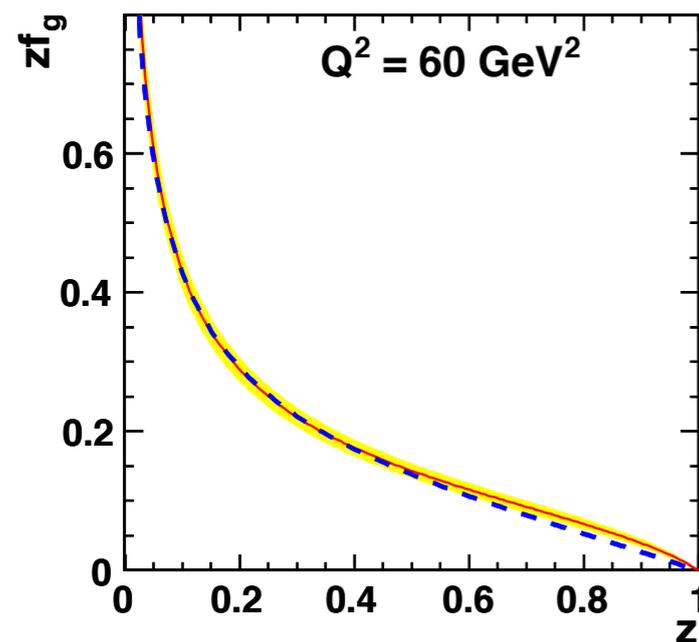
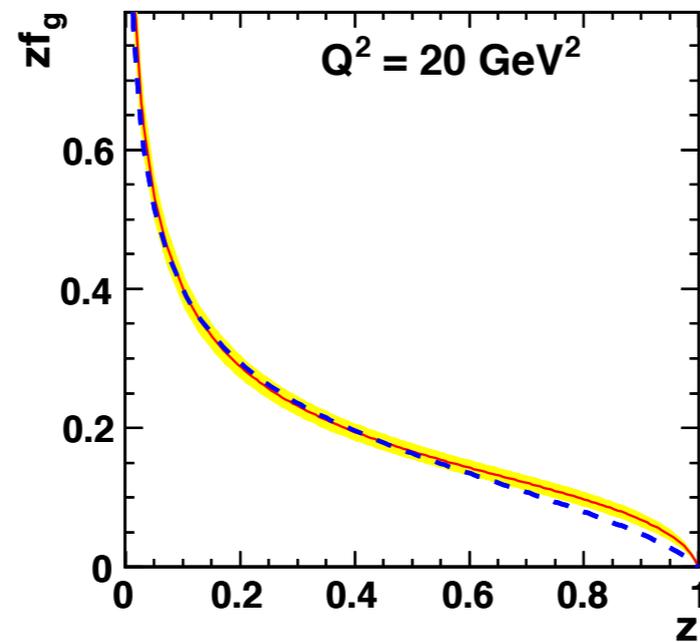
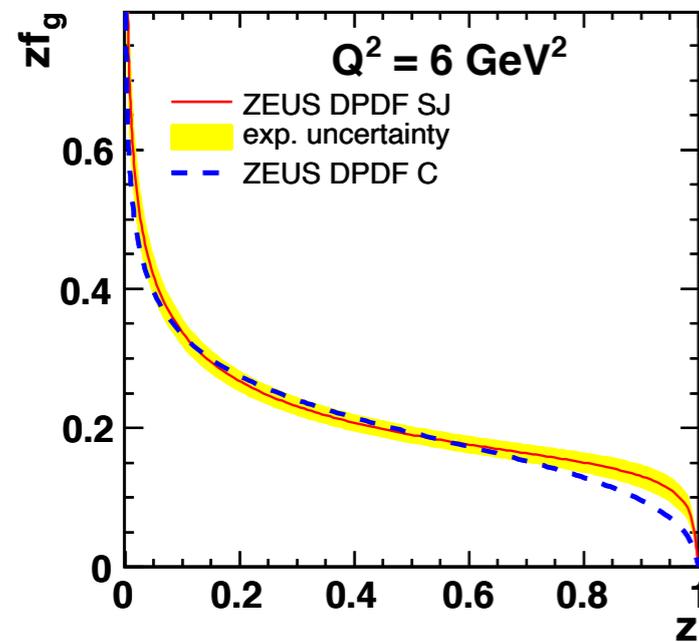
Parameter	ZEUS S	ZEUS C	ZEUS SJ	H1 A	H1 B
B_q	1.34 ± 0.05	1.25 ± 0.03	1.23 ± 0.04	2.3 ± 0.36	1.5 ± 0.12
C_q	0.34 ± 0.043	0.358 ± 0.043	0.332 ± 0.049	0.57 ± 0.15	0.45 ± 0.09
B_g	-0.422 ± 0.066	0	-0.161 ± 0.051	0	0
C_g	-0.725 ± 0.082	0	-0.232 ± 0.058	-0.95 ± 0.20	0
$\alpha_{\text{IP}}(0)$	1.12 ± 0.02	1.11 ± 0.02	1.11 ± 0.02	1.118 ± 0.008	1.111 ± 0.007
$\alpha_{\text{IR}}(0)$	0.732 ± 0.031	0.668 ± 0.040	0.699 ± 0.043	0.5	0.5
α'_{IP}	0	0 GeV ⁻²	0 GeV ⁻²	0.06 GeV ⁻²	0.06 GeV ⁻²
α'_{IR}	0.9 GeV ⁻²	0.9 GeV ⁻²	0.9 GeV ⁻²	0.3 GeV ⁻²	0.3 GeV ⁻²
B_{IP}	7 GeV ⁻²	7 GeV ⁻²	7 GeV ⁻²	5.5 GeV ⁻²	5.5 GeV ⁻²
B_{IR}	2 GeV ⁻²	2 GeV ⁻²	2 GeV ⁻²	1.6 GeV ⁻²	1.6 GeV ⁻²

