Basics of Color Glass Condensate

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Literature

Textbook: Quantum Chromodynamics at high energy

by kovechegov and Levin

Reviews:

The Color Glass Condensate and high energy scattering in QCD

Iancu and Venugopalan

The Color Glass Condensate

Gelis, Iancu, JJM and Venugopalan

………………………………

Morreale and Salazar

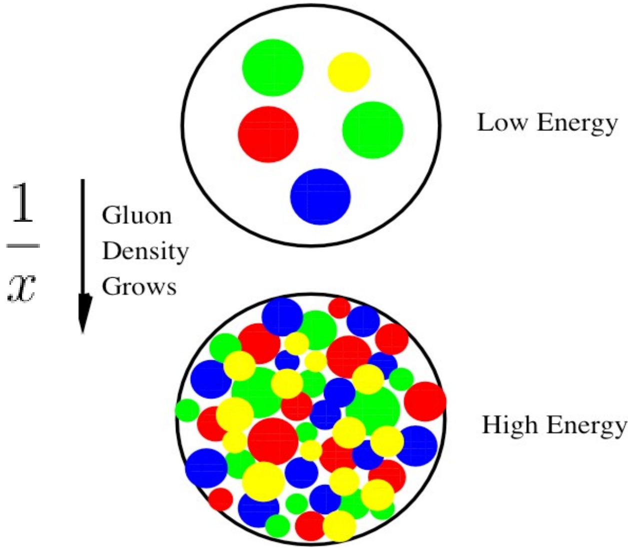
Mining for gluon saturation at colliders

Last time

A high energy proton/nucleus is a dense system of gluons

MV model:

- **large x partons as sources of color charge**
- **small x gluons represented as a classical field**

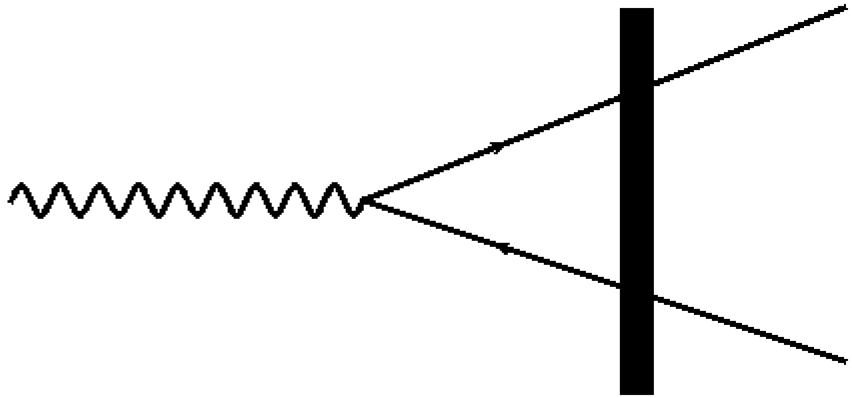


Multiple scatterings from this dense system

- **eikonal approximation**
  - **Wilson lines**

DIS total cross section (structure functions)

quark anti-quark production in DIS at small x



$$\frac{d\sigma^{\gamma^* A\to q\bar{q}X}}{d^2\mathbf{p}\, d^2\mathbf{q}\, dy_1\, dy_2} = \frac{e^2 Q^2 (z_1 z_2)^2 N_c}{(2\pi)^7} \delta(1-z_1-z_2) \int d^8 x_\perp e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)} e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}$$

$$[S_{122'1'} - S_{12} - S_{1'2'} + 1]$$

$$\Big\{ 4 z_1 z_2 K_0(|\mathbf{x}_{12}|Q_1) K_0(|\mathbf{x}_{1'2'}|Q_1) +$$

$$(z_1^2 + z_2^2)\, \frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2'}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2'}|}\, K_1(|\mathbf{x}_{12}|Q_1) K_1(|\mathbf{x}_{1'2'}|Q_1) \Big\}$$

with

$$\mathrm{S}_{122'1'} \equiv \frac{1}{N_c} Tr\, V(\mathbf{x}_1)\, V^\dagger(\mathbf{x}_2)\, V(\mathbf{x}_{2'})\, V^\dagger(\mathbf{x}_{1'}) \qquad \mathrm{S}_{12} \equiv \frac{1}{N_c} Tr\, V(\mathbf{x}_1)\, V^\dagger(\mathbf{x}_2)$$

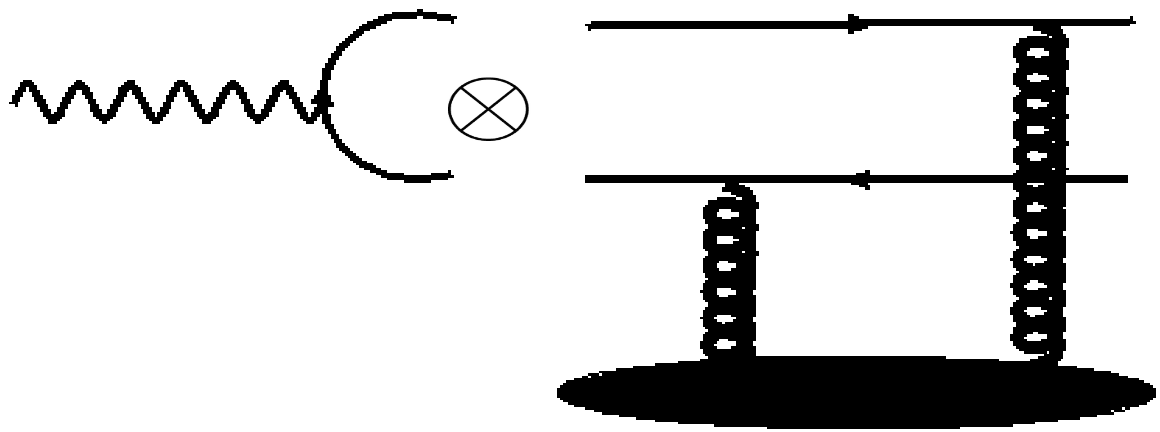
DIS total cross section

$$\sigma_{\mathbf{tot}}^{\gamma^\star\,\mathbf{T}} = \mathbf{2}\int_{\mathbf{0}}^{\mathbf{1}} \mathbf{dz} \int \mathbf{d^2x_t\, d^2y_t}\, \left|\boldsymbol{\Psi}(\mathbf{k^{\pm}}, \mathbf{k_t}|\mathbf{z}, \mathbf{x_t}, \mathbf{y_t})\right|^{\mathbf{2}} \sigma_{\mathrm{dipole}}(\mathbf{x_t}, \mathbf{y_t})$$

$$\sigma_{\mathrm{dipole}}(\mathbf{x_t}, \mathbf{y_t}) \equiv \frac{\mathbf{1}}{\mathbf{N_c}} \mathrm{Tr}\left\langle \mathbf{1} - \mathbf{V}(\mathbf{x_t})\mathbf{V}^{\dagger}(\mathbf{y_t})\right\rangle = \mathbf{1} - \mathbf{S}(\mathbf{x_t}, \mathbf{y_t})$$

$$\mathbf{r_t} \equiv \mathbf{x_t} - \mathbf{y_t}$$

given in terms of Bessel functions K_0, K_1

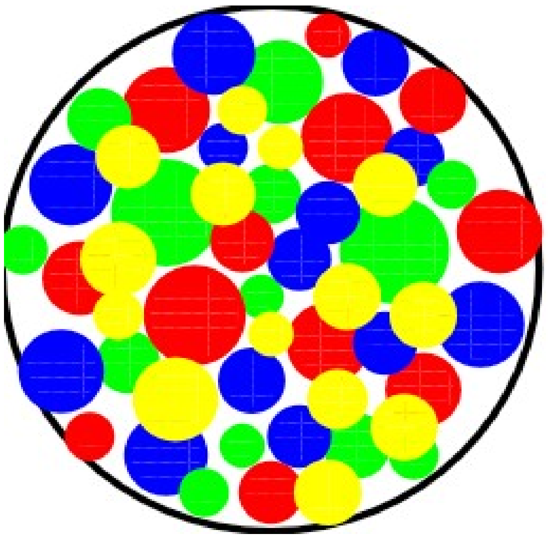


$$\mathbf{F_2} \sim \frac{\mathbf{Q^2}}{\alpha_{\mathbf{em}}}\, \sigma_{\mathbf{tot}}^{\gamma^\star\,\mathbf{T}}$$

