Introduction to physics of EIC
QCD and DIS

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Basic books

- S. Weinberg, *Foundation of Modern Physics*, Cambridge Univ. Press 2021
- D. Griffiths, *Introduction to Elementary Particles*, WILLEY-WCH, 2008
- F. Halzen, A. D. Martin, *Quarks and Leptons*, WILEY and SONS 1984
- R. K. Ellis, W.J. Stirling, B. R. Webber, *QCD and Collider Physics*, Cambridge University Press 1996
- R. G. Roberts, *The Structure of the Proton*, Cambridge Univ. Press 1990

Quantum Chromodynamics

Gell-Man - Zweig quark model

- Quarks - provide fundamental representations 3 and $\bar{3}$ of $SU(3)_f$

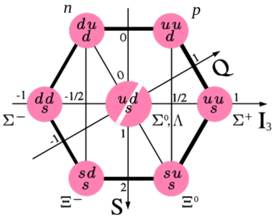
Quark flavour	I_3	B	S	$Y = B + S$	$Q = I_3 + Y/2$
u	$1/2$	$1/3$	0	$1/3$	$2/3$
d	$-1/2$	$1/3$	0	$1/3$	$-1/3$
s	0	$1/3$	-1	$-2/3$	$-1/3$

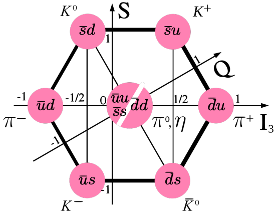
- Hadrons from $SU(3)_f$ representation composition:

$$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus \bar{8} \oplus 10 \qquad \textit{baryons } B = 1$$

$$3 \otimes \bar{3} = 1 \oplus 8 \qquad \textit{mesons } B = 0$$

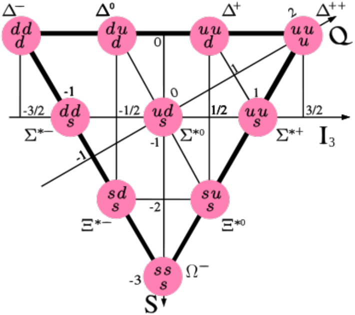
- Baryon (spin 1/2) octet and meson (spin 0) nonet





Arguments for QCD construction

- Baryon (spin 3/2) decuplet



- Fermionic state $|\Delta^{++}, J_3 = 3/2\rangle = |u\uparrow u\uparrow u\uparrow\rangle$ is symmetric - need of additional quantum number which restores antisymmetry.
- Deep Inelastic *ep* Scattering (DIS) provides an evidence for quark existence - elastic scattering on a point-like and free particle with spin 1/2.

ADVANTAGES OF THE COLOR OCTET GLUON PICTURE ☆

H. FRITZSCH*, M. GELL-MANN and H. LEUTWYLER**
California Institute of Technology, Pasadena, Calif. 91109, USA

Received 1 October 1973

It is pointed out that there are several advantages in abstracting properties of hadrons and their currents from a Yang–Mills gauge model based on colored quarks and color octet gluons.

- For each flavour $f = u, d, s, c, b, t$, quark fields form $SU(3)_c$ color triplet

$$\Psi_f(x) = \begin{pmatrix} \psi_{f1} \\ \psi_{f2} \\ \psi_{f3} \end{pmatrix} \qquad\qquad (1,2,3) = (R,G,B)$$

- Quarks interact with colored gluons - octet of $SU(3)_c$ gauge fields

$$\hat{A}_\mu(x) = \sum_{a=1}^{8} A^a_\mu(x)\,\hat{T}^a = \begin{pmatrix} A^3_\mu + A^8_\mu/\sqrt{3} & A^1_\mu - iA^2_\mu & A^4_\mu - iA^5_\mu \\ A^1_\mu + iA^2_\mu & -A^3_\mu + A^8_\mu/\sqrt{3} & A^6_\mu - iA^7_\mu \\ A^4_\mu + iA^5_\mu & A^6_\mu + iA^7_\mu & -2A^8_\mu/\sqrt{3} \end{pmatrix}$$

- Local gauge transformation of quark fields and their covariant derivatives, $U(x) \in SU(3)\,, x = (x^\mu)\,, \partial_\mu = \partial/\partial x^\mu$,

$$\Psi'_f(x) = U(x)\,\Psi_f(x)$$
$$(\partial_\mu + ig\hat{A}'_\mu(x))\,\Psi'_f(x) = U(x)\,(\partial_\mu + ig\hat{A}_\mu(x))\,\Psi_f(x)$$

