Basics of Color Glass Condensate

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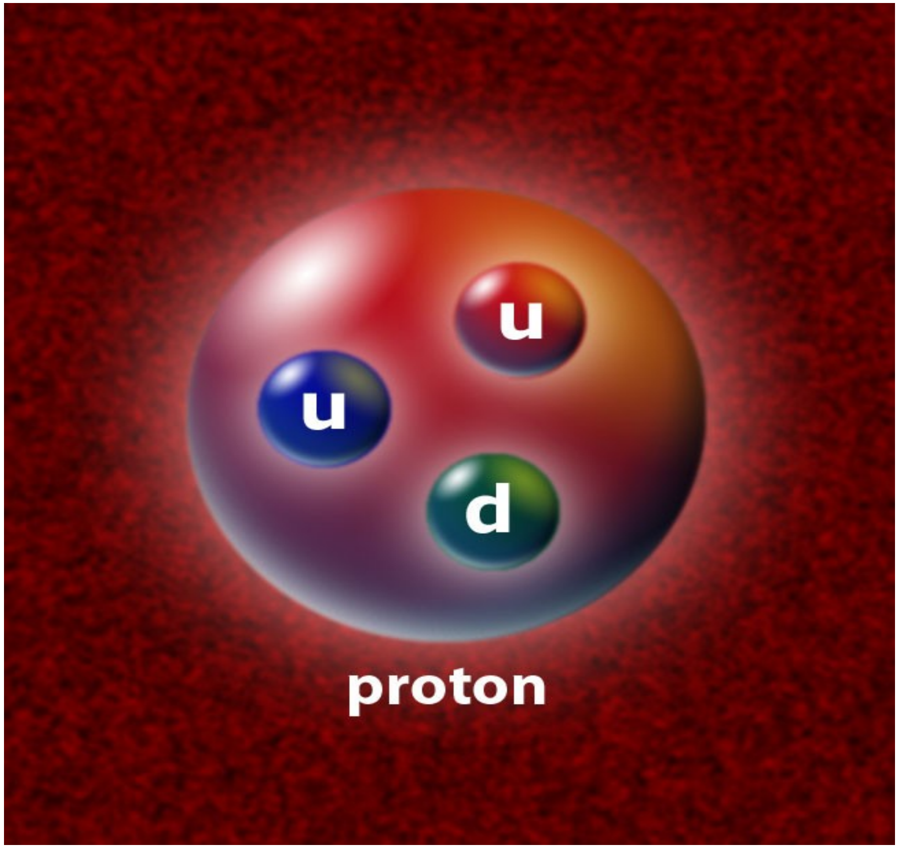
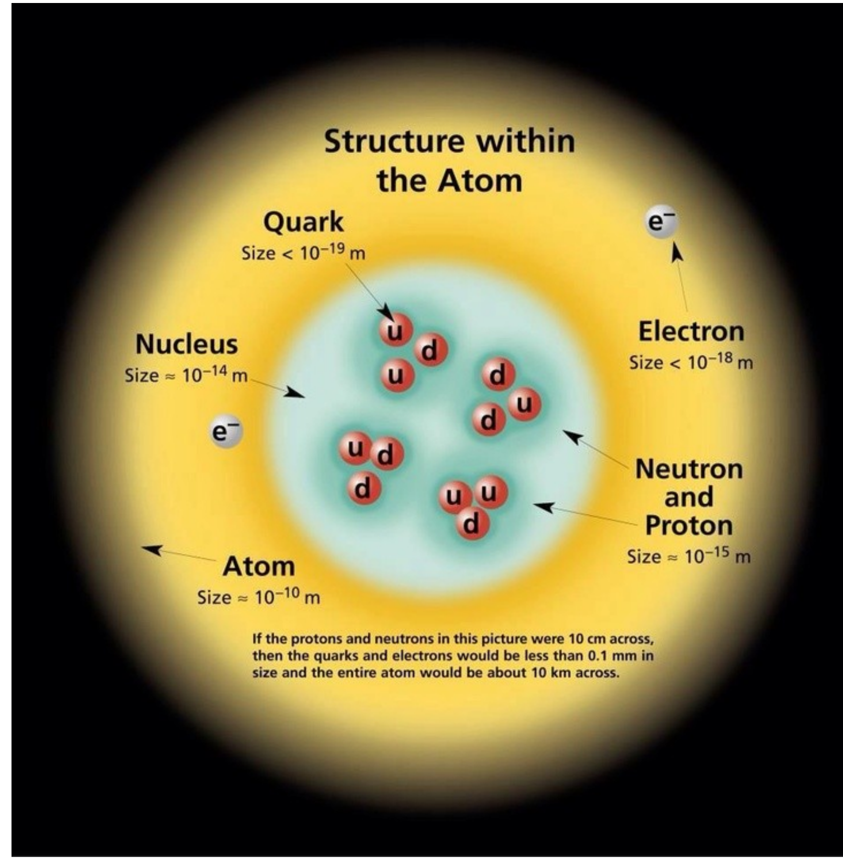
Quantum ChromoDynamics (QCD)

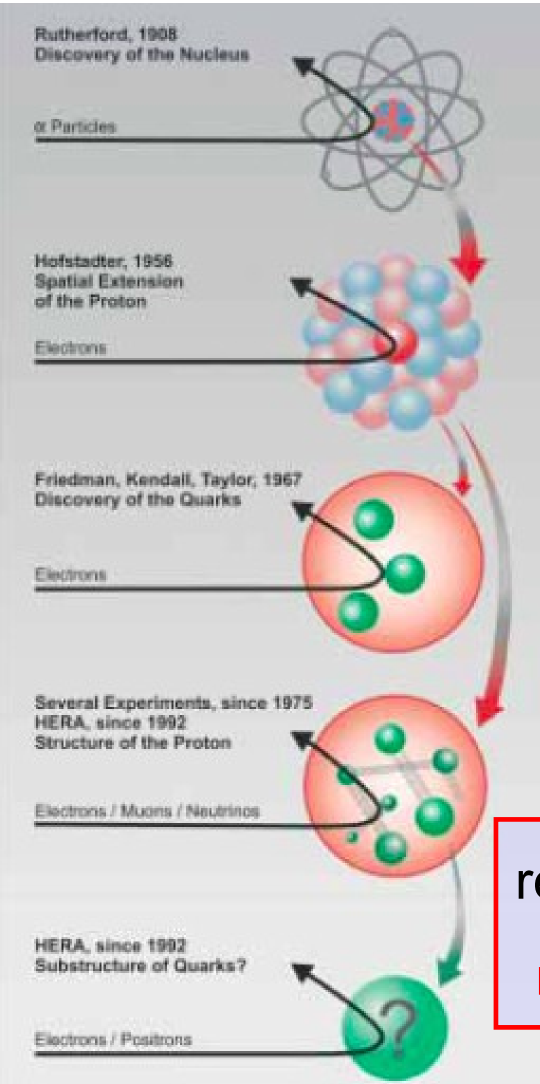




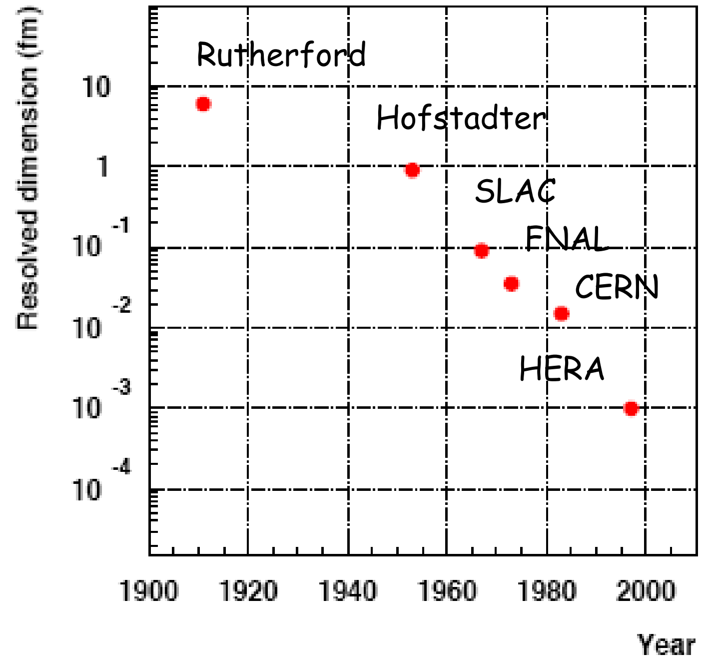
strong force confining quarks inside a proton

(and keeping protons inside a nucleus)

Deep Inelastic Scattering (DIS)

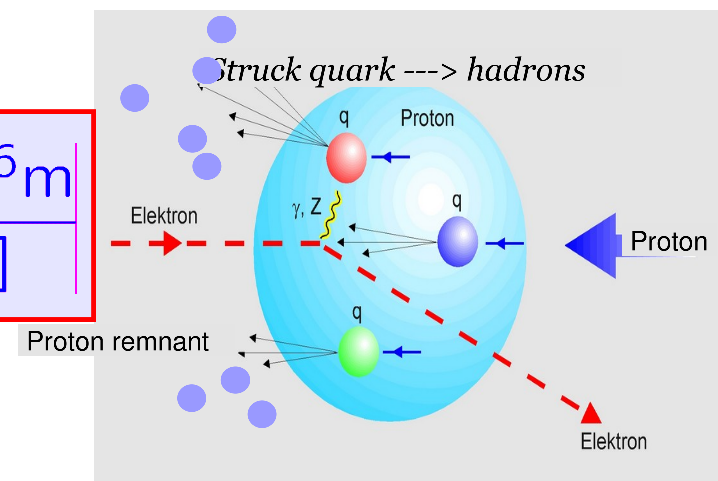


Probing
smaller distances
requires
larger momentum
transfer q
(small wavelength)

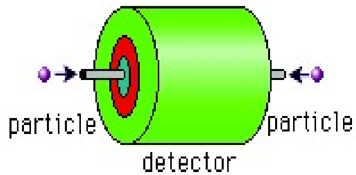


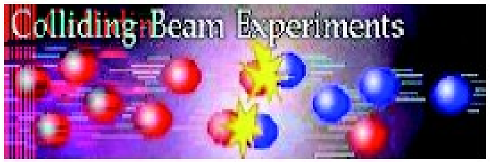
resolution: $\dfrac{\hbar}{Q} \approx \dfrac{2 \times 10^{-16}\mathrm{m}}{Q[\mathrm{GeV}]}$

r » 1/Q



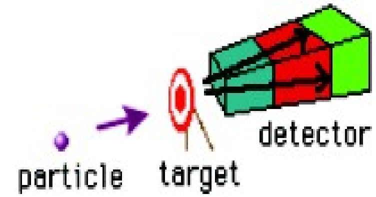
Collider experiment: Electron-Proton collisions at HERA (DESY, Hamburg, Germany)





Equivalent to fixed target of

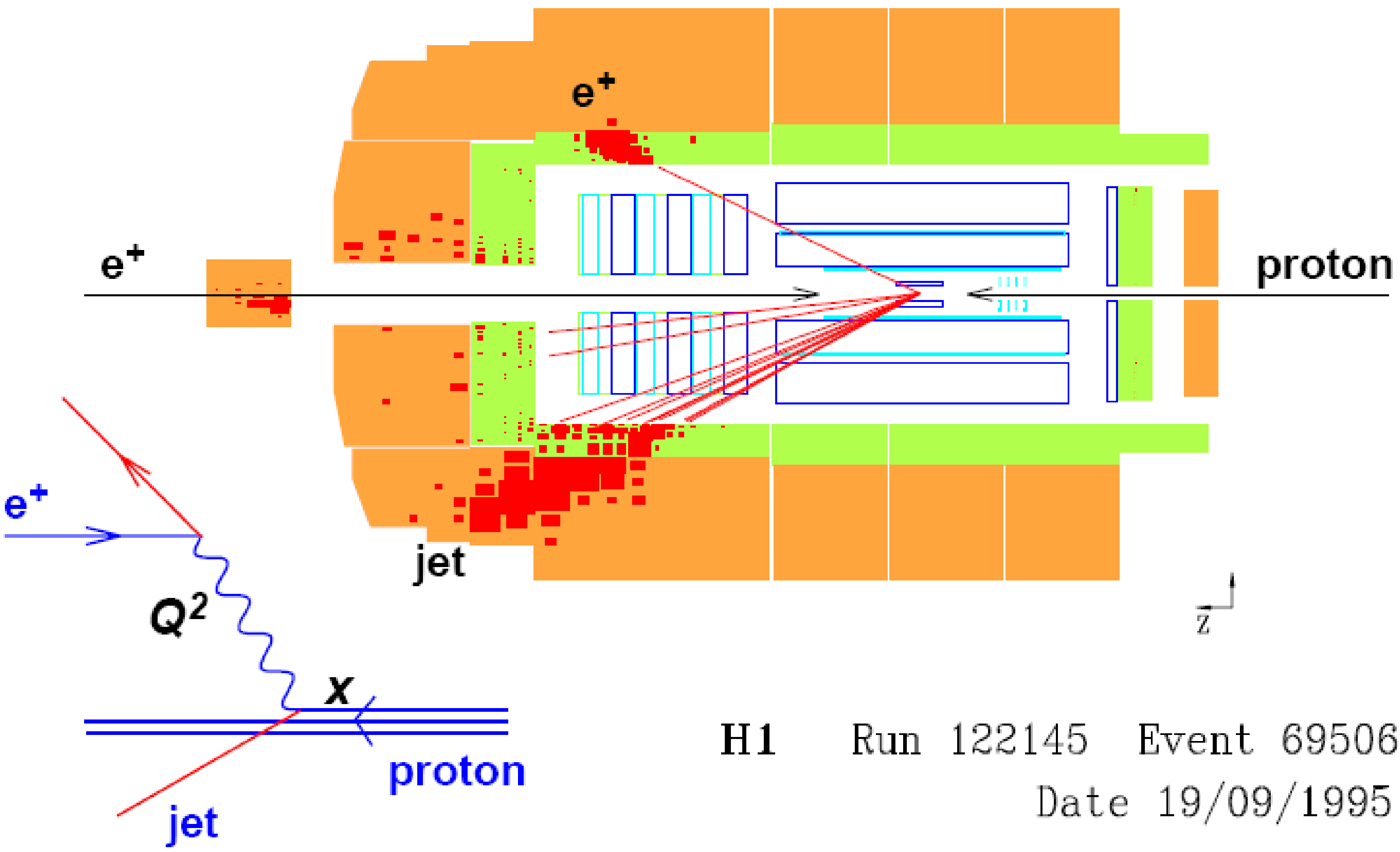
E_e = 50600 GeV:



E_e = 27.5 GeV E_p = 920 GeV

A DIS event

Q^2 = 25030 GeV^2; y = 0.56; **x=0.50**

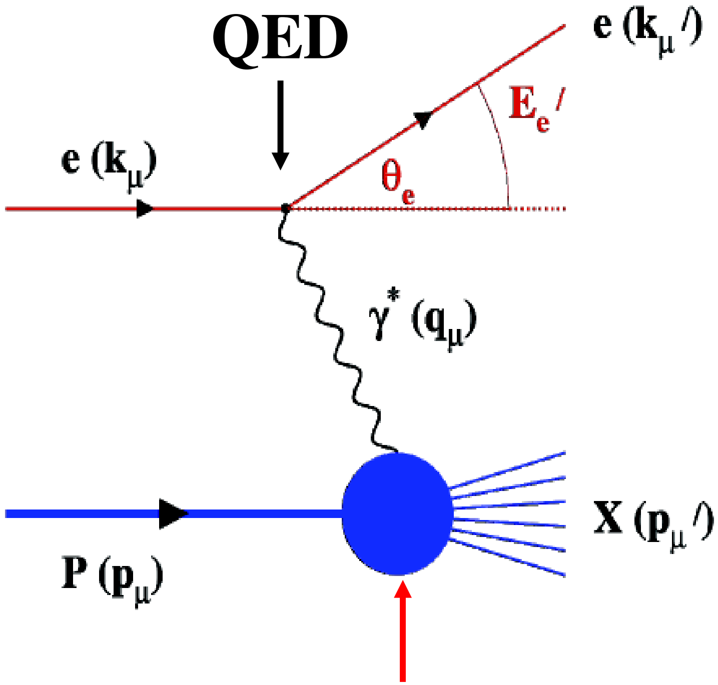


H1 Run 122145 Event 69506

Date 19/09/1995

Deep Inelastic Scattering (DIS)

probing hadron structure



Kinematic Invariants

$$Q^2 = -q^2 = -(k_\mu - k'_\mu)^2$$

$$Q^2 = 4E_e E'_e \sin^2\left(\frac{\theta'_e}{2}\right)$$

Measure of resolution power

$$y = \frac{pq}{pk} = 1 - \frac{E'_e}{E_e}\cos^2\left(\frac{\theta'_e}{2}\right)$$

Measure of inelasticity

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{sy}$$

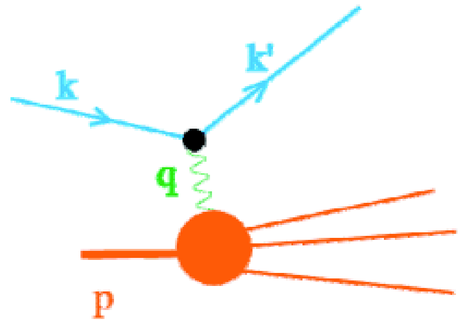
Measure of momentum fraction of struck quark

$$\mathbf{s} \equiv (\mathbf{p} + \mathbf{k})^2$$

Deep Inelastic Scattering

first analysis of DIS does not require any knowledge about QCD

electroweak theory tells us how
the virtual vector boson couples:
(let's assume only photon exchange)



$$d\sigma = \frac{4\alpha^2}{s}\frac{d^3\vec{k}'}{2|\vec{k}'|}\frac{1}{Q^4}L^{\mu\nu}(k,q)W_{\mu\nu}(p,q)$$

phase space scat. lepton

photon propagator²

leptonic tensor

hadronic tensor contains information about hadronic structure

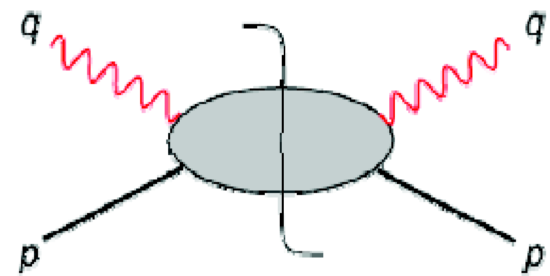
(can be easily generalized to W/Z-boson exchange)

with $$L_{\mu\nu} = 2(k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu} k\cdot k')$$

Deep Inelastic Scattering

Strong interactions: contained in the hadronic tensor $\mathbf{W}_{\mu\nu}(\mathbf{p}, \mathbf{q})$

to all orders in the strong interaction $W_{\mu\nu}$
is given by the square of $\gamma^*(q)\, h(p) \to X$



symmetries (parity, Lorentz), hermiticity & current conservation
tell us that $W^{\nu\mu} = W^{\mu\nu *}$ $\quad q_\mu W^{\mu\nu} = 0$

$$W_{\mu\nu}(p,q) = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1(x, Q^2) + \left(p_\mu - q_\mu \frac{p \cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p \cdot q}{q^2}\right) \frac{1}{p \cdot q} F_2(x, Q^2)$$

structure functions

space-time picture of DIS

light cone variables

advantages: boosting is easy

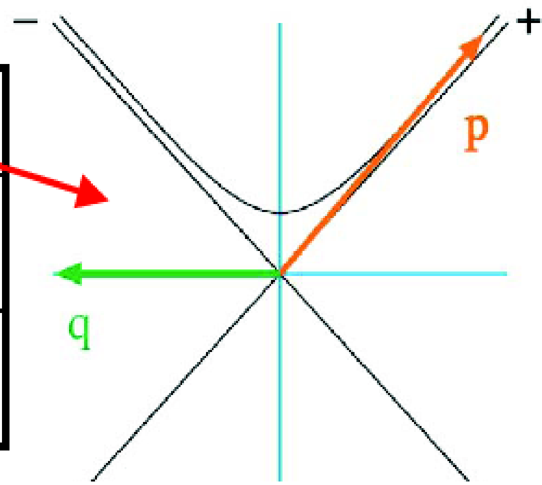
separation of large and small components of vectors

$$P^+ \equiv \frac{E + P_z}{\sqrt{2}}$$

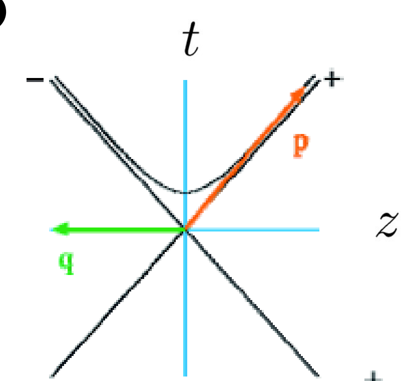
$$P^- \equiv \frac{E - P_z}{\sqrt{2}} \qquad (\mathbf{V}^+, \mathbf{V}^-, \mathbf{V_t}) \rightarrow (\mathbf{e}^{\omega}\mathbf{V}^+, \mathbf{e}^{-\omega}\mathbf{V}^-, \mathbf{V_t}) \text{ with } \mathbf{e}^{\omega} = \frac{\mathbf{Q}}{\mathbf{x\, m_h}}$$

$$P_t = P_t$$

4-vector	hadron rest frame	Breit frame
$(p^+, p^-, \vec{p}_T)$	$\frac{1}{\sqrt{2}}(m_h, m_h, \vec{0})$	$\frac{1}{\sqrt{2}}(\frac{Q}{x}, \frac{x m_h^2}{Q}, \vec{0})$
$(q^+, q^-, \vec{q}_T)$	$\frac{1}{\sqrt{2}}(-m_h x, \frac{Q^2}{m_h x}, \vec{0})$	$\frac{1}{\sqrt{2}}(-Q, Q, \vec{0})$



space-time picture of DIS

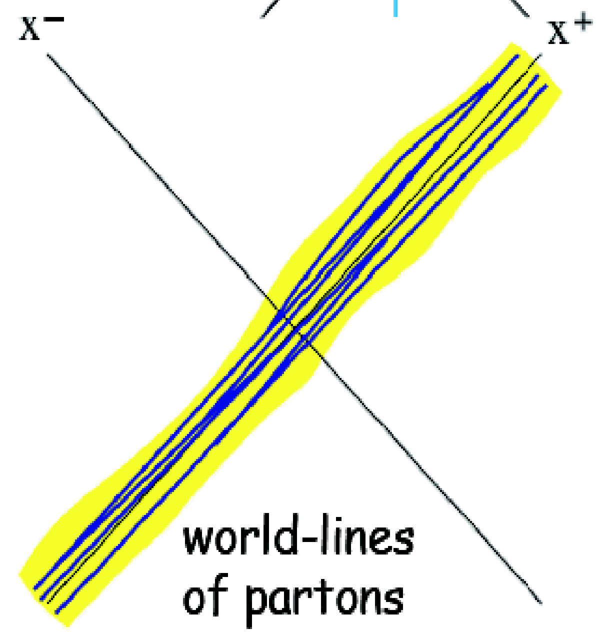


simple estimate for typical time-scale of interactions among the partons inside a fast-moving hadron:

rest frame: $\Delta x^+ \sim \Delta x^- \sim \frac{1}{m}$

Breit frame: $\Delta x^+ \sim \frac{1}{m}\frac{Q}{m} = \frac{Q}{m^2}$ large

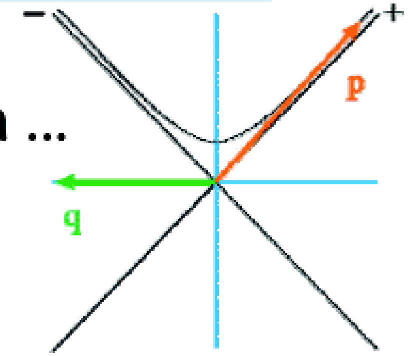
$\Delta x^- \sim \frac{1}{m}\frac{m}{Q} = \frac{1}{Q}$ small



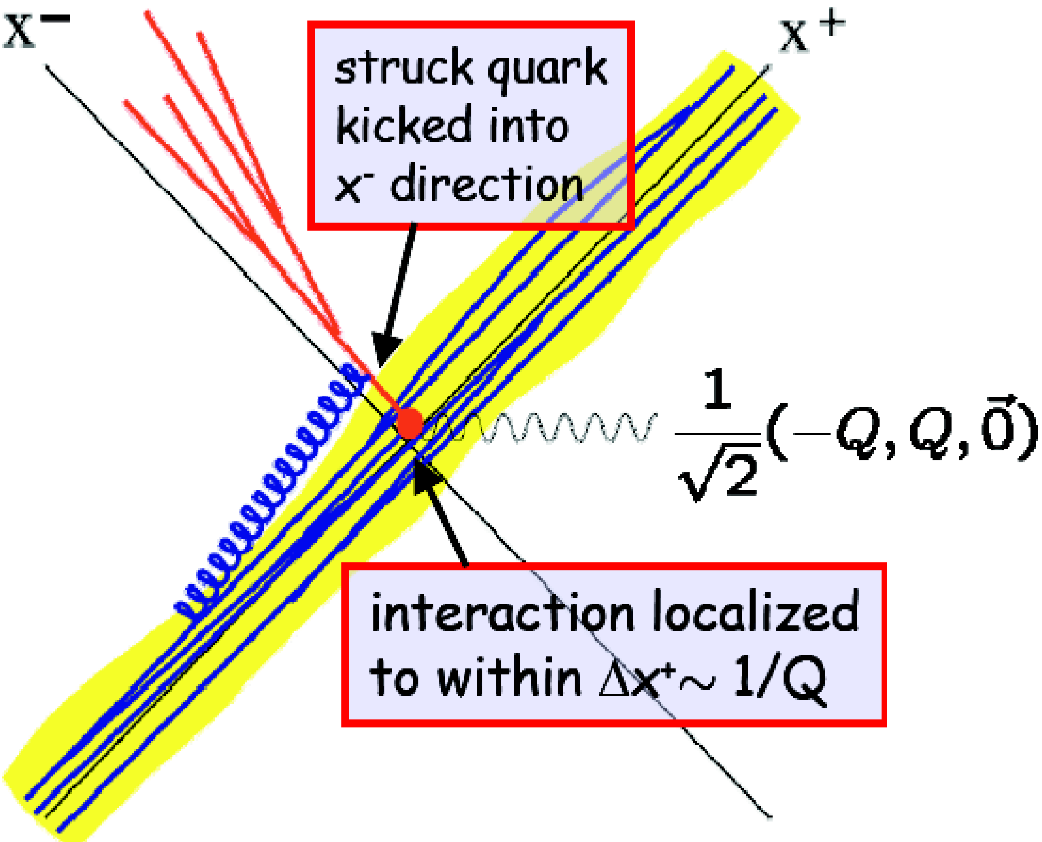
> interactions between partons are spread out inside a fast moving hadron

How does this compare with the time-scale of the hard scattering?

space-time picture of DIS



now let the virtual photon meet our fast moving hadron ...