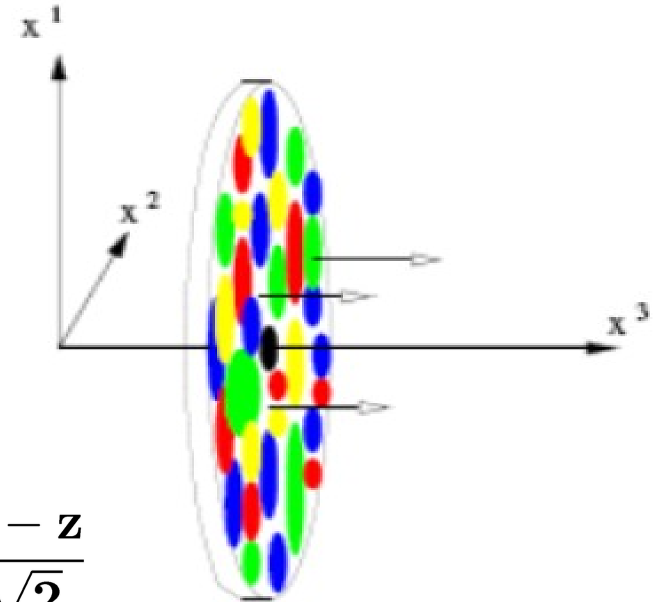
total cross section = *probability of photon decaying into a quark anti-quark pair* $\otimes$ *probability of the quark anti-quark "dipole" scattering on the target*

QED *QCD*

A model of nuclei at high energy



boost



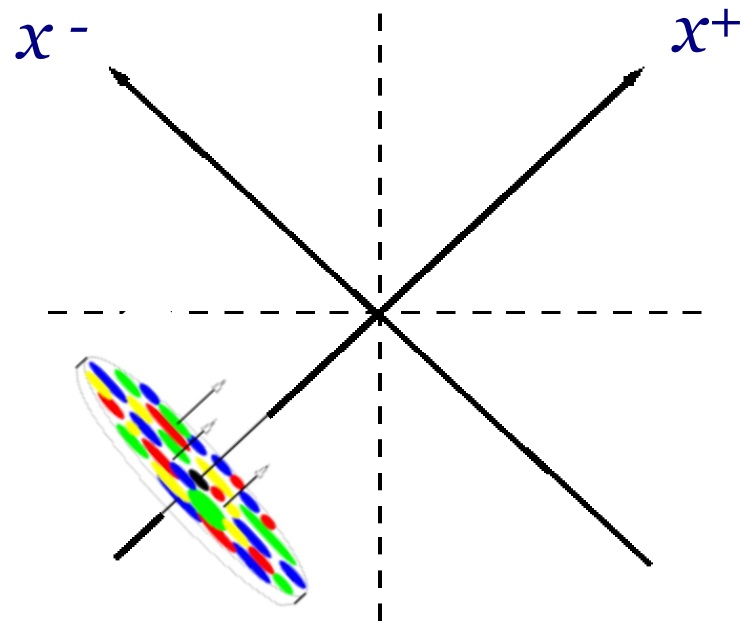
$$\mathbf{x^+} \equiv \frac{\mathbf{t+z}}{\sqrt{\mathbf{2}}} \qquad \mathbf{x^-} \equiv \frac{\mathbf{t-z}}{\sqrt{\mathbf{2}}}$$

sheet of color charge moving along x^+ and sitting at $x^- = 0$

$$\boxed{\mathbf{J^\mu_a(x)} \equiv \delta^{\mu\,+}\,\delta(\mathbf{x^-})\,\rho_\mathbf{a}(\mathbf{x_t})}$$

color current *color charge*

current has only one large component



static color charges ρ

classical equations of motion $$\mathbf{D}_\mu \mathbf{F}_\mathbf{a}^{\mu\nu} = \mathbf{g}\,\mathbf{J}_\mathbf{a}^\nu$$

solution *(in light cone gauge $A^+ = 0$) :*

$$\begin{aligned} A_a^- &= 0 \\ A_i^a &= \theta(x^-)\,\alpha_i^a(x_t) \end{aligned}$$

with

$$\begin{aligned} \partial_i \alpha_i^a(x_t) &= g\,\rho^a(x_t) \\ \alpha_i &= \frac{i}{g} U(x_t)\partial_i U^\dagger(x_t) \end{aligned}$$

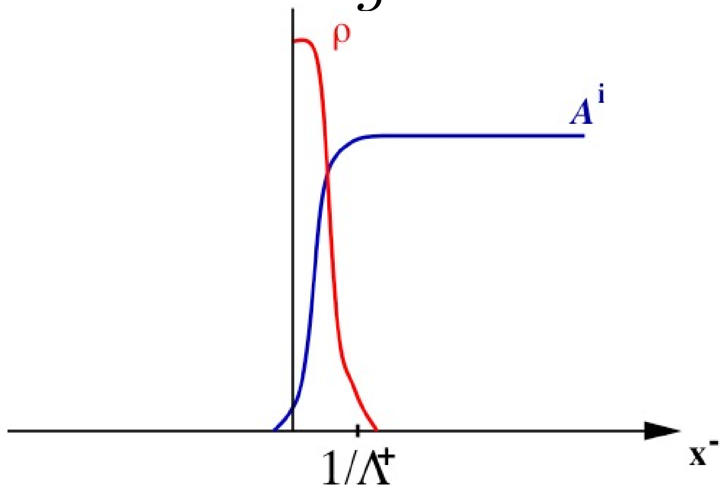
solution is a 2-d pure gauge

it is (LC) time-independent

the only "physical" color field is

$$\mathbf{F}^{+\,\mathbf{i}} \sim \delta(\mathbf{x}^-)\alpha^\mathbf{i} \neq \mathbf{0}$$

$$(\mathbf{F}^{\mathbf{i}\,\mathbf{j}} = \mathbf{0})$$



color averaging

McLerran-Venugopalan (93) $$< \mathbf{O}(\rho) > \equiv \int \mathbf{D}[\rho]\, \mathbf{O}(\rho)\, \mathbf{W}[\rho]$$

$$\mathbf{W}[\rho] \simeq \mathbf{e}^{-\int \mathbf{d^2 x_t} \frac{\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{a}}(\mathbf{x_t})}{\mathbf{2}\,\mu^{\mathbf{2}}}} \qquad \mu^{\mathbf{2}} \equiv \frac{\mathbf{g^2\, A}}{\mathbf{S}_\perp}$$

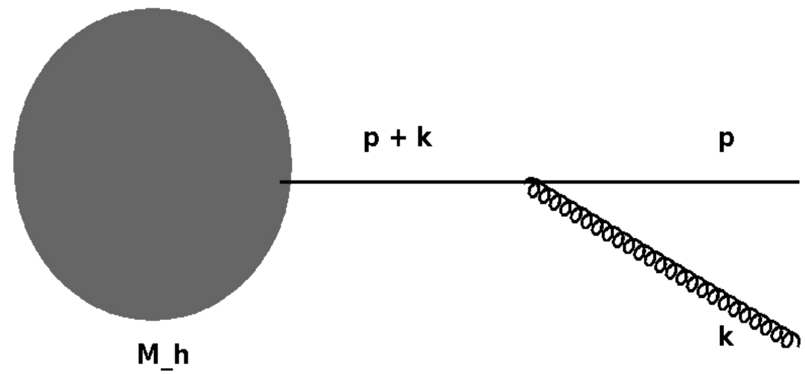
$$\mathbf{T(r_t)} \equiv \frac{\mathbf{1}}{\mathbf{N_c}} < \mathbf{Tr}\,[\mathbf{1} - \mathbf{V(r_t)^\dagger\, V(0)}] > \sim \mathbf{1} - \mathbf{e}^{-[\mathbf{r_t^2\, Q_s^2}]\,\mathbf{log}(\frac{\mathbf{1}}{\mathbf{r_t}\, \mathbf{\Lambda_{QCD}}})}$$

$$\mathbf{r_t} \equiv \mathbf{x_t} - \mathbf{y_t}$$

$r_t \ll \frac{1}{Q_s}$ $\quad T(r_t) \longrightarrow r_t^2\, Q_s^2\, log(\frac{1}{r_t\, \Lambda_{QCD}})$ *color transparency*