- This gives inhomogeneous transformation law for gluon fields

$$\hat{A}'_\mu = U\,\hat{A}_\mu U^\dagger + \frac{i}{g}(\partial_\mu U)U^\dagger$$

- Field strength (3×3 hermitian and traceless matrix)

$$\hat{F}_{\mu\nu} = \sum_{a=1}^{8} F^a_{\mu\nu}\,\hat{T}^a = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu + ig[\hat{A}_\mu, \hat{A}_\nu]$$
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\nu - gf^{abc}A^b_\mu A^c_\nu$$

- Homogeneous local gauge transformation law

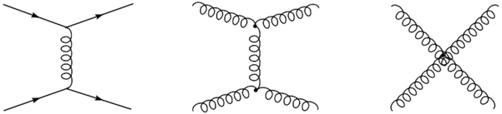
$$\hat{F}'_{\mu\nu} = U\,\hat{F}_{\mu\nu}\,U^\dagger$$

QCD interactions

- QCD lagrangian is invariant under local $SU(3)_c$ gauge transformations

$$\mathcal{L} = \sum_f i\left\{\overline{\Psi}_f\gamma^\mu\left(\partial_\mu + ig\hat{A}_\mu\right)\Psi_f - m_f\overline{\Psi}_f\Psi_f\right\} - \frac{1}{4}\mathrm{Tr}(\hat{F}_{\mu\nu}\hat{F}^{\mu\nu})$$

- Basic vertices:

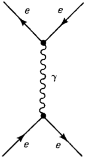


- Gluon self-interactions $\sim g$ and g^2 - highly non-linear theory
- Quark masses are generated by Higgs mechanism
- QED if the gauge group $SU(3) \to U(1) = \{U = \mathrm{e}^{i\phi}; \phi \in \mathbf{R}\}$

$$A^a_\mu(x) \to A_\mu(x) \to photon$$

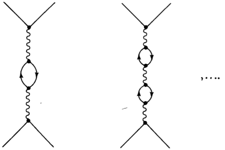
Running couplings in QFT

- In renormalized QFT couplings run: $g \to g(\mu)$, $m \to m(\mu), \ldots$
- Coupling constant e_0 renormalization in QED:



$$V(q) = \frac{e_0^2}{4\pi}\frac{1}{q^2} = \frac{\alpha_0}{q^2}$$

- Resummation is needed: $1 + x + x^2 + x^3 + \ldots = 1/(1-x)$



$$V(q) = \frac{e^2(q)}{4\pi}\frac{1}{q^2} = \frac{\alpha(q)}{q^2}$$

- Running coupling constant:

$$\alpha(q) = \frac{\alpha_0}{1 - \frac{\alpha_0}{3\pi}\ln\left(\frac{m^2 - q^2}{\Lambda_{UV}^2}\right)}$$

Coupling constant renormalization in QED

- Compute $\alpha(q)$ at spatial infinty $r = \infty \;\Leftrightarrow\; q^2 = 0$

$$\alpha(0) = \frac{\alpha_0}{1 - \frac{\alpha_0}{3\pi} \ln\left(\frac{m^2}{\Lambda_{UV}^2}\right)} \approx \frac{1}{137}$$

- Renormalization: relate $\alpha(q)$ to $\alpha(0)$

$$\alpha(q) = \frac{\alpha(0)}{1 - \frac{\alpha(0)}{3\pi} \ln\left(\frac{m^2 - q^2}{m^2}\right)}$$

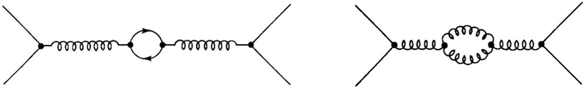
- With $-q^2 \to \infty$ we have $\alpha(q) \to \infty$. Landau pole for

$$\frac{\alpha(0)}{3\pi} \ln\left(\frac{m^2 - q^2}{m^2}\right) = 1 \qquad \Rightarrow \qquad \frac{-q^2}{m^2} = 1 + \exp\left(3\pi/\alpha_0\right) \approx 10^{555}$$

- QED interactions are getting stronger with decreasing distance.