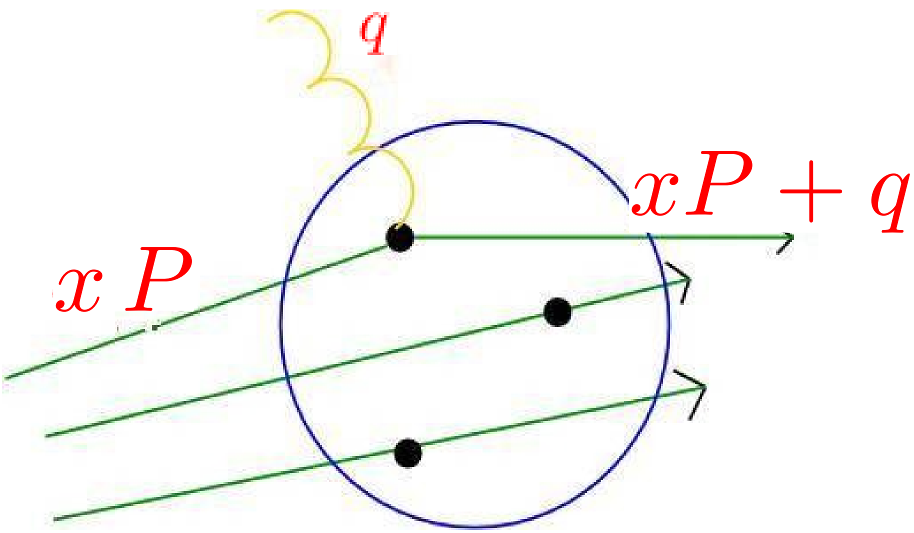
upshot:

- partons are free during the hard interaction
- hadron effectively consists of partons that have momenta $(p_i^+, p_i^-, \vec{p}_i)$
- convenient to introduce momentum fractions
  $0 < \xi_i \equiv p_i^+/p^+ < 1$

what is inside a hadron: parton model

Bjorken limit $\mathbf{Q^2, S \to \infty}$ $\mathbf{x_{Bj} = \frac{Q^2}{S}}$

structure functions depend only on x_{Bj}



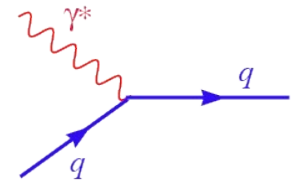
Feynman:

parton constituents of proton are "free" on time scale 1/Q << 1/Λ (interaction time scale between partons)

$$\mathbf{F_2(x) \equiv \sum^{f} e_f^2\, x[q_f(x) + \bar{q}_f(x)]}$$

DIS in the QCD-improved parton model

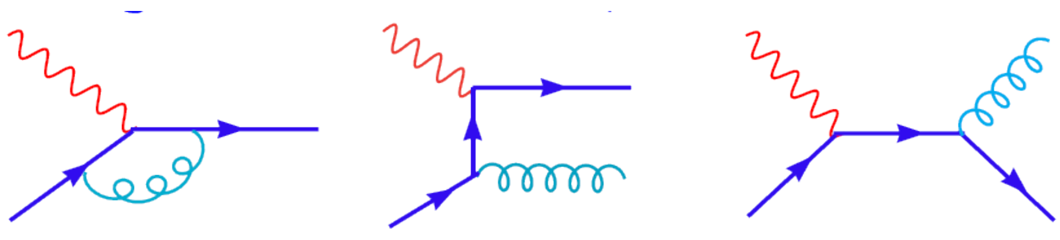
we got a long way (parton model) *without* invoking QCD



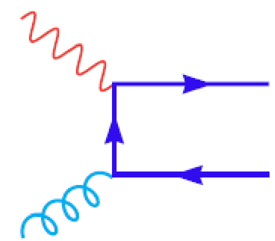
now we have to study **QCD dynamics in DIS**

– this leads to similar problems already encountered in e^+e^-

let's try to compute the **$O(\alpha_s)$ QCD corrections** to the naive picture



α_S corrections to the LO process



photon-gluon fusion

caveat: ***expect divergencies***

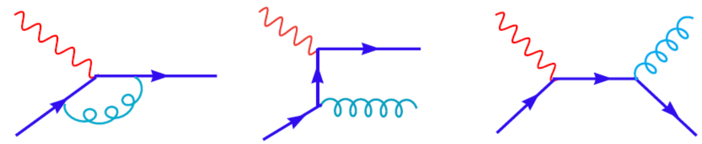
related to soft/collinear emission or from loops

what to do with infinities?

introduce "**regulator**" in the intermediate stages, remove it at the end

general structure of the QCD corrections [O(α_s)]

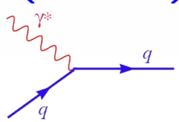
using small quark/gluon mass as a regulator:



$$\left.\frac{d^2\hat{\sigma}}{dx\,dQ^2}\right|_{F_2} \equiv \hat{F}_2^q$$

LO

$$= e_q^2 x \left[\delta(1-x) + \frac{\alpha_s(\mu_r)}{4\pi}\left[P_{qq}(x)\ln\frac{Q^2}{m_g^2} + C_2^q(x)\right]\right]$$

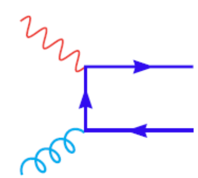


large logarithms (collinear emission)

finite coefficients

$$\left.\frac{d^2\hat{\sigma}}{dx\,dQ^2}\right|_{F_2} \equiv \hat{F}_2^g$$

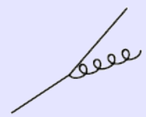
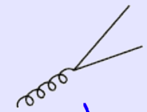
$$= \sum_q e_q^2 x \left[0 + \frac{\alpha_s(\mu_r)}{4\pi}\left[P_{qg}(x)\ln\frac{Q^2}{m_q^2} + C_2^g(x)\right]\right]$$



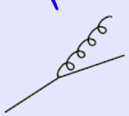
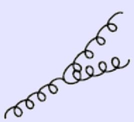
divergences absorbed into pdf

$$\mathbf{F_2(x, Q^2) \equiv \sum^{f} e_f^2\, x[q_f(x, Q^2) + \bar{q}_f(x, Q^2)]}$$

DGLAP "evolution" equation

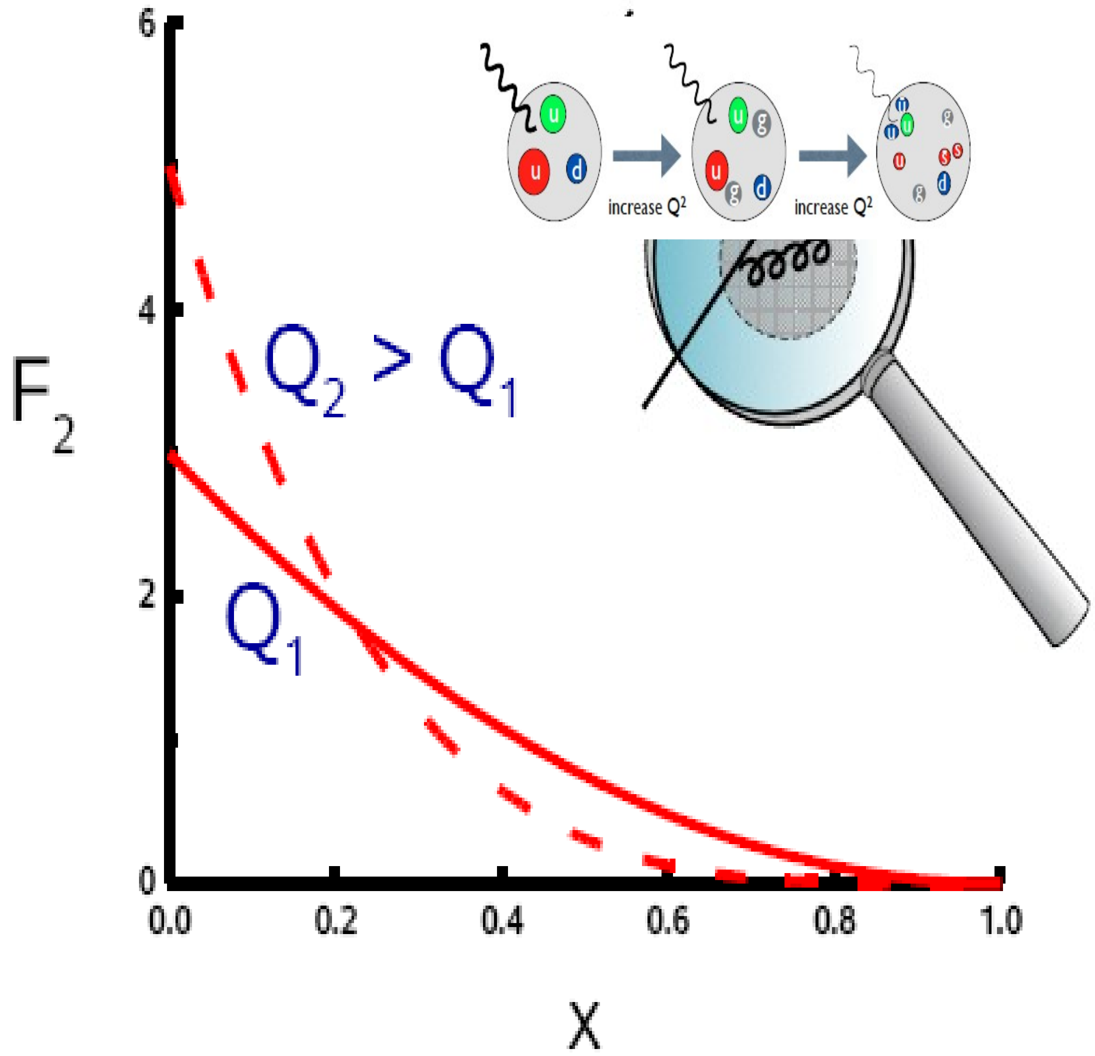
$$\frac{d}{d\ln\mu}\begin{pmatrix} q(x,\mu) \\ g(x,\mu) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix}_{(z,\alpha_s)} \cdot \begin{pmatrix} q(x/z,\mu) \\ g(x/z),\mu) \end{pmatrix}$$