$r_t \gg \frac{1}{Q_s}$ $\quad T(r_t) \longrightarrow 1$ *perturbative unitarization*

One-loop corrections: soft gluon radiation (k << p)



consider a very energetic quark emerging from a hard process M_h

$$M = \bar{u}(p)\, igt^a \not\epsilon^\lambda(k) \frac{i(\not p + \not k)}{(p+k)^2 + i\epsilon} M_h$$

$$\simeq -2gt^a \bar{u}(p)\, \epsilon^\lambda(k)\cdot p \frac{1}{2p\cdot k}$$

$$\simeq -2gt^a \frac{\vec{k}_\perp \cdot \vec{\epsilon}^{\,\lambda}_\perp}{k_\perp^2} \bar{u}(p)\, M_h$$

using

$$\not\epsilon \not p = -\not p \not\epsilon + 2\epsilon\cdot p$$

$$\bar{u}(p) \not p = 0$$

$$\epsilon^\lambda_\mu(k) = \left(0, \frac{\vec{k}_\perp \cdot \vec{\epsilon}^{\,\lambda}_\perp}{k^+}, \vec{\epsilon}^{\,\lambda}_\perp\right)$$

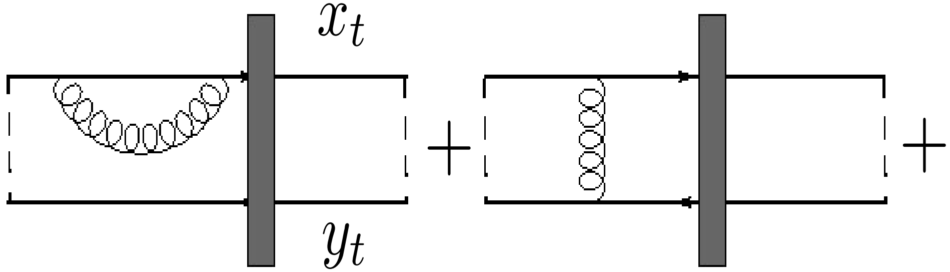
in coordinate space

$$\int \frac{d^2k_\perp}{(2\pi)^2} e^{ik_\perp\cdot(x_\perp - z_\perp)} \frac{\epsilon^\lambda_\perp \cdot k_\perp}{k_\perp^2} = \frac{i}{2\pi} \frac{\epsilon^\lambda_\perp\cdot(x_\perp - z_\perp)}{(x_\perp - z_\perp)^2}$$

$\mathbf{x}_\perp$, $\mathbf{z}_\perp$ are coordinates of quark and gluon

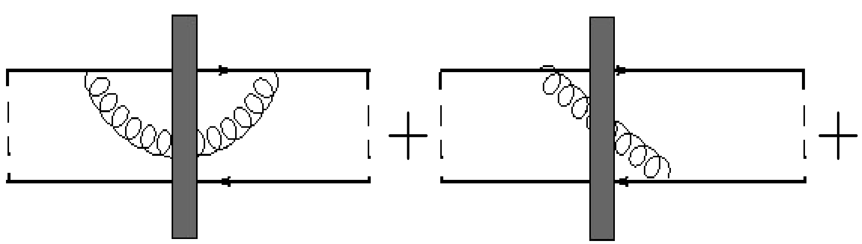
One-loop corrections: energy (x) dependence

virtual corrections:



$$\longrightarrow Tr\, V(x_t)\, V^\dagger(y_t)$$

real corrections:



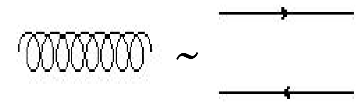
$$\longrightarrow Tr\, V(x_t)\, V^\dagger(z_t)\, Tr\, V(z_t)\, V^\dagger(y_t)$$

$$\frac{1}{(x_t - z_t)^2} \qquad \frac{(x_t - z_t)\cdot(y_t - z_t)}{(x_t - z_t)^2 (y_t - z_t)^2}$$

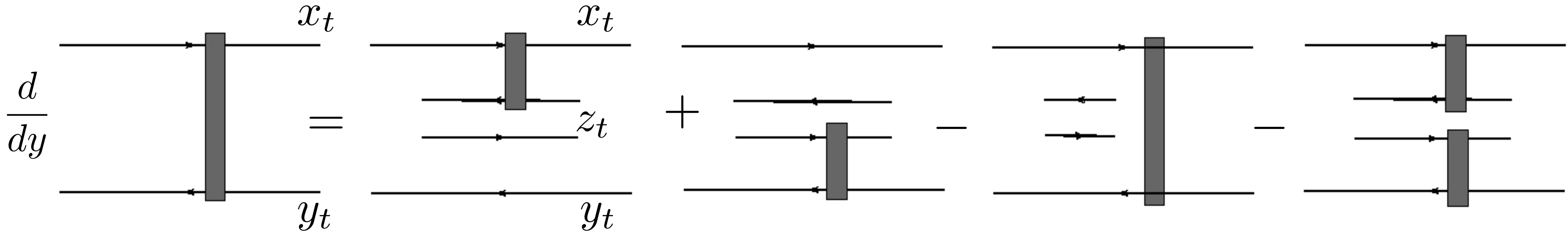
One-loop correction: Balitsky-Kovechegov (BK) equation

at large N_c

$$3 \otimes \bar{3} = 8 \oplus 1 \simeq 8$$



$$T \equiv 1 - S$$



$$\frac{d}{dy} T(x_t, y_t) = \frac{N_c \alpha_s}{2\pi^2} \int d^2 z_t \frac{(x_t - y_t)^2}{(x_t - z_t)^2 (y_t - z_t)^2} \left[T(x_t, z_t) + T(z_t, y_t) - T(x_t, y_t) - T(x_t, z_t) T(z_t, y_t) \right]$$

linear (BFKL) non-linear

both T = 0 and T = 1 are fixed points

unstable ↗ (T = 0)

stable ↖ (T = 1)

Solution of BFKL evolution equation

$$\frac{d}{dy}T(x_t,y_t)=\frac{N_c\alpha_s}{2\pi^2}\int d^2z_t\frac{(x_t-y_t)^2}{(x_t-z_t)^2(y_t-z_t)^2}\left[T(x_t,z_t)+T(z_t,y_t)-T(x_t,y_t)\right]$$

expand T in terms of gluon field A and write in momentum space

unintegrated gluon distribution $\phi(x,k_\perp)$

$$\phi(\mathbf{x},\mathbf{k}_\perp)\sim(\frac{\mathbf{1}}{\mathbf{x}})^{\alpha_\mathbf{P}-\mathbf{1}}\,\mathbf{exp}\left\{-\frac{\ln^{\mathbf{2}}(\frac{\mathbf{k}_\perp}{\mathbf{\Lambda_{QCD}}})}{\bar{\alpha}\,\ln(\mathbf{1}/\mathbf{x})}\right\}$$