Asymptotic freedom in QCD

- In QCD we have to include gluon diagrams with self-interactions



- Result of the Nobel Prize calculation - Gross, Politzer, Wilczek, 1974

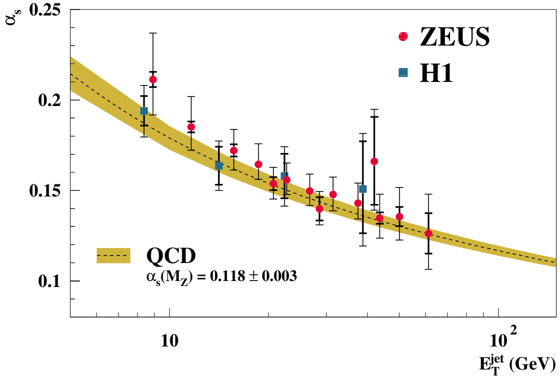
$$\alpha_s(Q) = \frac{g^2(Q)}{4\pi} = \frac{\alpha_s(\mu)}{1 + \frac{\alpha_s(\mu)}{12\pi}(11N_c - 2n_f)\ln\left(\frac{Q^2}{\mu^2}\right)} \quad \xrightarrow{Q^2\to\infty} \quad 0$$

- QCD interactions are getting weaker with decreasing distance.
- Intrinsic scale of QCD: $\Lambda_{QCD} \sim 300 - 400\,\mathrm{MeV}$

$$\alpha_s(Q) = \frac{12\pi}{(11N_c - 2n_f)\ln(Q^2/\Lambda^2_{QCD})}$$

- Perturbative QCD makes sense only for $Q^2 \gg \Lambda^2_{QCD}$ when $\alpha_s < 1$.
- Infrared Landau pole for $Q = \Lambda_{QCD}$ in the confinement region.

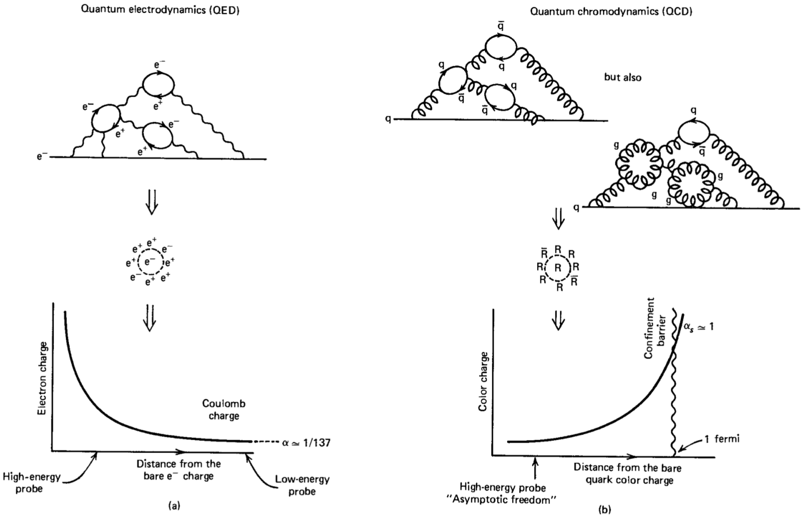
- From high p_T jet production in DIS at HERA, 1992-2007



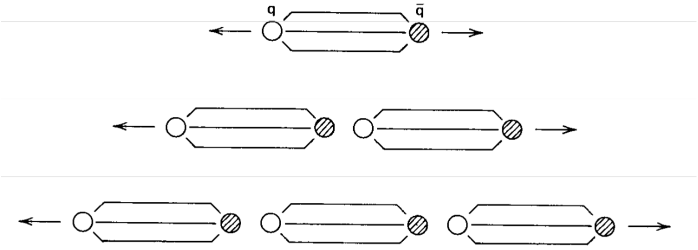
- In here $Q = E_T^{jet}$ while $\mu = M_z$.

QED versus QCD

From F. Halzen, A. D. Martin, *Quarks and Leptons*



- At large distances (at the detectors) only color singlets are observed



- Decay: $\Delta^{++} \to \pi^{+} \; + \; p$

$$uuu \;\to\; u\bar{d} \;+\; d\,uu$$

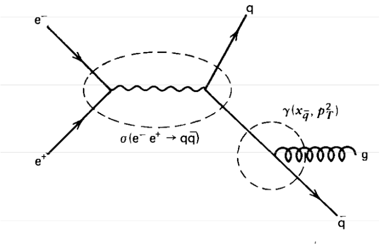
- Mathematical proof of color confinement for gluons (pure YM theory without quarks) is one of the 6 still unsolved millennium problems.