Parameters in **bold font** were fixed in the fits

DPDFs: ZEUS fits

Gluon

$$f_i^D(x, Q^2, x_{IP}, t) = f_{IP/p}(x_{IP}, t) f_i(\beta = x/x_{IP}, Q^2)$$



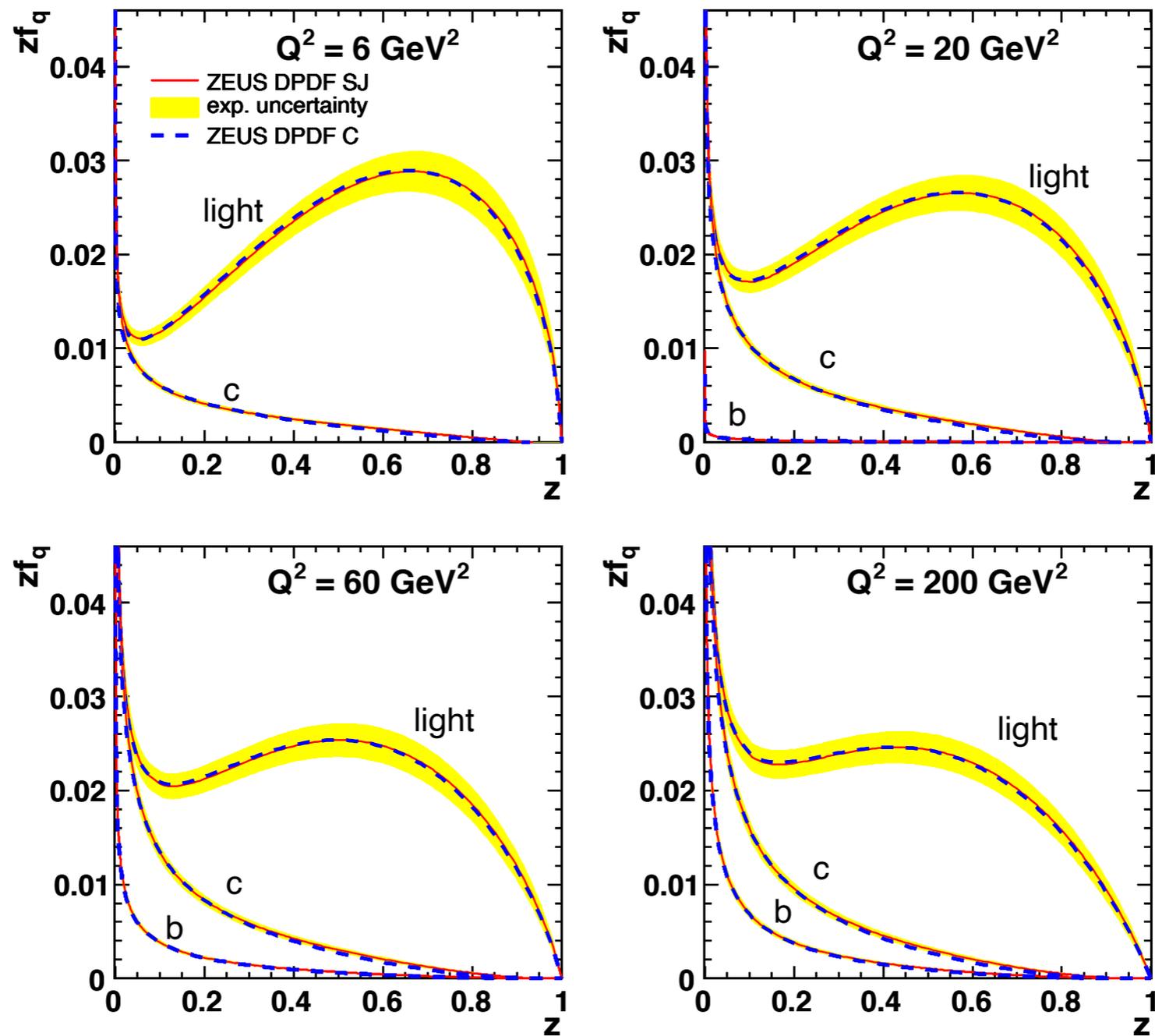
Inclusive data alone cannot fix the diffractive gluon distribution at large z .