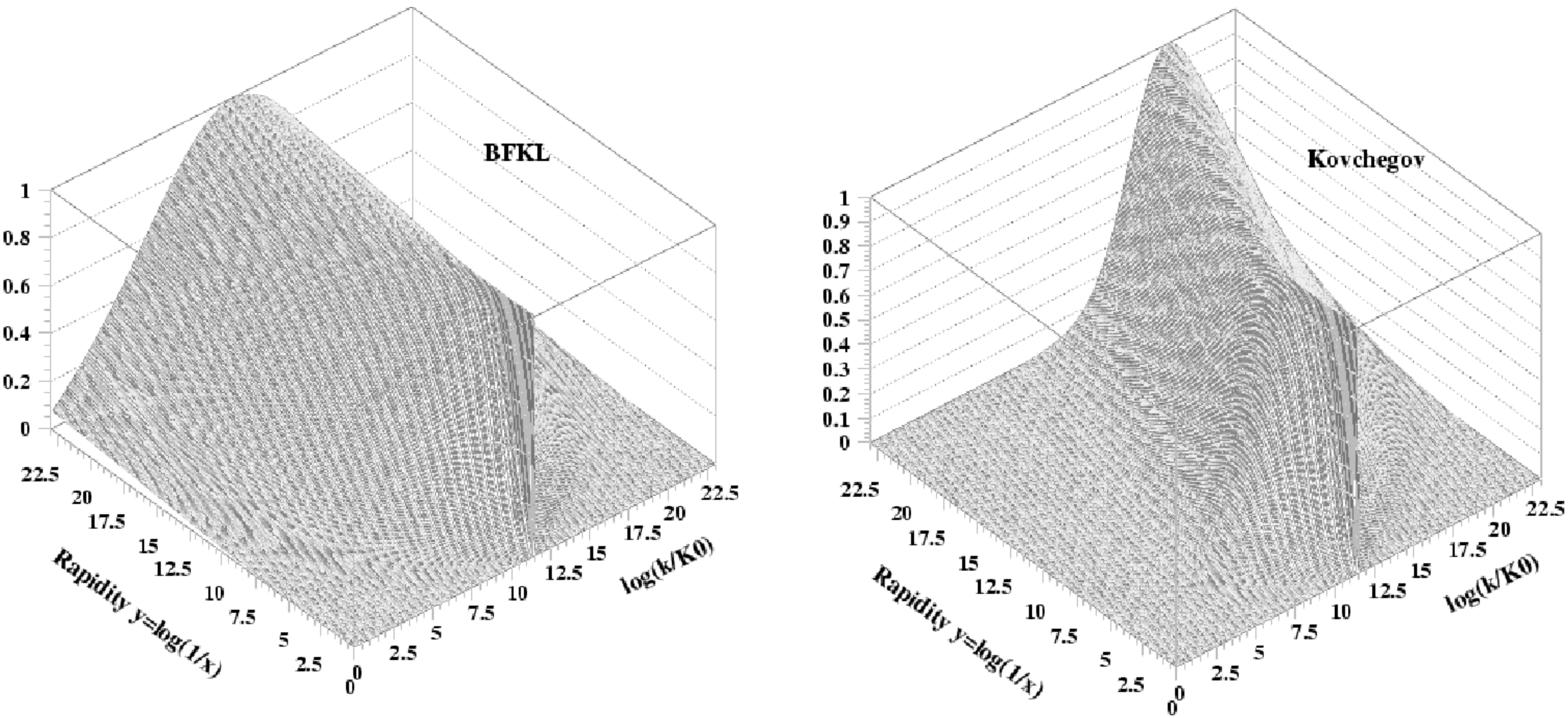
with

$$\alpha_\mathbf{P}-\mathbf{1}=\frac{\mathbf{4}\,\mathbf{N_c}\,\alpha_\mathbf{s}}{\pi}\,\ln\mathbf{2}$$

BFKL predicts a fast rise of gluon distribution: unitarity?

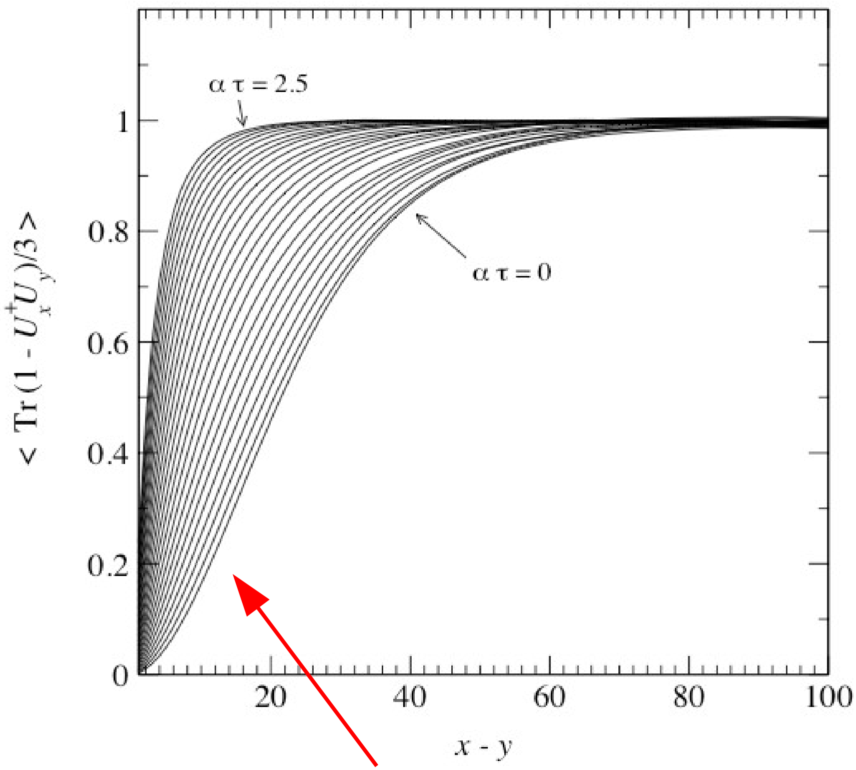
diffusion of momenta into infrared?

Solution of BK evolution equation



A. Stasto et al.

Solution of BK evolution equation



$\sim \mathbf{r_t^2\, xG(x, 1/r_t^2)}$

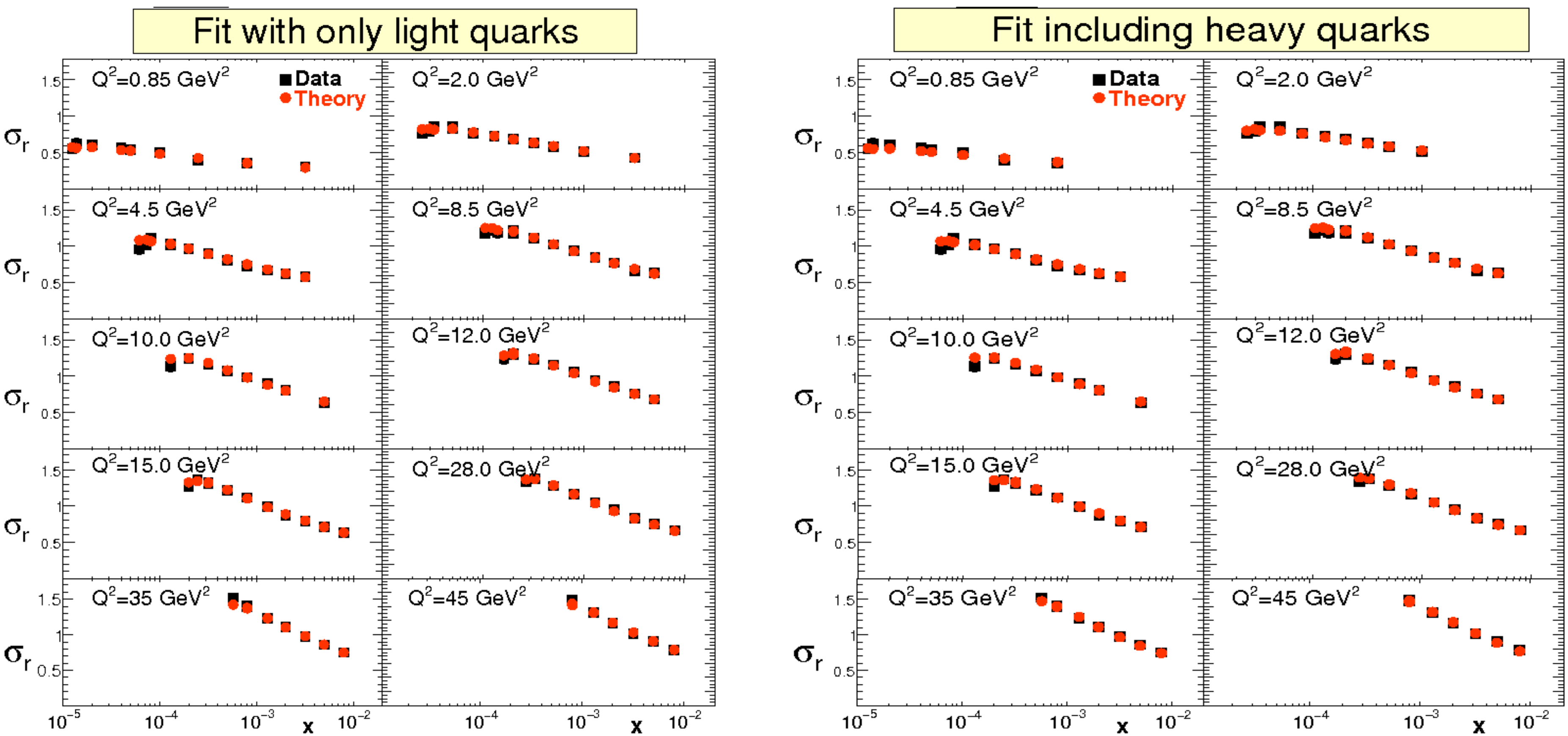
$$\tilde{T}(p_t) \sim \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right] \qquad Q_s^2 \ll p_t^2$$

$$\tilde{T}(p_t) \sim \log\left[\frac{Q_s^2}{p_t^2}\right] \qquad Q_s^2 \gg p_t^2$$

$$\tilde{T}(p_t) \sim \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right]^{\gamma} \qquad Q_s^2 < p_t^2$$