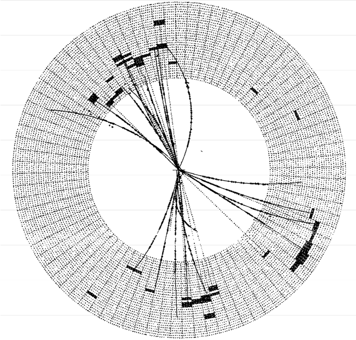
Jets

- Only colorless hadrons are in physical spectrum of QCD
- Quark and gluons are "visible"in the form of jets, e.g. in

$$e^+ + e^- \to q + \bar{q} + g$$

- From PETRA e^+e^- collider at DESY, 1979

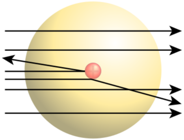




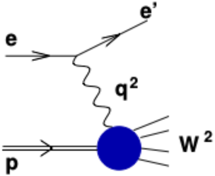
Deep Inelastic Scattering

Basic idea of DIS

- Rutherford 1912 - scattering of ^4He on gold: $\alpha + Au \rightarrow \alpha + Au$



- SLAC 1967-69 - scattering of leptons on nucleons: $e + p \rightarrow e + X$

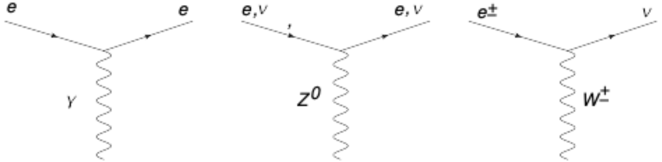


- Photon virtuality: $Q^2 \equiv -q^2 = -(k_e - k_{e'})^2 \; > \; 0$
- Invariant mass squared: $W^2 \equiv (P+q)^2 = M_X^2 \; \geqslant \; m_p^2 \simeq 1 \text{ GeV}^2$
- Deep Inelastic Scattering (DIS): $Q^2, W^2 \; > \; m_p^2$

- Scattering amplitude (λ, s are polarizations):

$$A(l\,N \to l'X) = \underbrace{\bar{u}_{l'}(k',\lambda)\,\Gamma^{\mu}\,u_l(k,\lambda)}_{leptonic\ \ current}\,\frac{ig_{\mu\nu}}{Q^2+m_B^2}\,\underbrace{\langle\,X\,|\,J^{\nu}(0)|\,P,s\,\rangle}_{hadronic\ \ ME}$$

- Leptonic current: neutral currents (NC) + charged currents (CC)



$$\Gamma^{\mu} = \qquad e\,\gamma^{\mu} \qquad e\,\gamma^{\mu}(v_l - a_l\,\gamma_5) \qquad g\,\gamma^{\mu}(1-\gamma_5)$$

- Electromagnetic interactions: P-parity conserving, $m_{\gamma} = 0$
- Weak interactions: P-parity violating, $m_Z = 91.19\ GeV$, $m_{W^{\pm}} = 80.38\ GeV$
- Strong interaction in hadronic ME

Leptonic and hadronic tensor

- Cross section for $lN \to l'X$ unpolarized DIS with γ exchange only

$$\frac{d\sigma}{dE'_e\, d\Omega'_e} \sim \frac{1}{(Q^2)^2}\, L^{\mu\nu}(k,k')\, W_{\mu\nu}(q,P)$$

- Leptonic tensor for $m_l = 0$

$$L^{\mu\nu}(k,k') = \left[k^\mu k'^{\,\nu} + k^\mu k'^{\,\nu} - g^{\mu\nu}(k\cdot k')\right] \qquad (+\, i\lambda\, \epsilon_{\mu\nu\alpha\beta}\, k^\alpha k'^\beta)$$

- Hadronic tensor

$$W_{\mu\nu}(q,P) = \sum_s \int d^4x\, e^{iq\cdot x} \langle\, P,s|\, J_\mu(x)\, J_\nu(0)\, |P,s\,\rangle$$

- Current conservation and P-parity conservation

$$q^\mu W_{\mu\nu} = 0 \qquad\qquad W_{\mu\nu} = W_{\nu\mu}$$

Structure functions

- Having q_μ, P_ν and $g_{\mu\nu}$ at the disposal:

$$W_{\mu\nu} = -\left(g_{\mu\nu} + \frac{q_\mu q_\nu}{Q^2}\right) F_1 + \frac{1}{P\cdot q}\left(p_\mu + q_\mu \frac{P\cdot q}{Q^2}\right)\left(p_\nu + q_\nu \frac{P\cdot q}{Q^2}\right) F_2$$