Fit SJ includes diffractive dijets

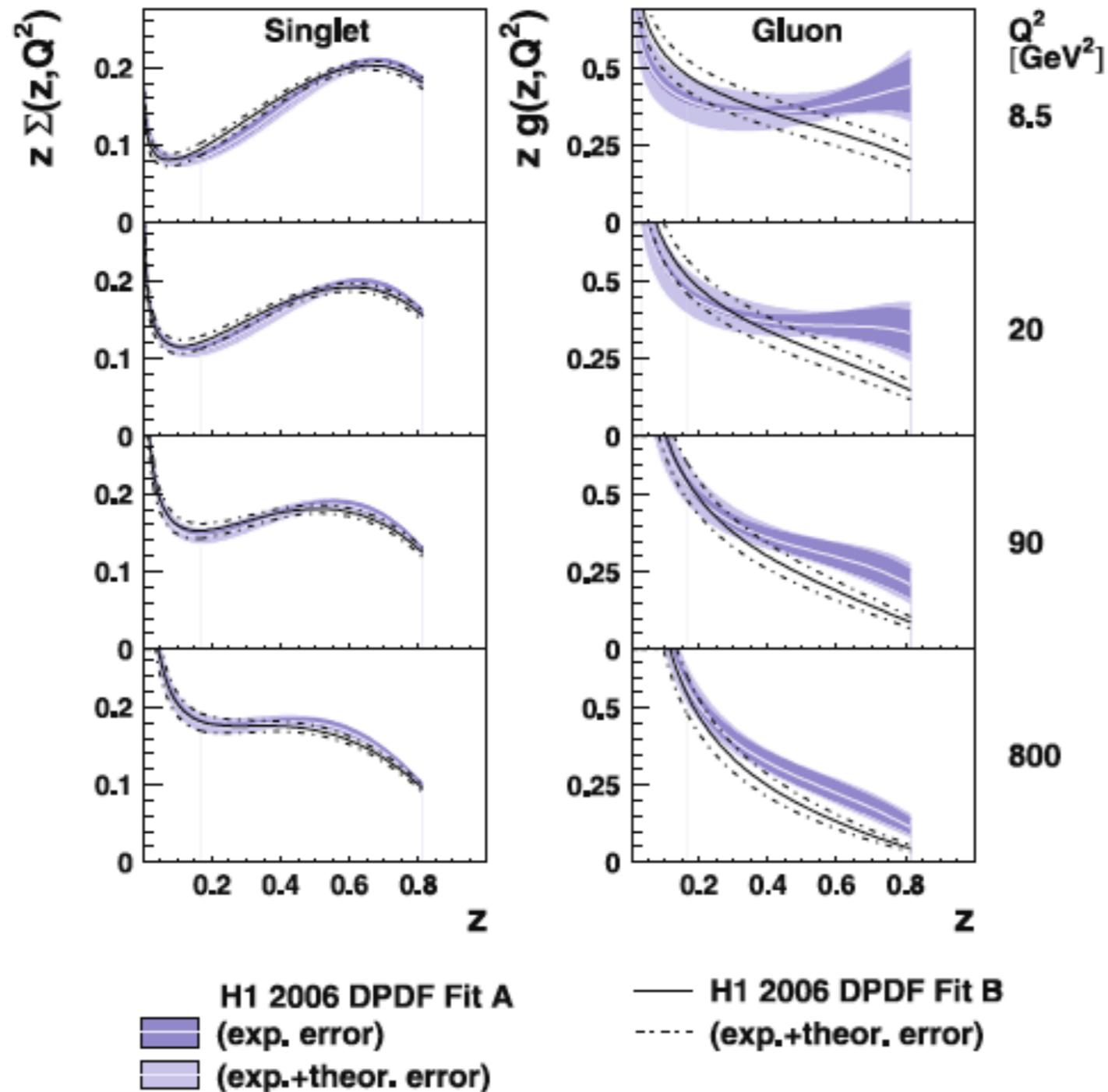
DPDFs: ZEUS fits

Quark

$$f_i^D(x, Q^2, x_{IP}, t) = f_{IP/p}(x_{IP}, t) f_i(\beta = x/x_{IP}, Q^2)$$



DPDFs: H1 fits



DGLAP fits fail for low values of Q^2

H1: $Q^2 < 8.5$ GeV²

ZEUS: $Q^2 < 5$ GeV²

Higher twists / saturation ... ?

Diffractive dijets in DIS

The factorization allows to use the diffractive parton distributions to predict other processes in diffraction with large scale present: universality

Examples include: **diffractive dijets**, **diffractive charm production**

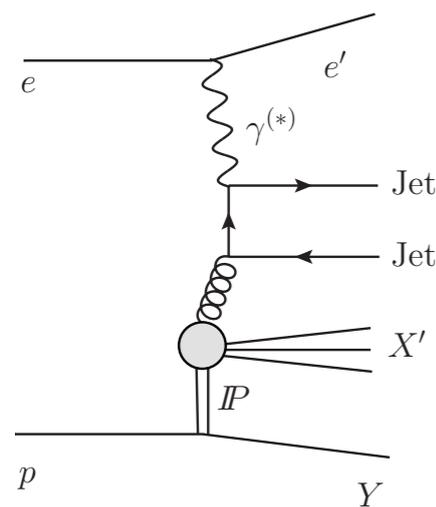
Factorization formula for diffractive dijets:

$$d\sigma(e+p \rightarrow e+2\text{jets}+X'+Y) = \sum_i \int dt \int d\xi \int dz d\hat{\sigma}(e+i \rightarrow e+2\text{jets}) f_i^{D(4)}(z, \xi, \mu^2, t)$$