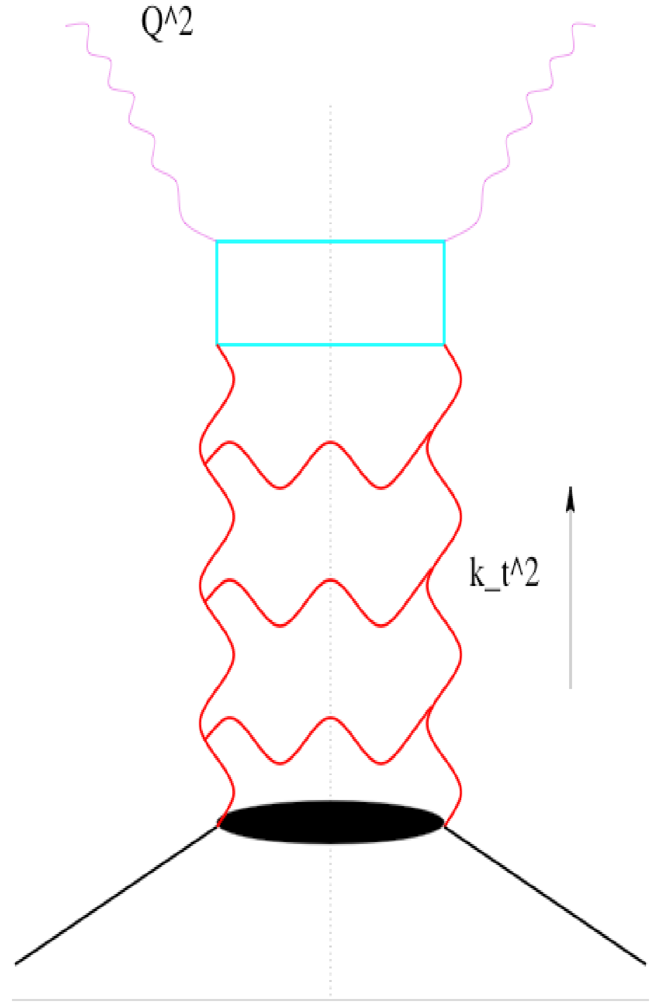



DGLAP "evolution" equation:

scale dependence of parton distribution functions

Dokshitzer-**G**ribov-**L**ipatov-**A**ltarelli-**P**arisi

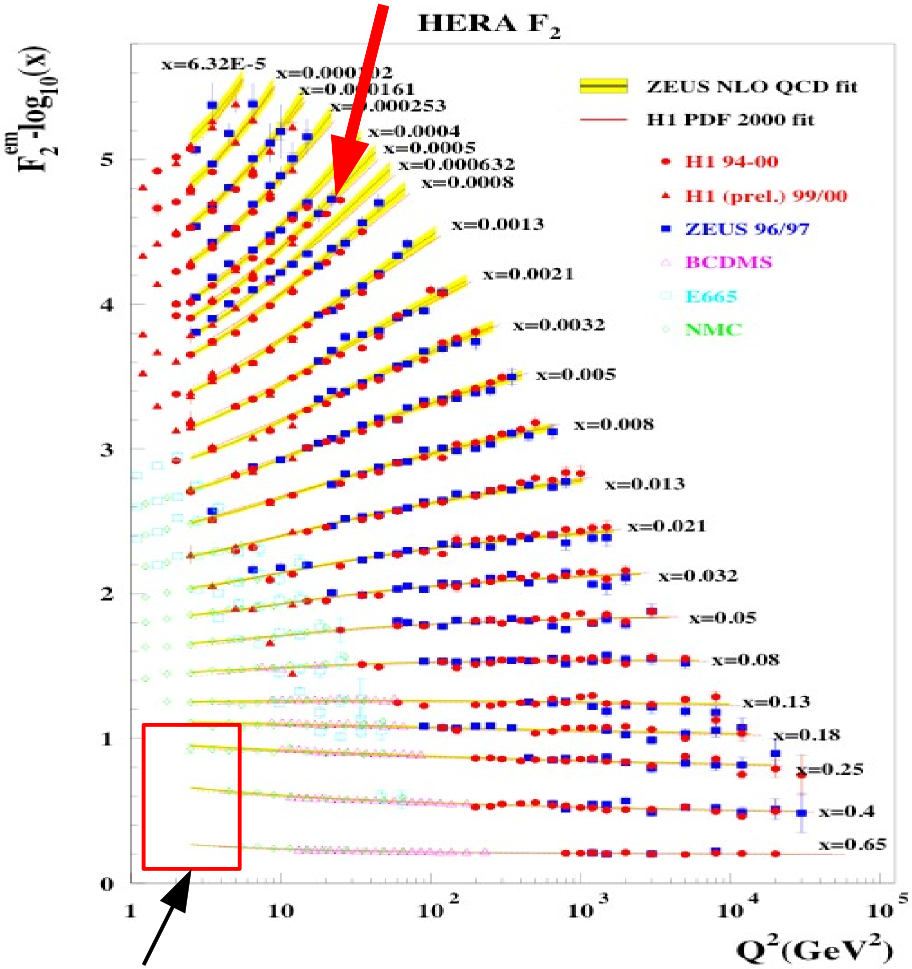




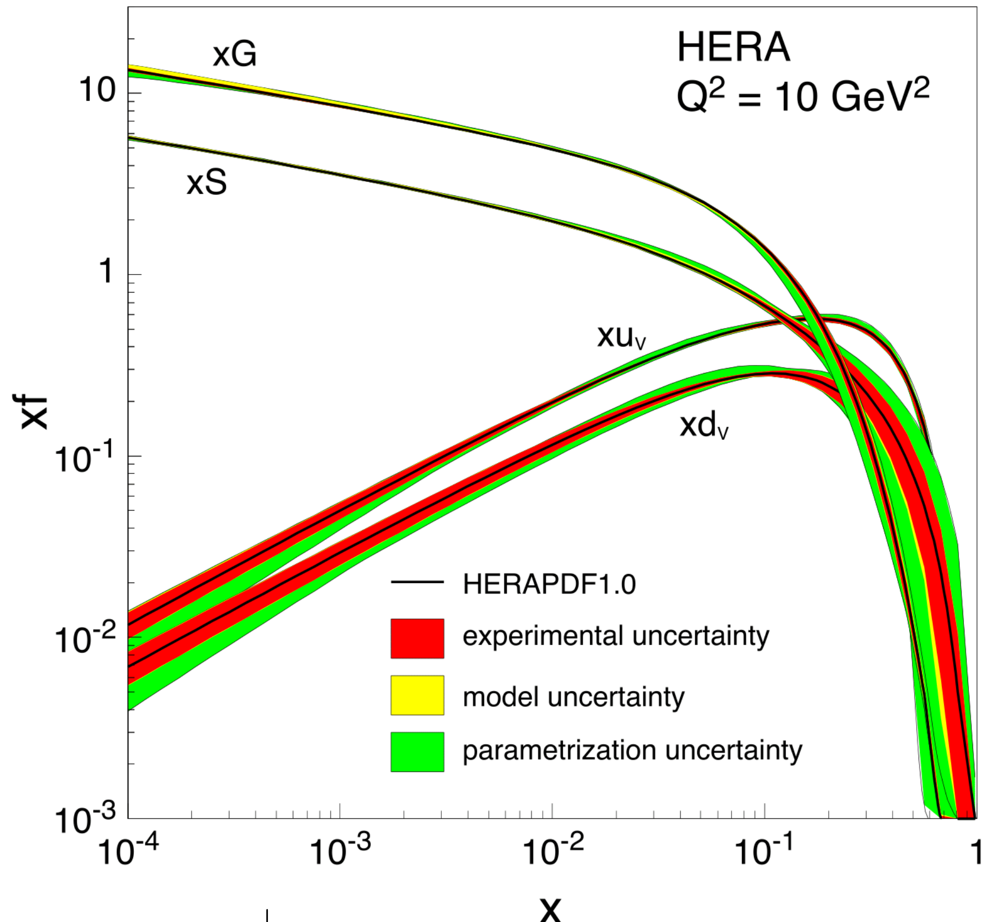
Deep Inelastic Scattering

$$F_2 \equiv \sum_{f=q,\bar{q}} e_f^2\, xq(x,Q^2)$$

QCD: scaling violations



early experiments (SLAC,...):
scale invariance of hadron structure



$$x = \frac{p^+}{P^+}$$

***x** is the fraction of hadron energy carried by a parton*

What drives the growth of parton distributions?

Splitting functions at leading order $O(\alpha_S{}^0)$ $\quad (x \neq 1)$

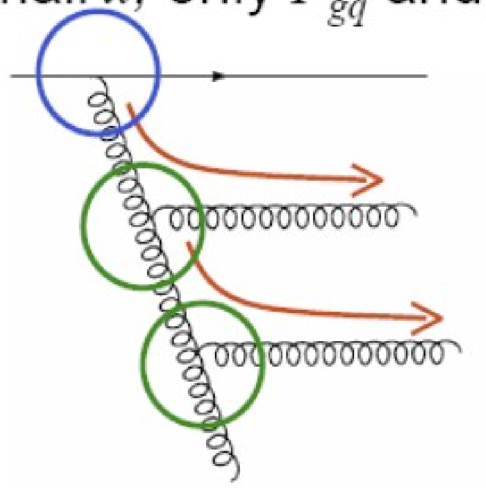
$$P_{qq}^{(0)}(x) = C_F \frac{1+x^2}{1-x}$$

$$P_{qg}^{(0)}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right]$$

$$P_{gq}^{(0)}(x) = C_F \frac{1+(1-x)^2}{x}$$

$$P_{gg}^{(0)}(x) = 2C_A \left[\frac{x}{1-x} + \frac{1-x}{x} + x(1-x)\right]$$

At small x, only P_{gq} and P_{gg} are relevant.



→ Gluon dominant at small *x*!

The double log approximation (DLA) of DGLAP is easily solved.

-- increase of gluon distribution at small x

$$\mathbf{xg(x, Q^2) \sim e^{\sqrt{\alpha_s\,(\log 1/x)\,(\log Q^2)}}}$$

QCD in the Regge-Gribov limit

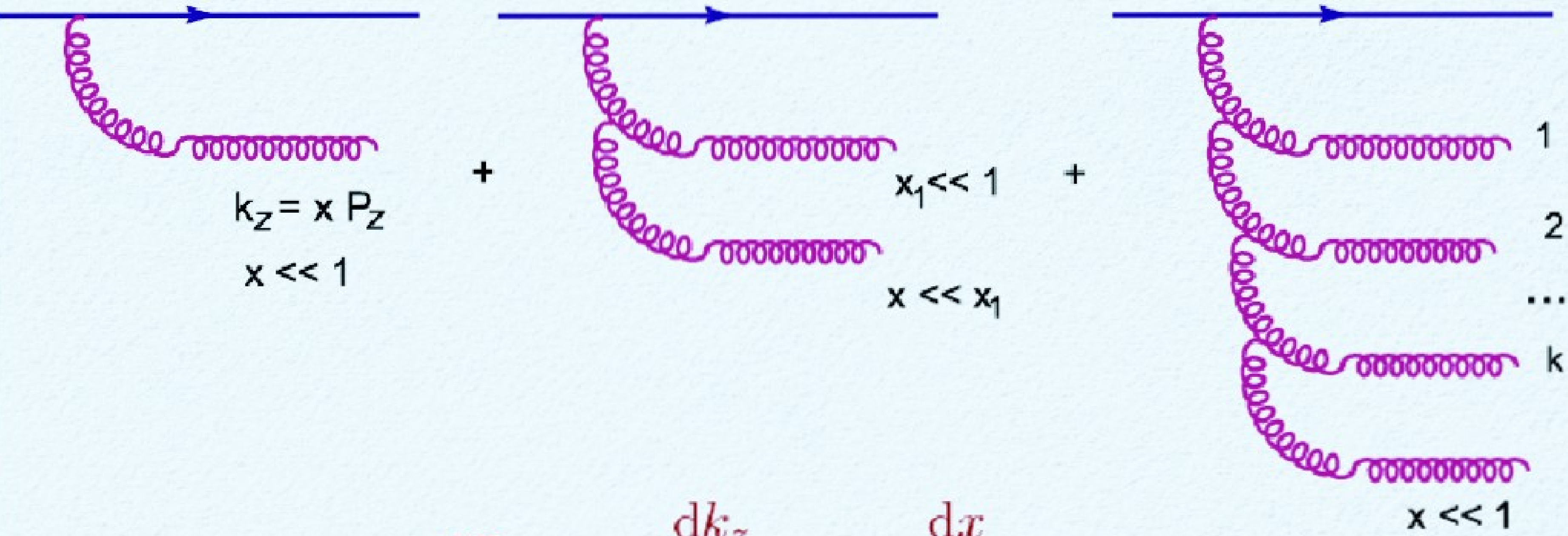
recall $x_{Bj} \equiv \frac{Q^2}{S}$ $\quad$ $S \to \infty\,,\ Q^2 \text{ fixed} : x_{Bj} \to 0$

Regge

Gribov

gluon radiation at small x :pQCD

The infrared sensitivity of bremsstrahlung favors the emission of 'soft' (= small–x) gluons $\mathbf{P_{gg}(x) \sim \frac{1}{x}}$ **for** $\mathbf{x \to 0}$



$$\mathrm{d}\mathcal{P} \propto \alpha_s \frac{\mathrm{d}k_z}{k_z} = \alpha_s \frac{\mathrm{d}x}{x}$$

The 'price' of an additional gluon:

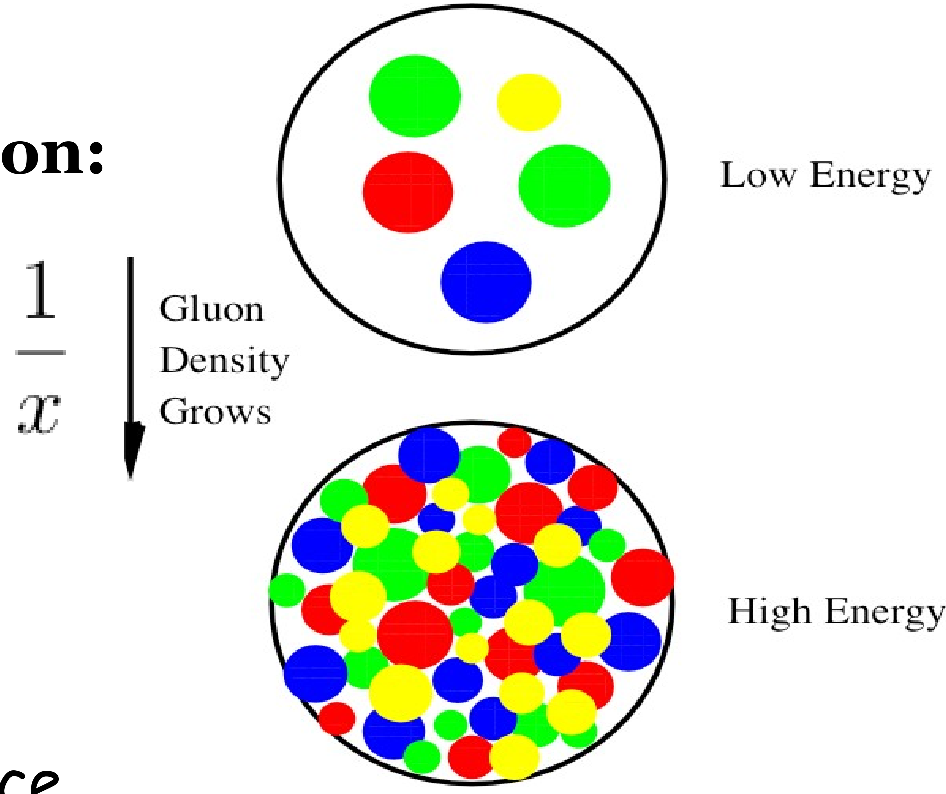
$$\mathcal{P}(1) \propto \alpha_s \int_x^1 \frac{\mathrm{d}x_1}{x_1} = \alpha_s \ln \frac{1}{x}$$

number of gluons grows fast

$$n \sim e^{\alpha_s ln 1/x}$$

Resolving the nucleus/hadron:

Regge-Gribov limit



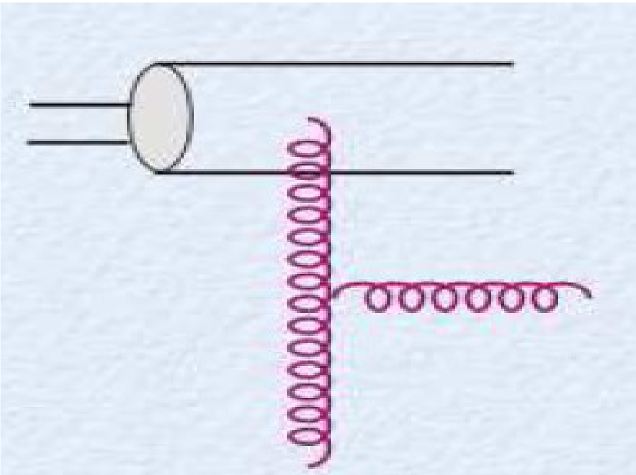
radiated gluons have the same size ($1/Q^2$) - the number of partons increase due to the increased longitudinal phase space

hadron/nucleus becomes a dense system of gluons: concept of a quasi-free parton is not useful

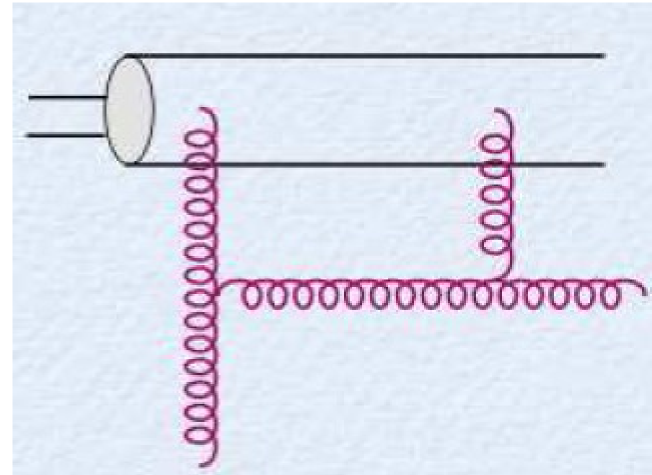
Physics of strong color fields in QCD, multi-particle production- possibly discover novel universal properties of theory in this limit

break down of pQCD at small x

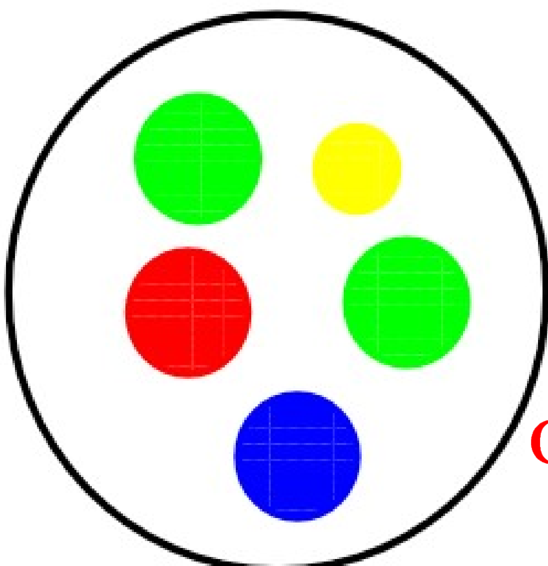
"attractive" bremsstrahlung *vs.* *"repulsive" recombination*



included in pQCD

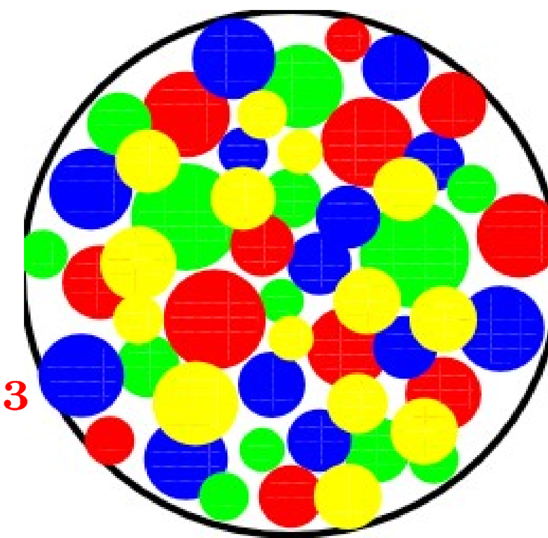


not included in pQCD
(collinear factorization)

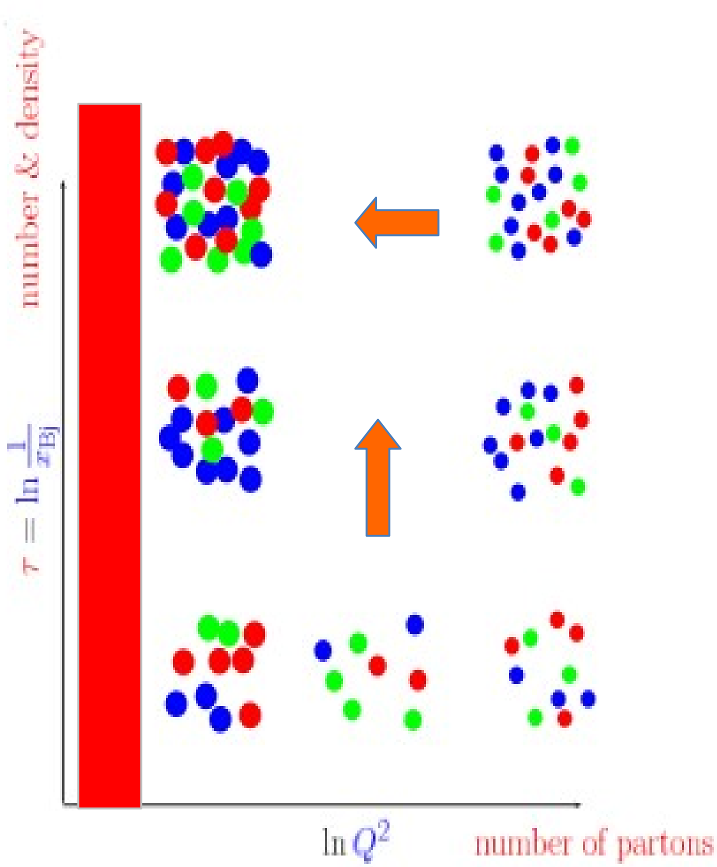


$$\frac{\alpha_s}{Q^2}\,\frac{xG(x,Q^2)}{\pi r^2} \sim 1$$

$$Q_s^2(x, b_t, A) \sim A^{1/3}\,\left(\frac{1}{x}\right)^{0.3}$$



Low x QCD:
many-body dynamics of universal gluonic matter (CGC)



How does this happen ?

How do correlation functions of these evolve ?

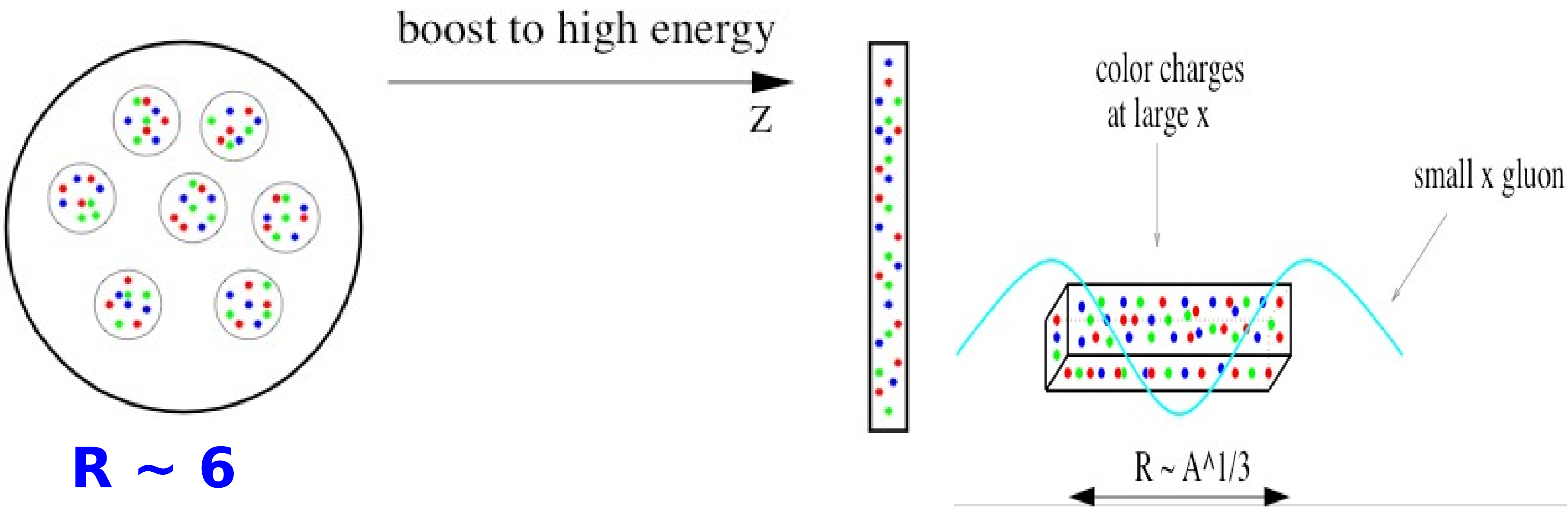
Are there scaling laws ?

Can CGC explain aspects of HEC ?

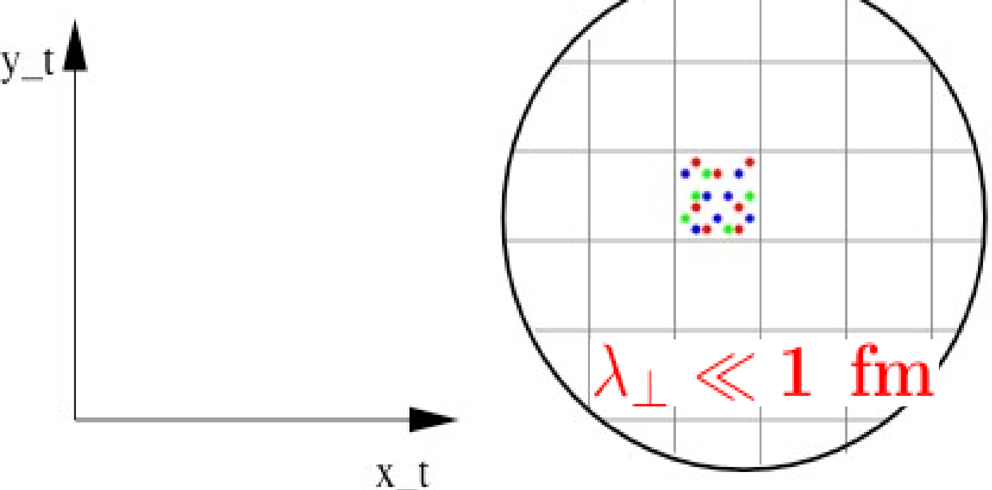
Initial conditions for hydro?
Thermalization ?
Long range rapidity correlations ?
Azimuthal angular correlations ?
Nuclear modification factor ?