Rummukainen-Weigert, NPA739 (2004) 183

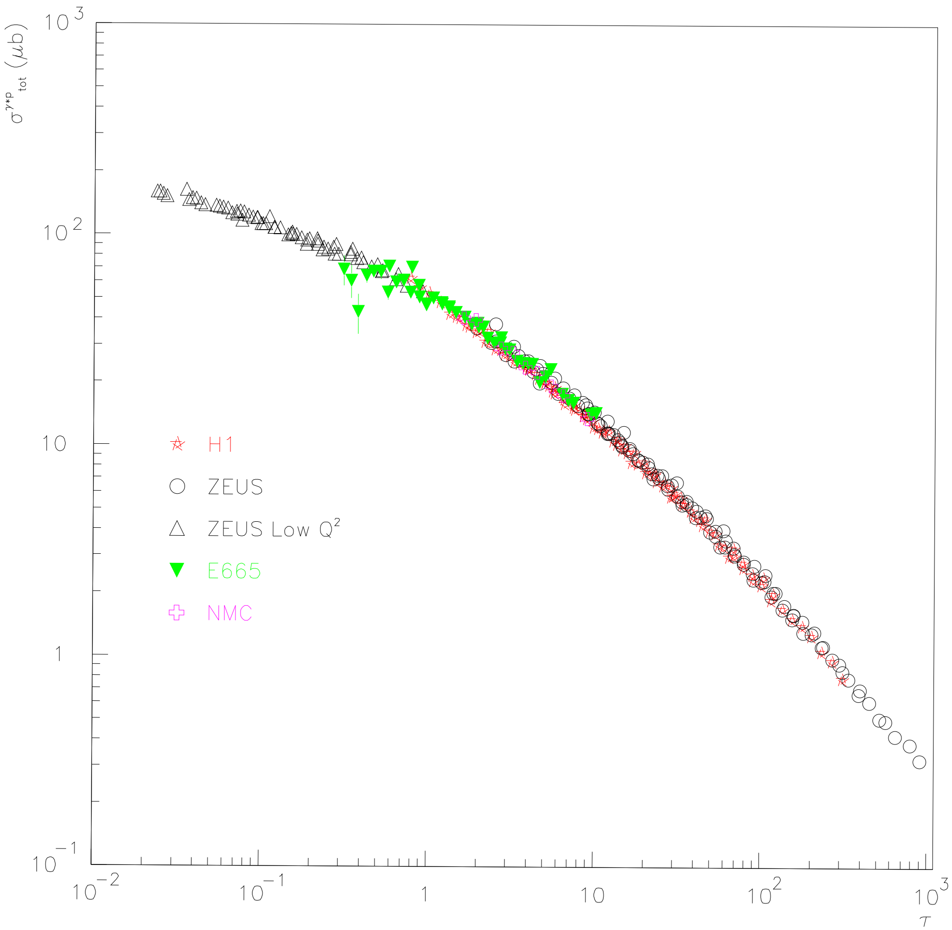
Structure functions at HERA



AAMQS(2010)

PQCD: DGLAP-based approaches also "work" : need more discriminatory observables

CGC at HERA? Extended scaling



SG-BK
PRL86 (2001) 596

Collinear Fact.:

$$\sigma = \sigma(\mathbf{x}, \mathbf{Q^2})$$

CGC:

$$\sigma = \sigma(\mathbf{Q^2}/\mathbf{Q_s^2})$$

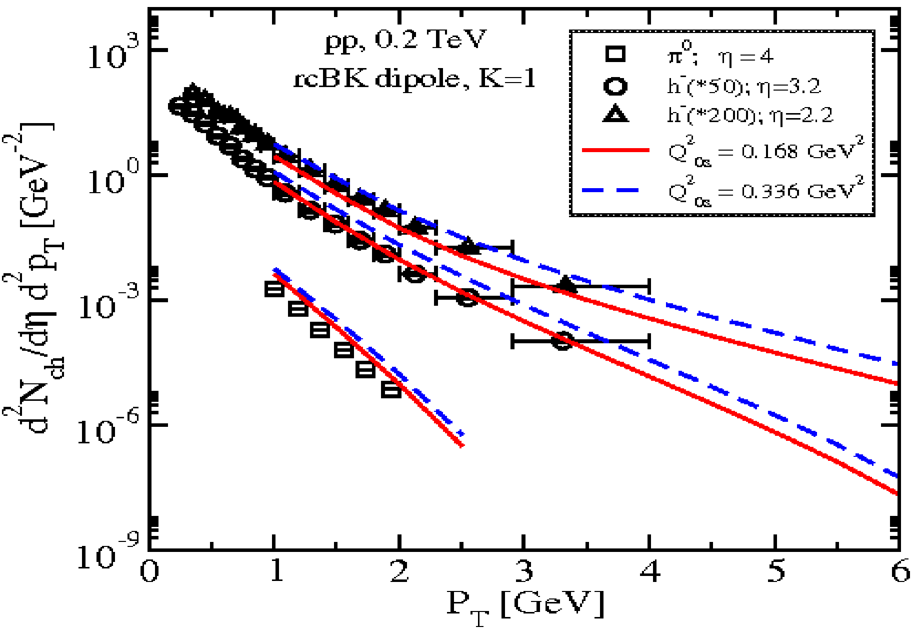
$$Q_s << Q << \frac{Q_s^2}{\Lambda}$$

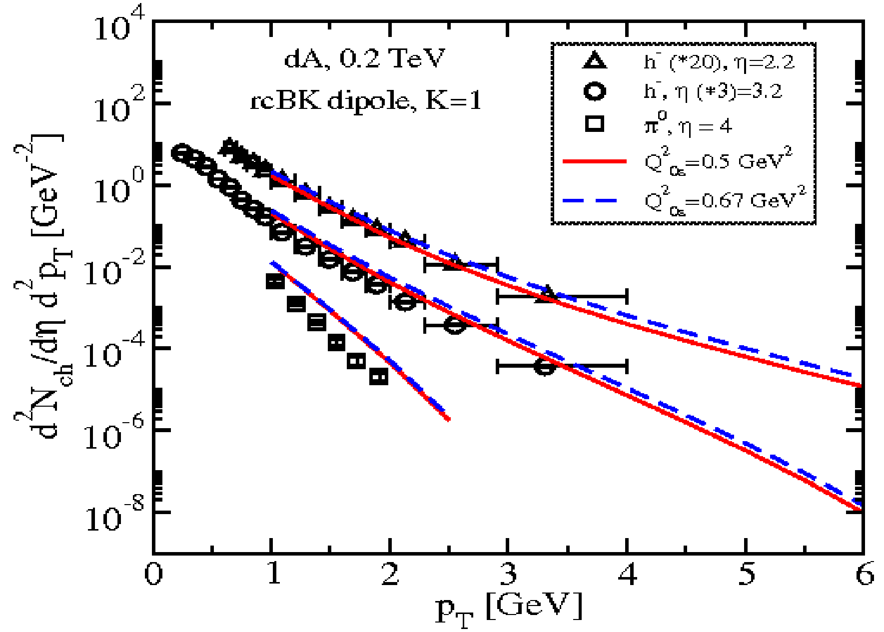
$$\mathbf{Q_s^2 = 1\,GeV^2[x_0/x]^{\lambda}}$$