- Two unknown, dimesionless structure functions: $F_{1,2} = F_{1,2}(Q^2, W^2, m_p^2)$

- Weak boson exchanges: P-parity violating tensor $\epsilon_{\mu\nu\alpha\beta}$ at the disposal

$$W_{\mu\nu} \;\to\; W_{\mu\nu} + i\epsilon_{\mu\nu\alpha\beta}\frac{P^\alpha q^\beta}{P\cdot q}\, F_3$$

- Reduced cross section, e.g. for NC DIS

$$\frac{1}{\sigma_0}\frac{d\sigma^{NC}}{dE'_e\, d\Omega'_e}(e^\pm p) \;=\; \tilde{F}_2 - \frac{y^2}{Y_+}\tilde{F}_L \mp \frac{xY_-}{Y_+}\tilde{F}_3$$

- Above: $\tilde{F}_L = \tilde{F}_2 - 2x\tilde{F}_1$ and x, y are kinematic variables

$$0 < x, y < 1\,, \qquad\qquad Y_\pm = 1 \pm (1-y)^2$$

Kinematic variables in DIS

- Lorentz invariant Bjorken variable: $(P_\mu \stackrel{\text{Lab}}{=} (m_p, 0, 0, 0))$

$$x = \frac{Q^2}{2P\cdot q} \stackrel{\text{Lab}}{=} \frac{Q^2}{2m_p E_\gamma} > 0$$

- Bound $x \leqslant 1$ from $W^2 > m_p^2$:

$$(P+q)^2 > m_p^2 \quad => \quad (q^2 + 2P\cdot q) > 0 \quad => \quad x = \frac{-q^2}{2P\cdot q} < 1$$

- Lorentz invariant inelasticity

$$y = \frac{P\cdot q}{P\cdot k_e} \stackrel{\text{Lab}}{=} \frac{E_\gamma}{E_e} = \left(1 - \frac{E_e'}{E_e}\right) \in (0,1)$$

- Important relation with Mandelstam $S = (k_e + P)^2$:

$$x\, y\, (S - m_p^2) = Q^2$$

Kinematic variables

- By measuring scattered electron energy E_e' and scattering angle θ_e':

$$Q^2 = 2E_e E_e'(1 - \cos\theta_e') \qquad x = \frac{E_e'}{E_p}\left[\frac{1-\cos\theta_e'}{2-(E_e'/E_e)(1+\cos\theta_e')}\right]$$

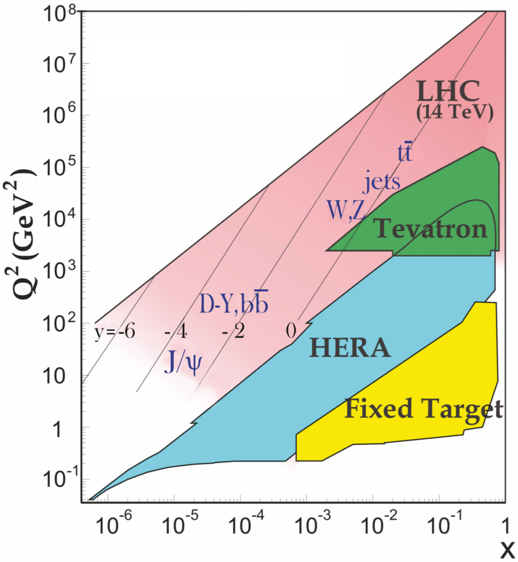
- Equivalent descriptions of DIS with

$$(E_e', \theta_e') \leftrightarrow (x, Q^2) \leftrightarrow (y, Q^2) \leftrightarrow (x, y)$$

- Photoproduction limit: $Q^2 \to 0$ for $\theta_e' \to 0$

Kinematic plane

(PDG Book)



Summary

- Nobel Prize 1969:
  Murray Gell-Mann

  "For his contributions and discoveries concerning the classification of elementary particles and their interactions"

- Nobel Prize 1990:
  Jerome I. Friedman, Henry W. Kendall, Richard E. Taylor

  "For pioneering investigations concerning deep inelastic scattering of electrons on protons which have been essential for the development of the quark model."

- Nobel Prize 2004:
  David J. Gross, David Politzer, Frank Wilczek

  "For the discovery of asymptotic freedom in the theory of the strong interactions "