A model of nuclei at high energy

(a system of color charges)



R ~ 6 fm

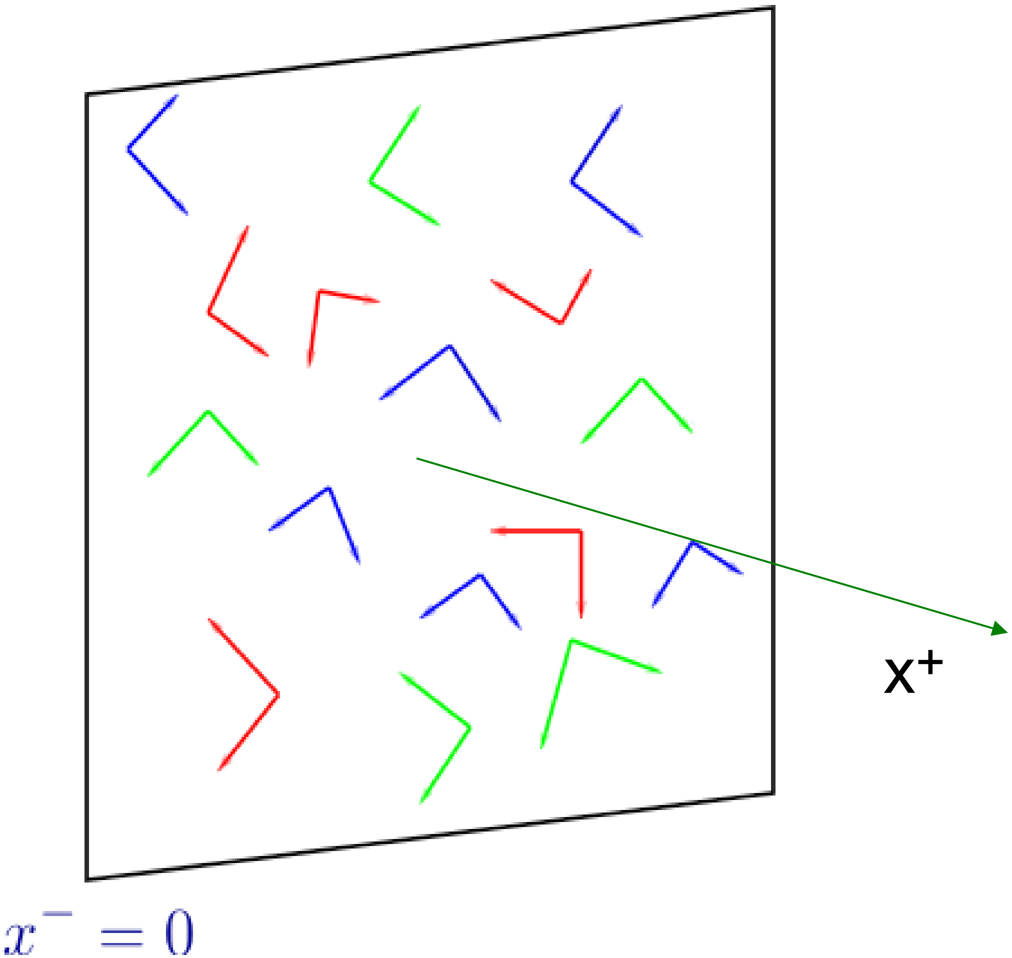


$$R \rightarrow \frac{R}{\gamma}$$

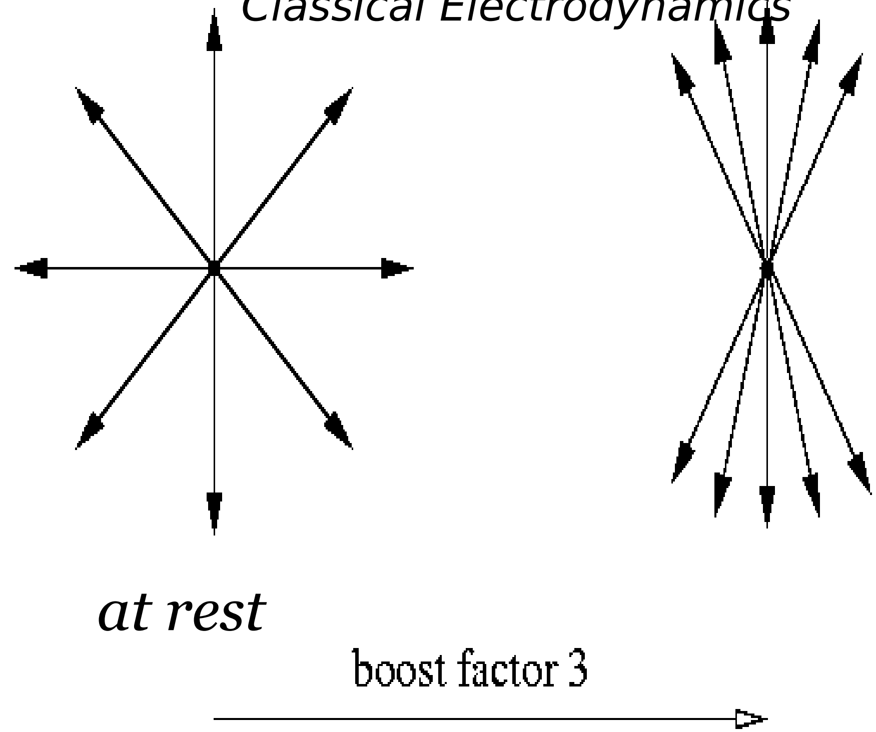
$$\gamma = 100 \quad RHIC$$

$$\gamma = 5500 \quad LHC$$

a very large nucleus at high energy: MV model



J.D. Jackson
Classical Electrodynamics

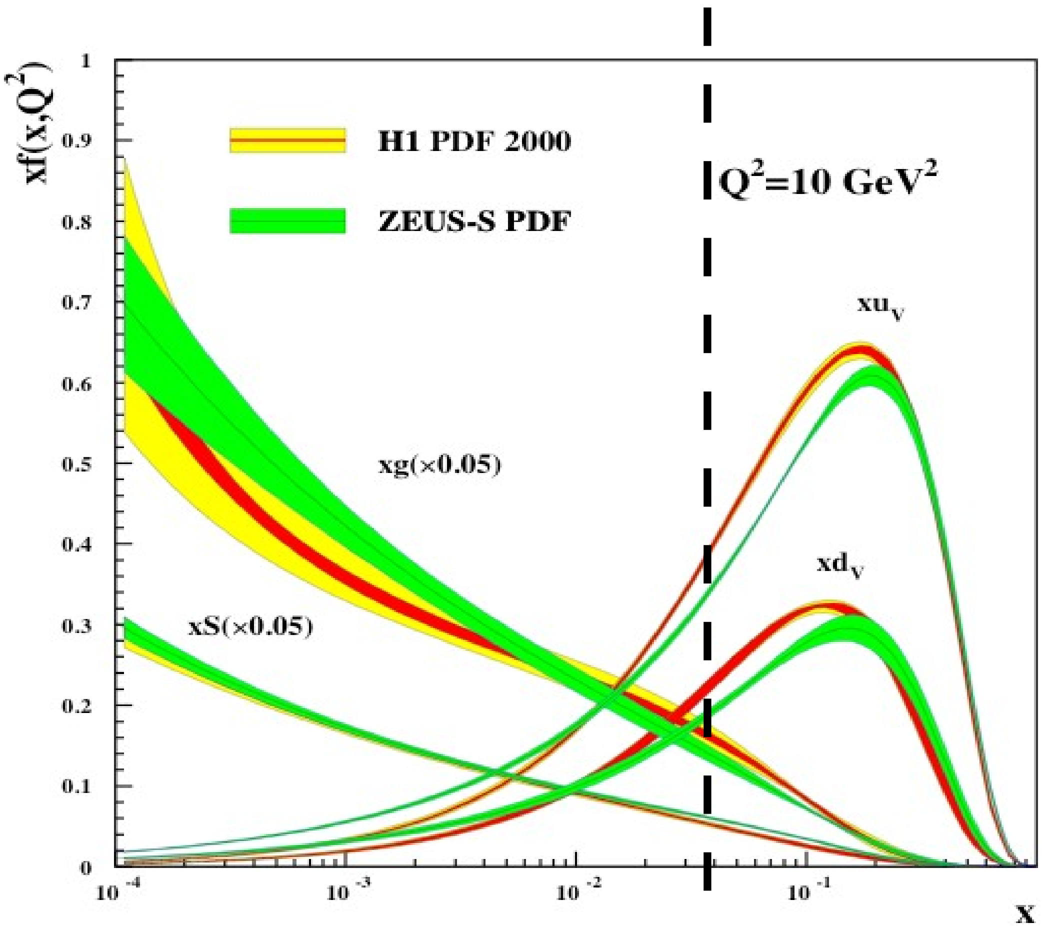


Electric field *of a point charge at high energy*

random ***color Electric & Magnetic fields*** *in the plane of the fast moving nucleus*

$$F_a^{+i} \sim \delta(x^-)\,\alpha_a^i(x_t)$$

high x partons as <u>static</u> color charges ρ

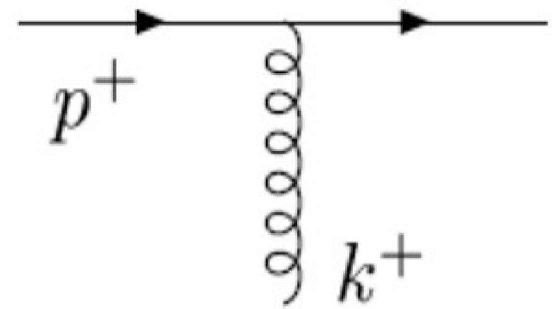


"wee" "valence"

X_o

recall for any 4-momentum

$$\mathbf{p^2 = 2p^+p^- - p_t^2}$$

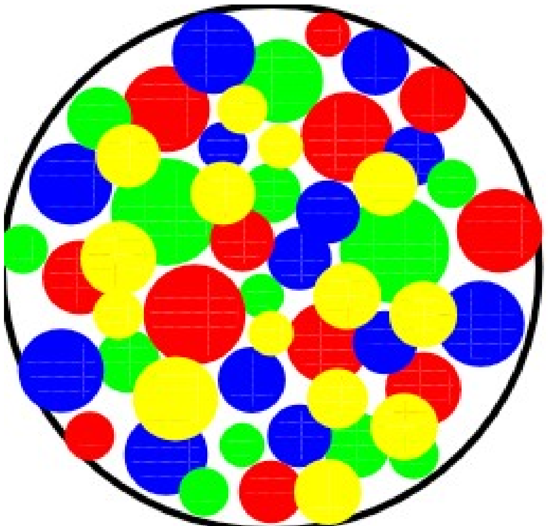


small x: $\mathbf{k^+ << p^+}$

$$\tau_{\mathrm{val}} \sim \frac{1}{p^-} = \frac{2p^+}{k_t^2} \gg \tau_{\mathrm{wee}} \sim \frac{1}{k^-} = \frac{2k^+}{k_t^2}$$

natural time scale for "valence" partons is much larger than that of "wee" partons