$$\mathbf{x_0 = 3X10^{-4}}$$

Single inclusive hadrons at RHIC

J. Jalilian-Marian, A. Rezaeian
PRD85 (2012) 0140017





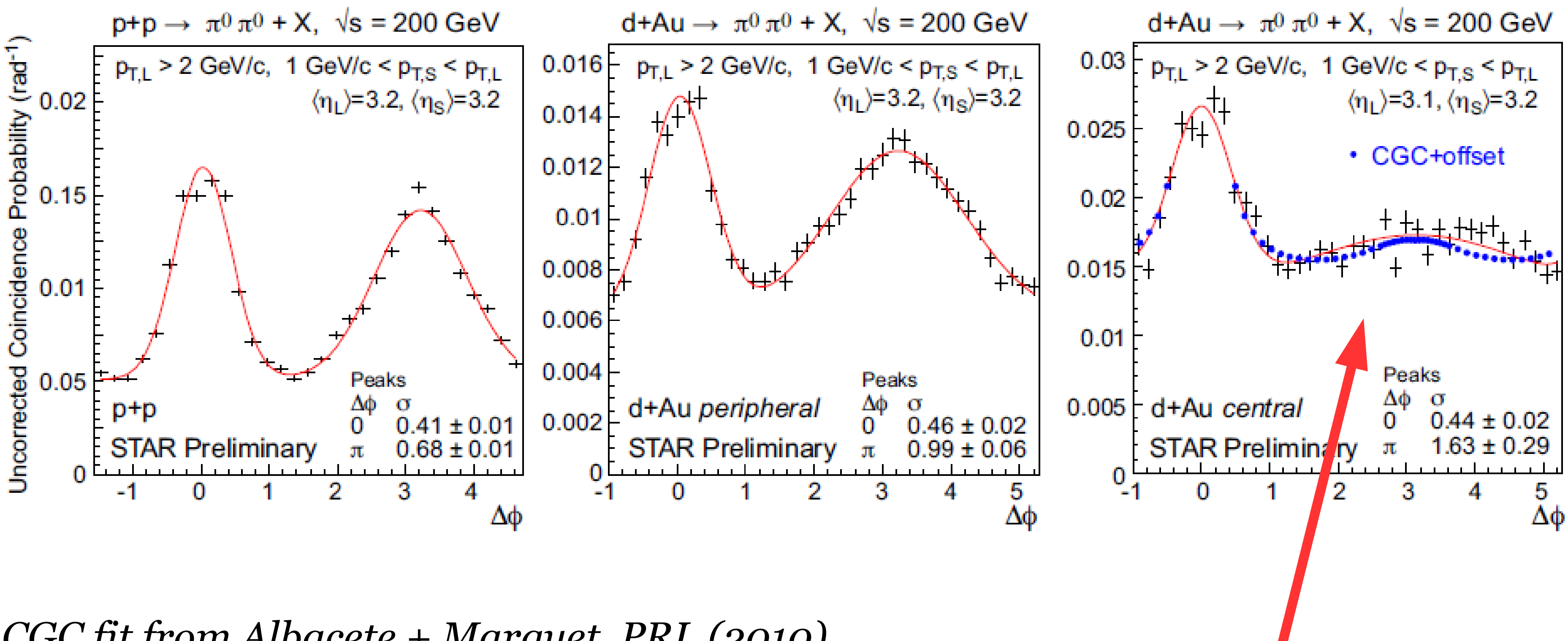
how about centrality dependence?:
Woods-Saxon, fluctuations

role of initial conditions in rcBK,
many CGC-based models also fit the data,
k-factors

cold matter energy loss?
Kopeliovich, Frankfurt and Strikman
Neufeld,Vitev,Zhang, PLB704 (2011) 590

disappearance of back to back hadrons in pA collisions

Recent STAR measurement (arXiv:1008.3989v1):

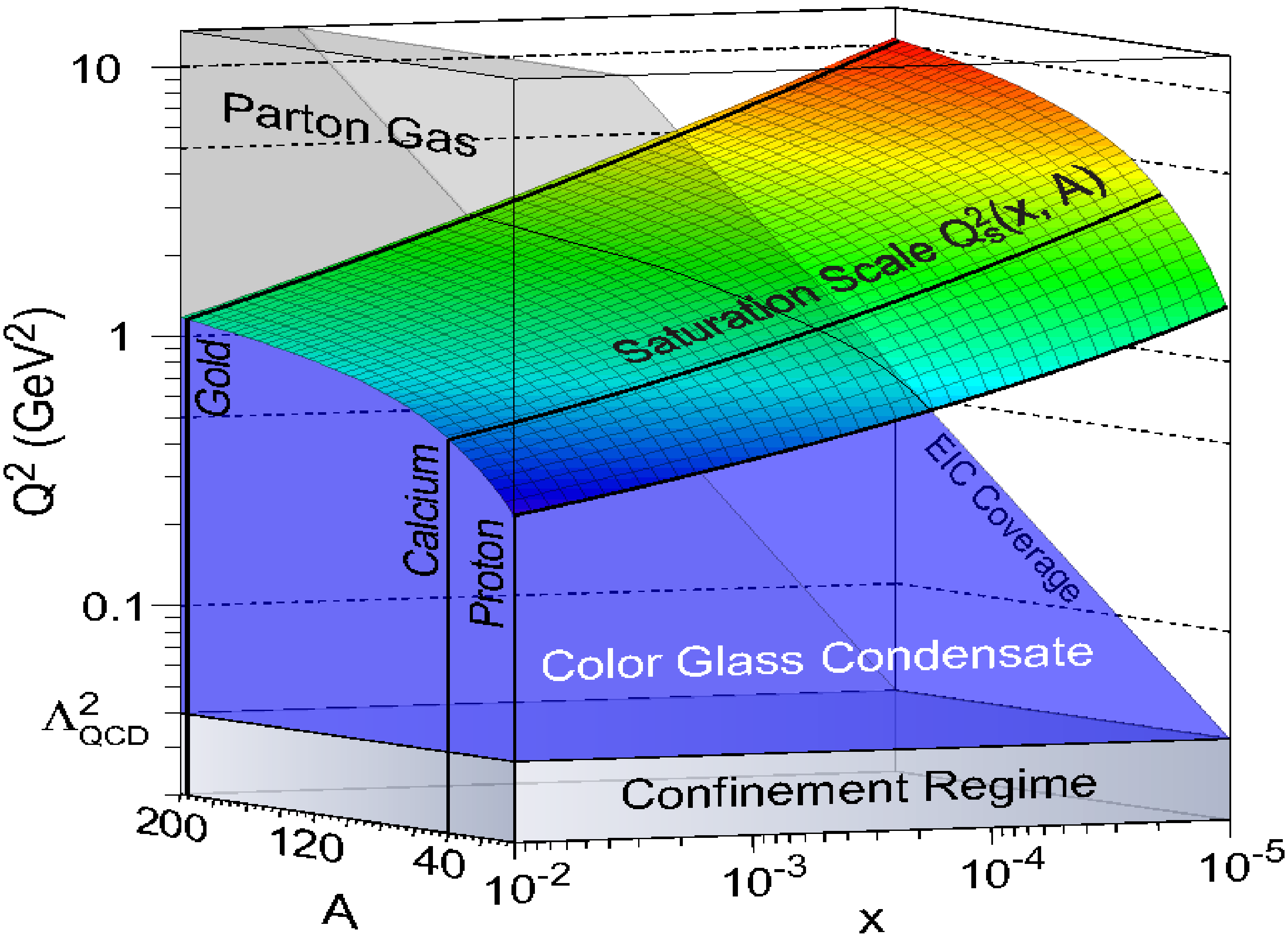


broadening + reduction

CGC fit from Albacete + Marquet, PRL (2010)
Tuchin, NPA846 (2010)
A. Stasto, B-W. Xiao, F. Yuan, PLB716 (2012)
T. Lappi, H. Mantysaari, NPA908 (2013)