μ^2 factorization scale

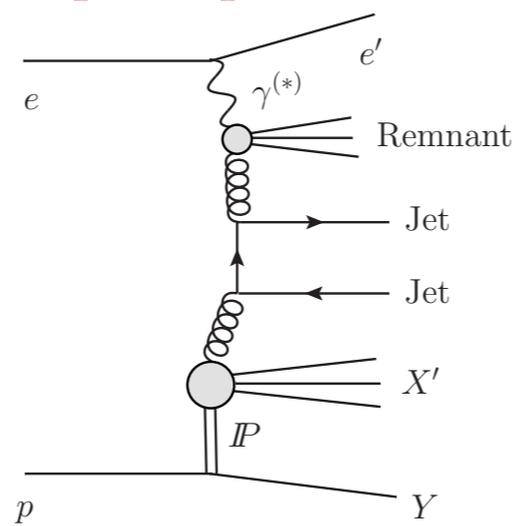
X' part of the diffractive system that does not include the jets

DIS, photoproduction



(a)
Direct

photoproduction



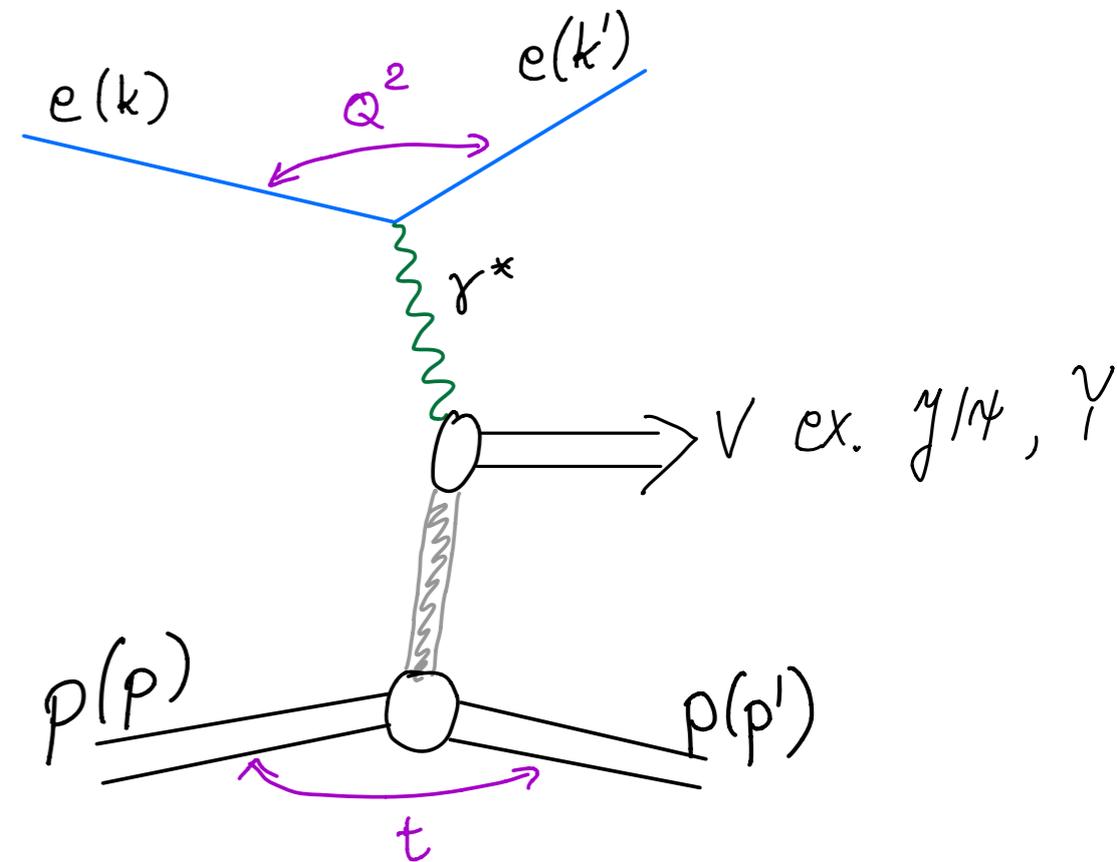
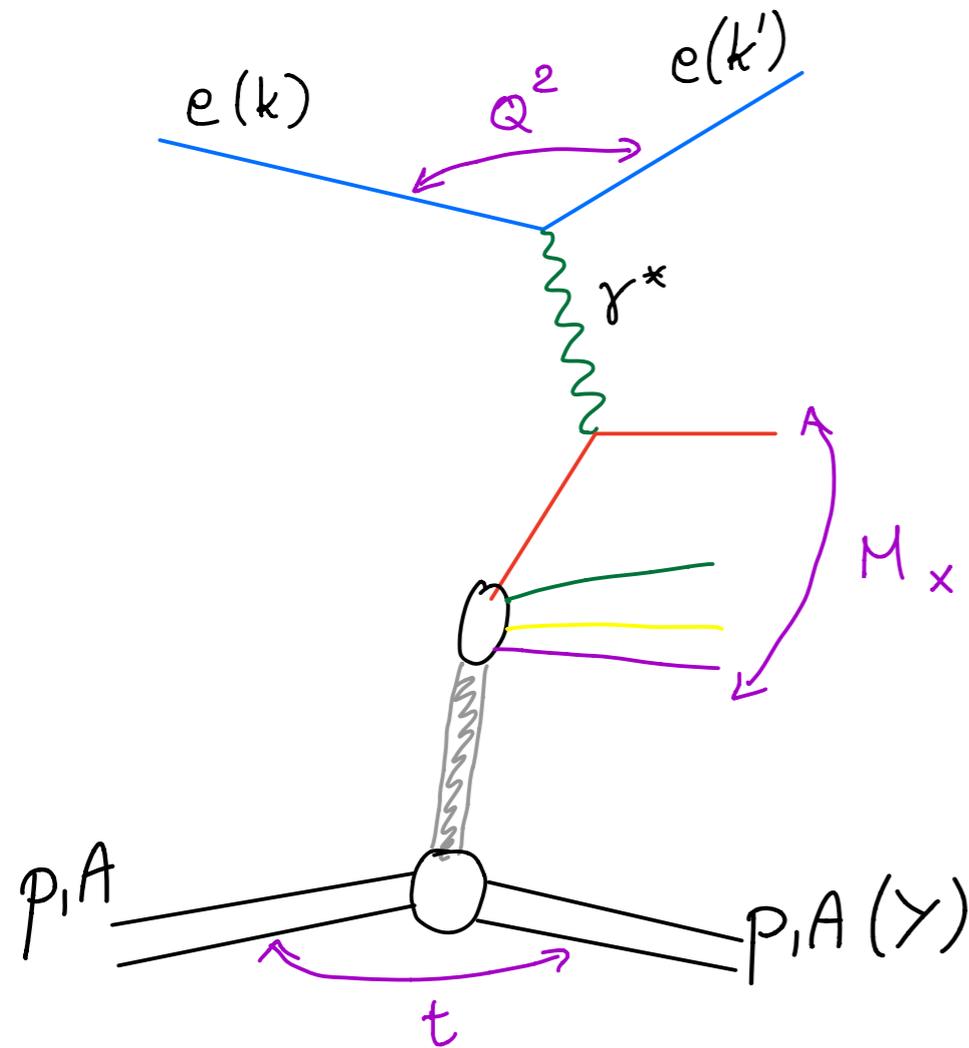
(b)
Resolved

DIS diffractive dijets consistent with factorization. Used in ZEUS SJ fits for example.

Photoproduction diffractive dijets: ZEUS consistent with factorization within errors, H1 data lower than predictions

Diffractive elastic vector meson production

Final state contains only vector meson, scattered lepton and proton



J/ψ vector meson: charm -anti charm system

$$m = 3.09 \text{ GeV}$$

Upsilon vector meson: bottom - anti bottom system

$$m = 9.46 \text{ GeV}$$

Thank you!