a very large nucleus at high energy: MV model

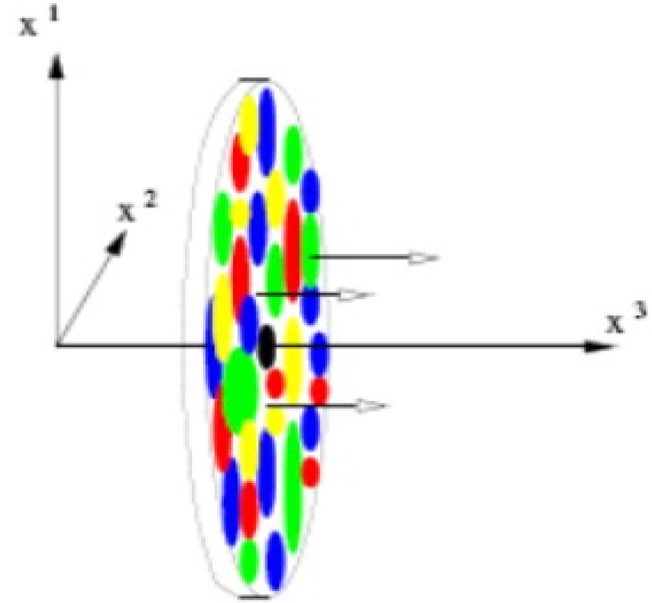


boost

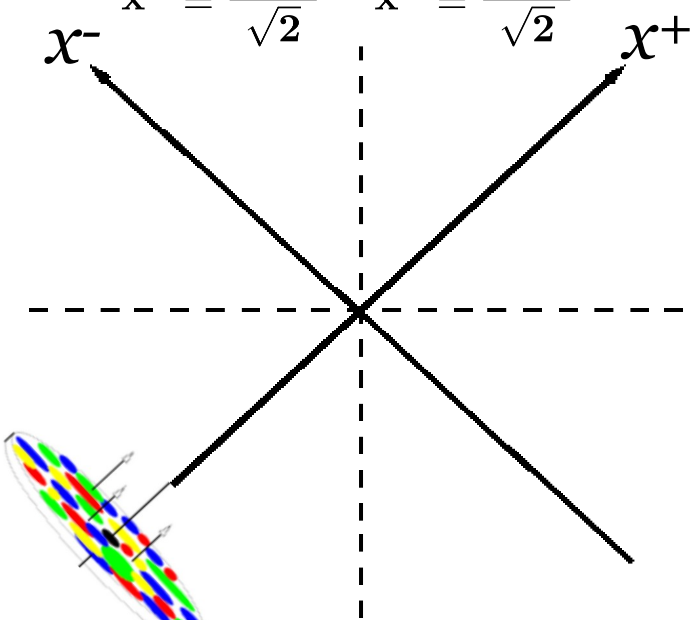
$$R \to \frac{R}{\gamma}$$

$\gamma \sim 100$ RHIC

$\gamma \sim 2500$ LHC



$$\mathbf{x^+} \equiv \frac{\mathbf{t+z}}{\sqrt{2}} \qquad \mathbf{x^-} \equiv \frac{\mathbf{t-z}}{\sqrt{2}}$$



sheet of color charge moving along x^+ and sitting at $x^- = 0$

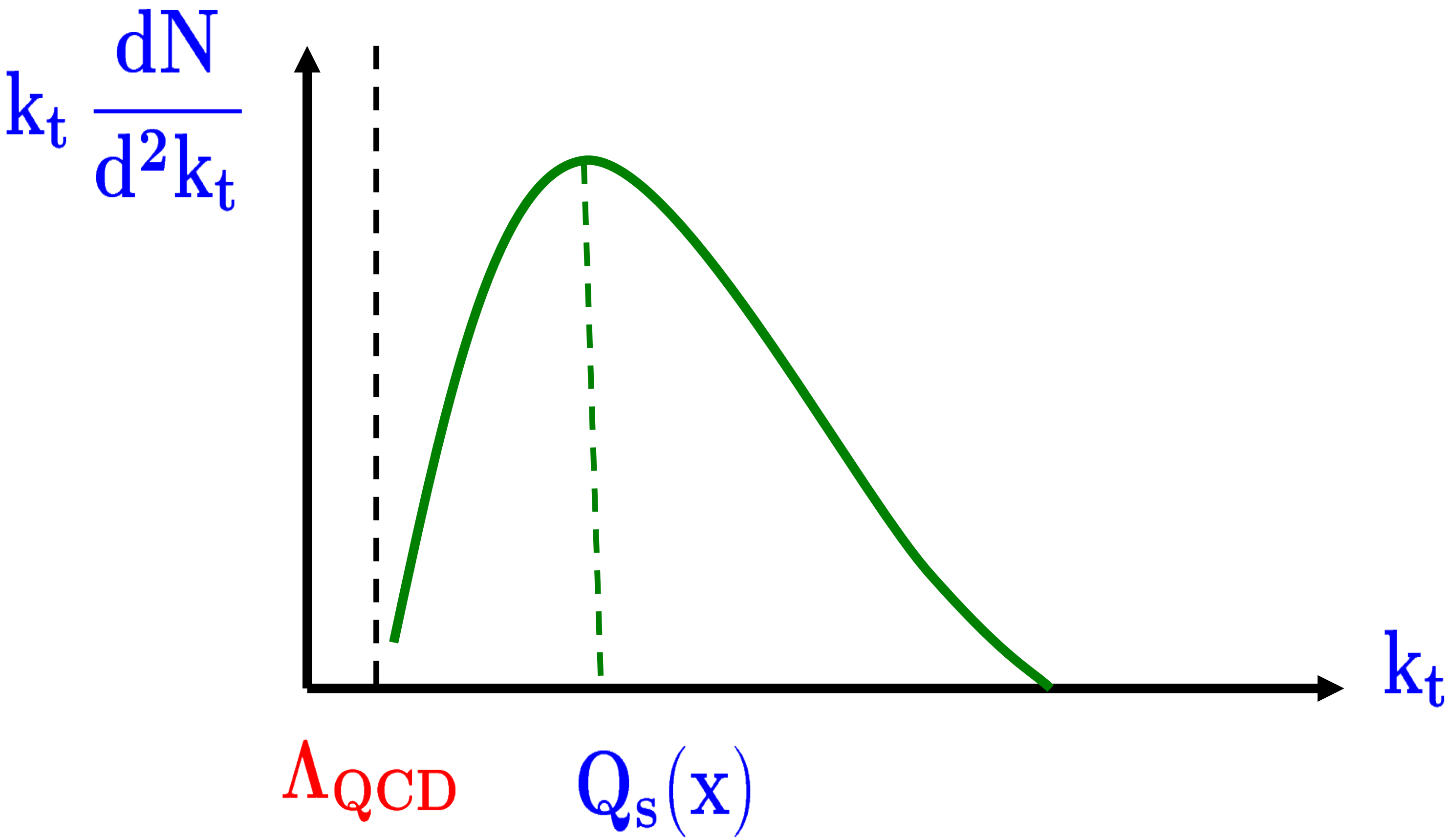
$$\mathbf{J^\mu_a(x)} \equiv \delta^{\mu\,+}\,\delta(\mathbf{x^-})\,\rho_\mathbf{a}(\mathbf{x_t})$$

color current *color charge*

$$\mathbf{A^a_i(x^-, x_t)} = \theta(\mathbf{x^-})\,\alpha^\mathbf{a}_\mathbf{i}(\mathbf{x_t})$$

with $\partial_i\,\alpha^a_i = g\,\rho^a$

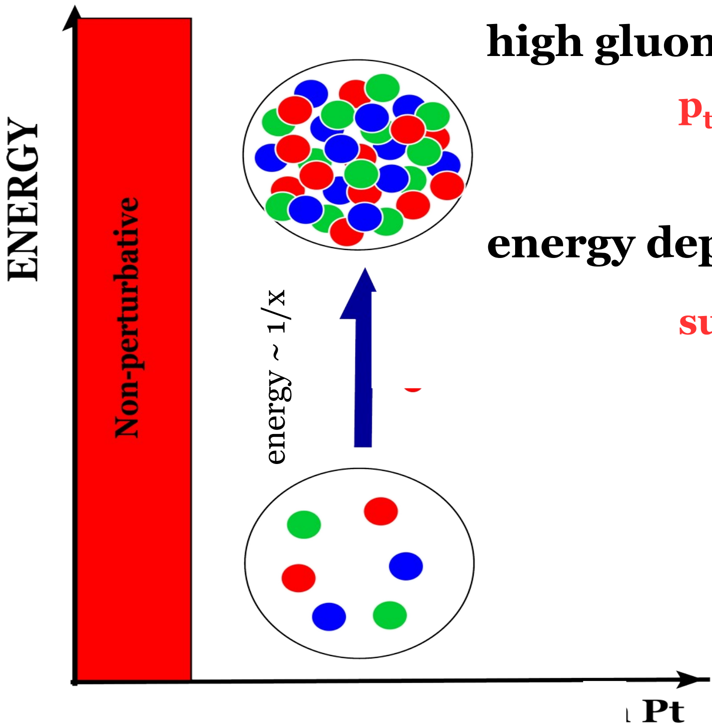
small x gluons in a hadron



most gluons in the wave function of a hadron have momentum Q_s

$$Q_s(x, b_t, A) \gg \Lambda_{QCD}$$

QCD at high energy/small x: gluon saturation



high gluon density: **Eikonal multiple scattering**

p_t broadening (generic to multiple scattering)

energy dependence: **x-evolution via JIMWLK/BK**

suppression of spectra/away side peaks

$$\mathbf{Q_s^2(x, b_t, A) \sim A^{1/3}\,(\frac{1}{x})^{0.3}}$$

$$Q_s^2(x = 3 \times 10^{-4}) \sim 1\, GeV^2$$

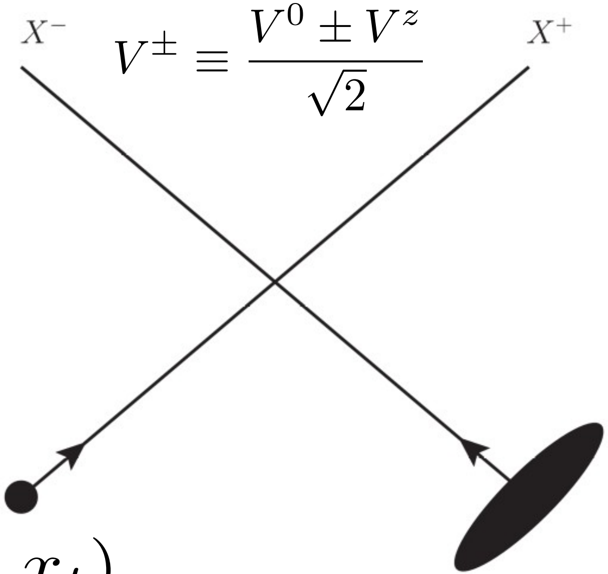
for a proton target (quarks)

a framework for multi-particle production in QCD at small x/low p_t

$$\mathbf{x \leq 0.01} \qquad \alpha_s \ln\,(x_v/x) \sim 1$$

scattering from a dense system of gluons

$$J_a^\mu \simeq \delta^{\mu -} \rho_a$$



$$D_\mu J^\mu = D_- J^- = 0$$

$$\partial_- J^- = 0$$ (in $A^+ = 0$ gauge)

does not depend on x^-

EOM:solution $$A_a^-(x^+, x_t) \equiv n^- S_a(x^+, x_t)$$

$$n^\mu = (n^+ = 0, n^- = 1, n_t = 0)$$

recall (eikonal approx): $$\bar{u}(q)\gamma^\mu u(p) \to \bar{u}(p)\gamma^\mu u(p) \sim p^\mu$$

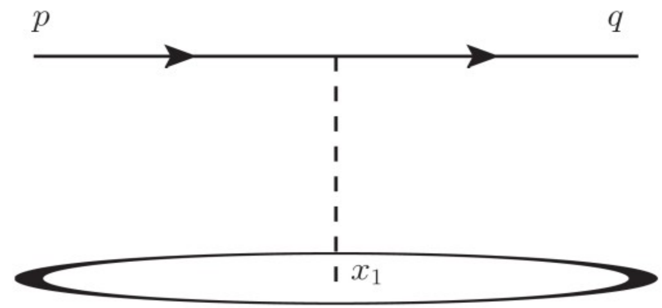
$$\bar{u}(q)\not{A}\, u(p) \to p \cdot A \sim p^+ A^-$$

scattering of a quark from background color field $A_a^-(x^+, x_t)$

$$i\mathcal{M}_1 = (ig)\int d^4x_1\, e^{i(q-p)x_1}\,\bar{u}(q)\,\left[\not{n}\,S(x_1)\right]\,u(p)$$

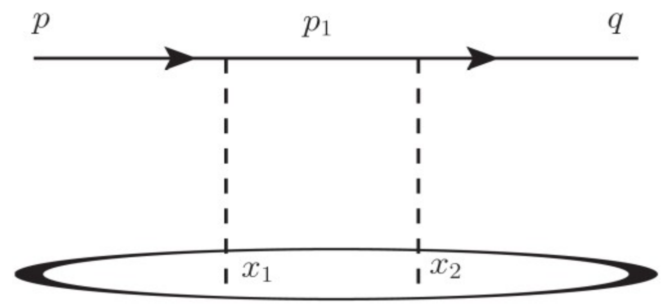
$$= (ig)(2\pi)\delta(p^+ - q^+)\int d^2x_{1t}\,dx_1^+\,e^{i(q^- - p^-)x_1^+}\,e^{-i(q_t-p_t)x_{1t}}$$

$$\bar{u}(q)\,\left[\not{n}\,S(x_1^+, x_{1t})\right]\,u(p)$$



$$i\mathcal{M}_2 = (ig)^2\int d^4x_1\,d^4x_2\int\frac{d^4p_1}{(2\pi)^4}\,e^{i(p_1-p)x_1}\,e^{i(q-p_1)x_2}$$

$$\bar{u}(q)\,\left[\not{n}\,S(x_2)\,\frac{i\not{p}_1}{p_1^2+i\epsilon}\,\not{n}\,S(x_1)\right]\,u(p)$$