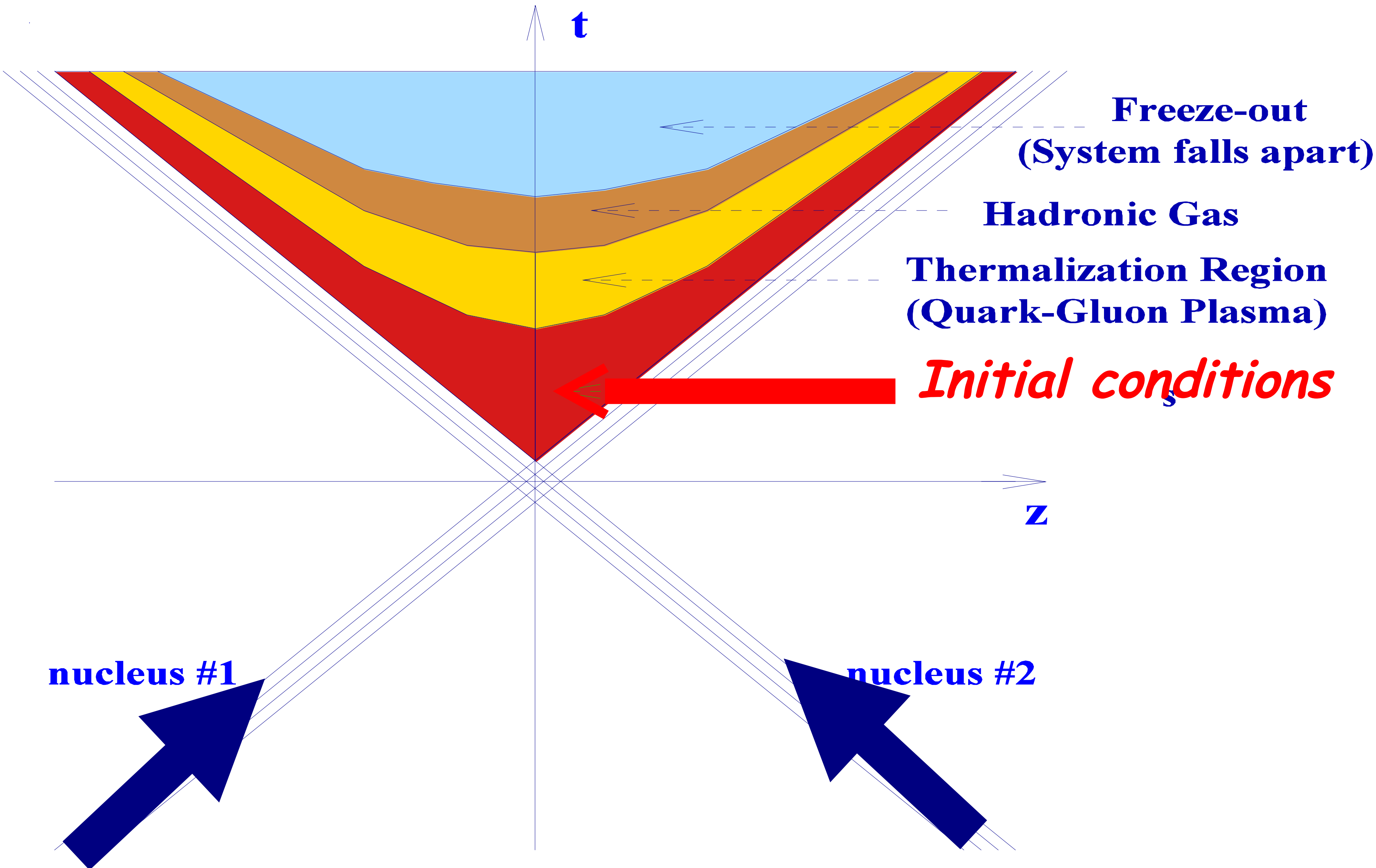
shadowing+energy loss: Z. Kang, I. Vitev, H. Xing, PRD85 (2012) 054024

The Saturation Scale Q_s

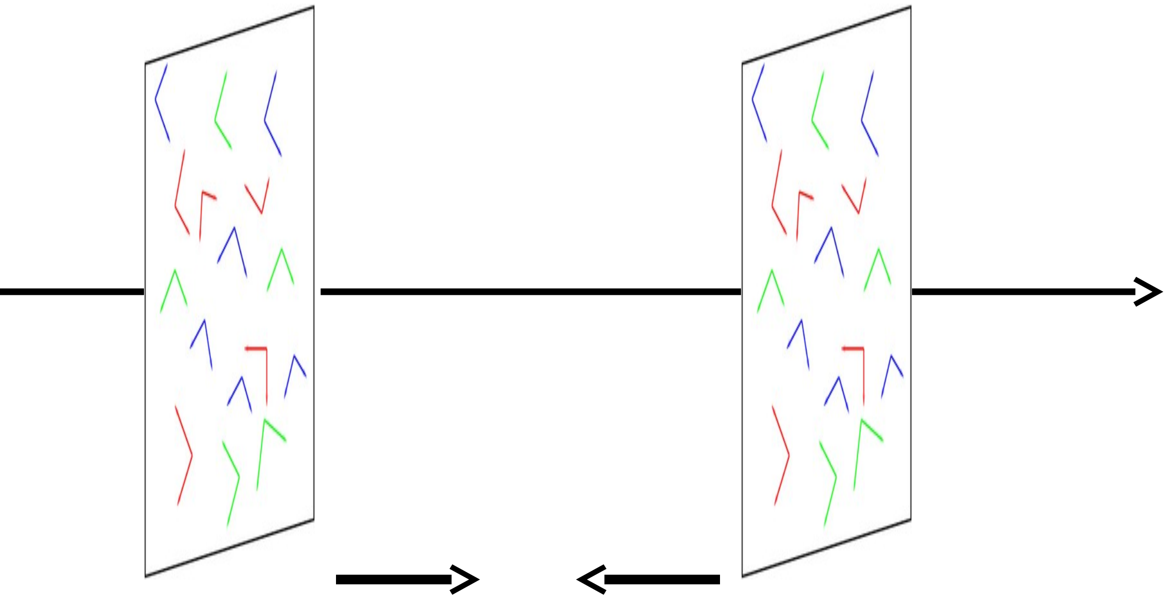


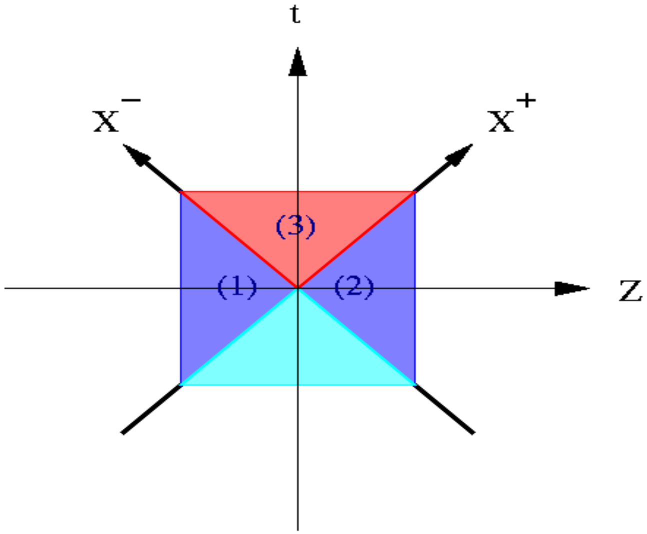
X 9/4 for gluons

Space-Time History of a Heavy Ion Collision



Heavy Ion Collisions at High Energy:
Colliding Sheets of Color Glass





before the collision:

$$\mathbf{A^+ = A^- = 0}$$

$$\mathbf{A^i = A^i_1 + A^i_2}$$

$$\mathbf{A^i_1} = \theta(\mathbf{x^-})\theta(-\mathbf{x^+})\alpha^\mathbf{i}_\mathbf{1}$$

$$\mathbf{A^i_2} = \theta(-\mathbf{x^-})\theta(\mathbf{x^+})\alpha^\mathbf{i}_\mathbf{2}$$

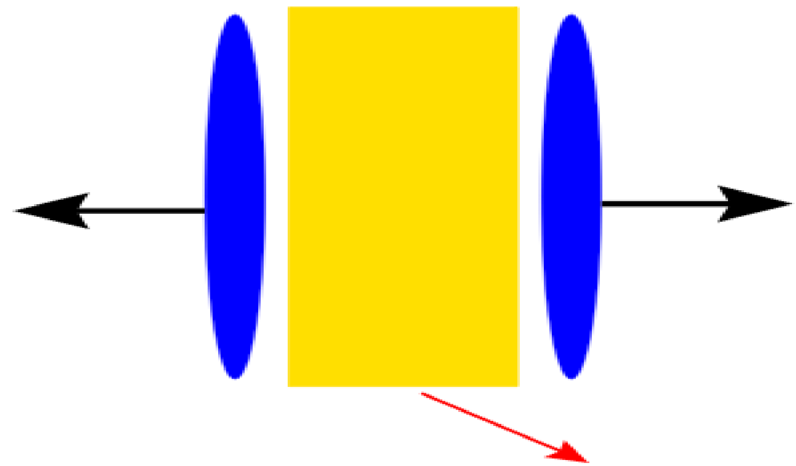
after the collision:

solve for A_μ

in the forward LC

Colliding Sheets of Color Glass at High Energies

solve the classical eqs. of motion in the forward light cone: subject to initial conditions given by one nucleus solution

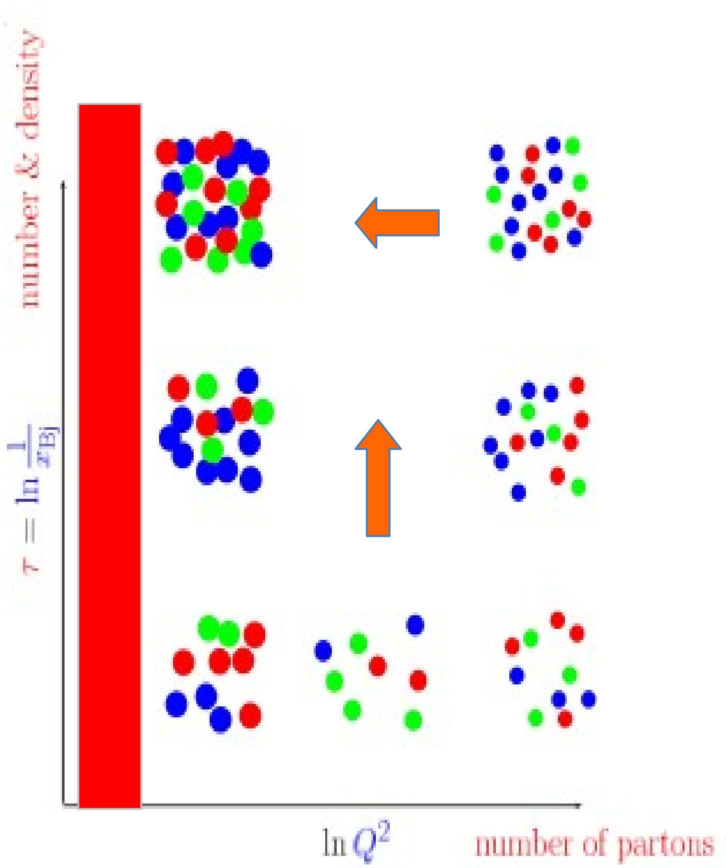


GLASMA:strong color fields with occupation number ~ $\frac{1}{\alpha_s}$

initial energy and multiplicity of produced gluons depend on Q_s

$$\frac{1}{A_\perp}\frac{dE_\perp}{d\eta} = \frac{0.25}{g^2}Q_s^3 \qquad\qquad \frac{1}{A_\perp}\frac{dN}{d\eta} = \frac{0.3}{g^2}Q_s^2$$

Low x QCD:
many-body dynamics of universal gluonic matter (CGC)



How does this happen ?

How do correlation functions of these evolve ?

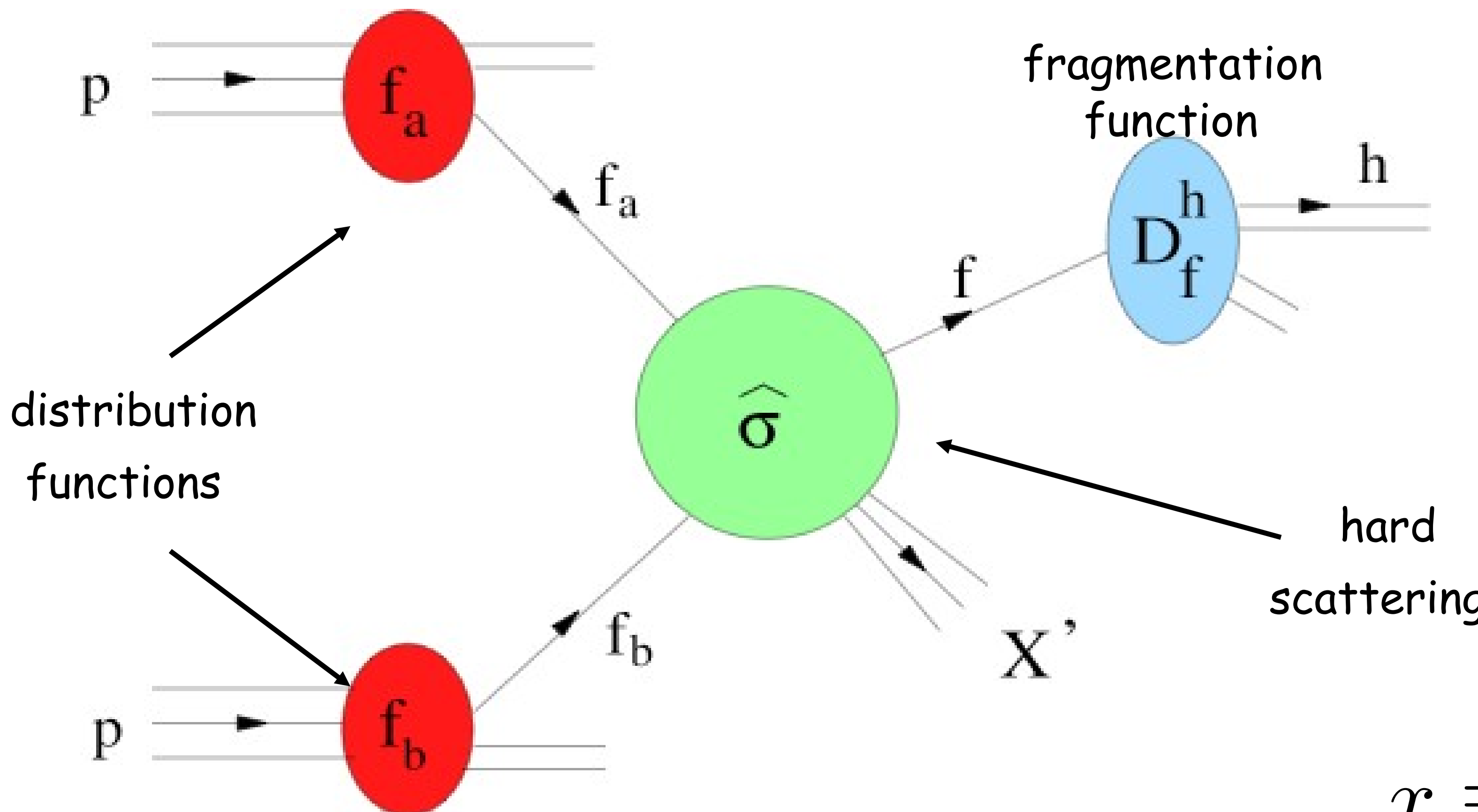
Are there scaling laws ?

Can CGC explain aspects of HEC ?

Initial conditions for hydro?
Thermalization ?
Long range rapidity correlations ?
Azimuthal angular correlations ?
Nuclear modification factor ?

pQCD in pp Collisions

collinear factorization: separation of soft (long distance) and hard (short distance)



$$x \equiv \frac{p}{P}$$

$$\frac{d\sigma^{pp\to h\,X}}{d^2p_t\,dy} \sim f_a(x_1) \otimes f_b(x_2) \otimes \hat{\sigma} \otimes D_f^h(z) + \cdots\cdots$$

power corrections