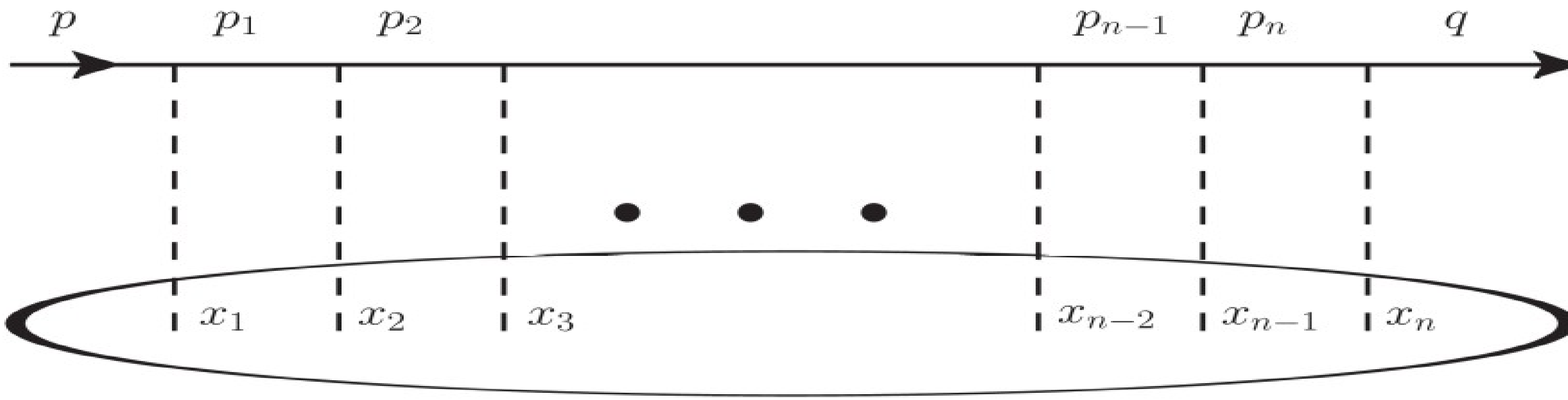
$$\int\frac{dp_1^-}{(2\pi)}\,\frac{e^{ip_1^-(x_1^+-x_2^+)}}{2p^+\left[p_1^- - \frac{p_{1t}^2 - i\epsilon}{2p^+}\right]} = \frac{-i}{2p^+}\,\theta(x_2^+ - x_1^+)\,e^{i\frac{p_{1t}^2}{2p^+}(x_1^+-x_2^+)}$$

contour integration over the pole leads to path ordering of scattering

ignore all terms: $O(\frac{p_t}{p^+}, \frac{q_t}{q^+})$ and use $\not{n}\,\frac{\not{p}_1}{2n\cdot p}\,\not{n} = \not{n}$

$$i\mathcal{M}_2 = (ig)^2\,(-i)(i)\,2\pi\delta(p^+ - q^+)\int dx_1^+\,dx_2^+\,\theta(x_2^+ - x_1^+)\int d^2x_{1t}\,e^{-i(q_t-p_t)\cdot x_{1t}}$$

$$\bar{u}(q)\,\left[S(x_2^+, x_{1t})\,\not{n}\,S(x_1^+, x_{1t})\right]\,u(p)$$

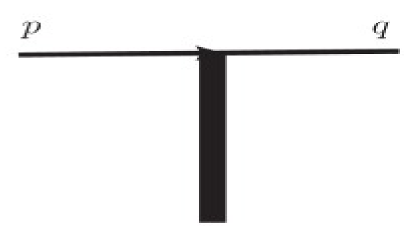


$$
\begin{aligned}
i\mathcal{M}_n \;=\;& 2\pi\delta(p^+ - q^+)\,\bar{u}(q)\,\not{n}\int d^2x_t\, e^{-i(q_t - p_t)\cdot x_t} \\
& \Big\{(ig)^n\,(-i)^n (i)^n \int dx_1^+\, dx_2^+ \cdots dx_n^+\, \theta(x_n^+ - x_{n-1}^+)\cdots\theta(x_2^+ - x_1^+) \\
& \big[S(x_n^+, x_t)\, S(x_{n-1}^+, x_t)\;\cdots S(x_2^+, x_t) S(x_1^+, x_t)\big]\Big\}\, u(p)
\end{aligned}
$$

sum over all scatterings $\quad i\mathcal{M} = \sum_n i\,\mathcal{M}_n$

$$
i\mathcal{M}(p,q) = 2\pi\delta(p^+ - q^+)\,\bar{u}(q)\,\not{n}\int d^2x_t\, e^{-i(q_t - p_t)\cdot x_t}\,\left[V(x_t) - 1\right]\, u(p) \equiv \bar{u}(q)\tau_f u(p)
$$

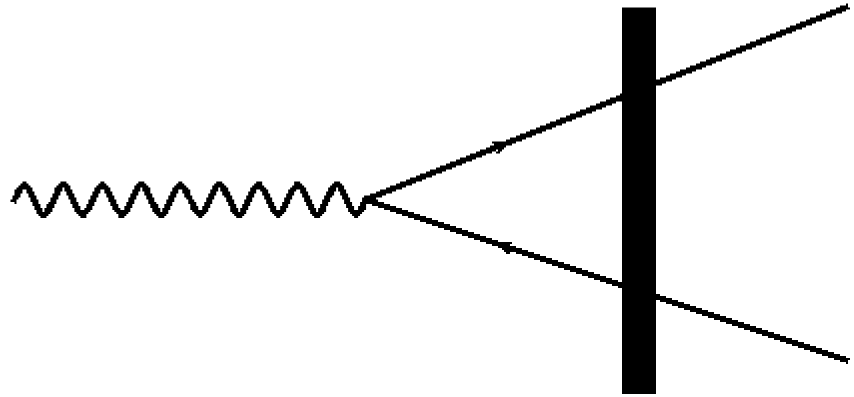
with $\;V(x_t) \equiv \hat{P}\,\exp\left\{ig\int_{-\infty}^{+\infty} dx^+\, n^-\, S_a(x^+, x_t)\, t_a\right\}$



$$
\frac{d\,\sigma^{q\,T\to q\,X}}{d^2p_t\,dy} \sim |i\mathcal{M}|^2 \sim \;F.T.\; < Tr\, V(x_t)\, V^\dagger(y_t) >
$$

quark anti-quark production in DIS at small x

$$\gamma^\star\, \mathbf{T} \to \mathbf{q(p)}\, \bar{\mathbf{q}}(\mathbf{q})\, \mathbf{X}$$



$$iM = \int \frac{d^4k}{(2\pi)^4} \bar{u}(p) \left[S_F^0(p)\right]^{-1} S_F(p,k) \left[ie\, \not\epsilon(l)\right] S_F(k-l,-q) \left[S_F^0(-q)\right]^{-1} v(q)$$

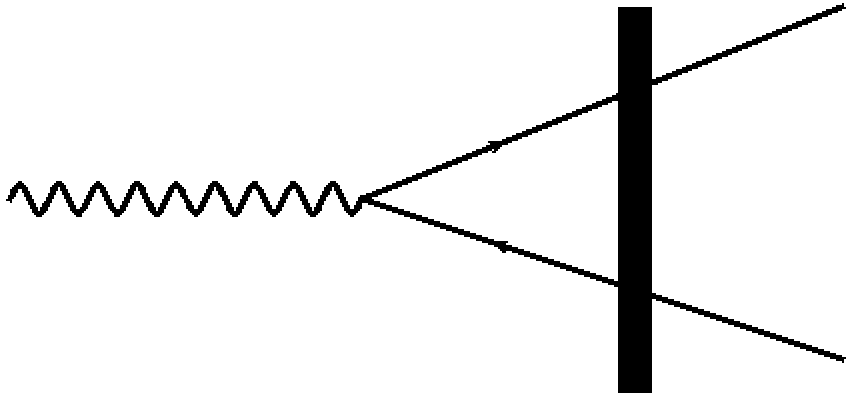
with

$$S_F(p,q) \equiv S_F^0(p)\, \tau_F(p,q)\, S_F^0(q)$$

$$\tau_\mathbf{F}(\mathbf{p},\mathbf{q}) \equiv (\mathbf{2}\pi)\delta(\mathbf{p}^+ - \mathbf{q}^+)\, \not n \int \mathbf{d^2x_t}\, \mathbf{e}^{-\mathbf{i}(\mathbf{q_t}-\mathbf{p_t})\cdot\mathbf{x_t}} \left[\theta(\mathbf{p}^+)\mathbf{V}(\mathbf{x_t}) - \theta(-\mathbf{p}^+)\mathbf{V}^\dagger(\mathbf{x_t})\right]$$

use spinor helicity methods to evaluate the Dirac algebra

quark anti-quark production in DIS at small x



$$\frac{d\sigma^{\gamma^* A \to q\bar{q}X}}{d^2\mathbf{p}\, d^2\mathbf{q}\, dy_1\, dy_2} = \frac{e^2 Q^2 (z_1 z_2)^2 N_c}{(2\pi)^7} \delta(1 - z_1 - z_2) \int d^8 x_\perp e^{i\mathbf{p}\cdot(\mathbf{x}_1' - \mathbf{x}_1)} e^{i\mathbf{q}\cdot(\mathbf{x}_2' - \mathbf{x}_2)}$$

$$[S_{122'1'} - S_{12} - S_{1'2'} + 1]$$

$$\Big\{ 4 z_1 z_2 K_0(|\mathbf{x}_{12}| Q_1) K_0(|\mathbf{x}_{1'2'}| Q_1) +$$

$$(z_1^2 + z_2^2) \frac{\mathbf{x}_{12} \cdot \mathbf{x}_{1'2'}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2'}|} K_1(|\mathbf{x}_{12}| Q_1) K_1(|\mathbf{x}_{1'2'}| Q_1) \Big\}$$

with

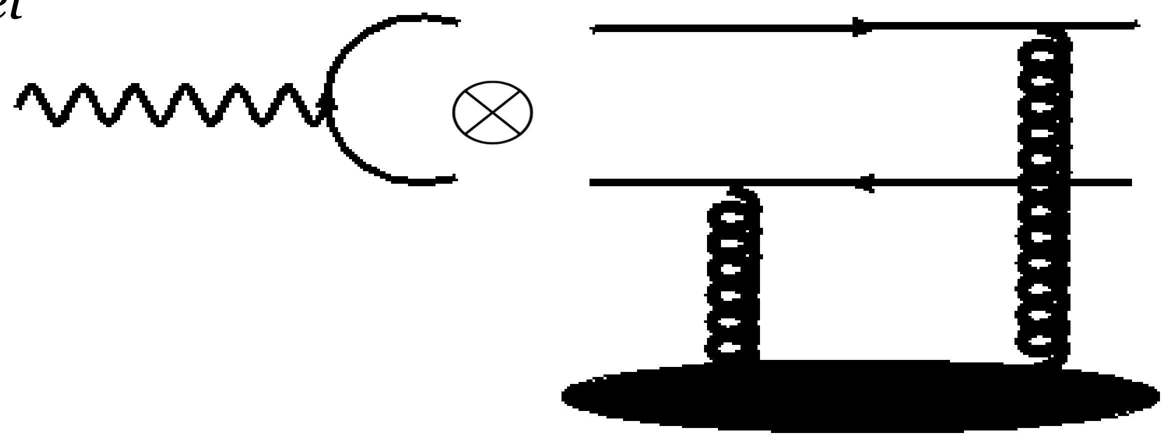
$$\mathrm{S}_{122'1'} \equiv \frac{1}{N_c} Tr\, V(\mathbf{x}_1)\, V^\dagger(\mathbf{x}_2)\, V(\mathbf{x}_{2'})\, V^\dagger(\mathbf{x}_{1'}) \qquad \mathrm{S}_{12} \equiv \frac{1}{N_c} Tr\, V(\mathbf{x}_1)\, V^\dagger(\mathbf{x}_2)$$

DIS total cross section

$$\sigma_{\rm DIS}^{\rm total} = 2\int_0^1 dz \int d^2x_t d^2y_t \left|\Psi(k^{\pm}, k_t | z, x_t, y_t)\right|^2 \sigma_{\rm dipole}(x_t, y_t)$$

$$\sigma_{\rm dipole}(x_t, y_t) \equiv \frac{1}{N_c} {\rm Tr} \left\langle 1 - V(x_t) V^{\dagger}(y_t) \right\rangle$$

can be written in closed form in terms of Bessel functions K_0, K_1



total cross section = probability of photon decaying into a quark anti-quark pair $\otimes$ *probability of the quark anti-quark "dipole" scattering on the target*

QED *QCD*

Next time

One-loop corrections to the total cross section

Evolution equations
solutions

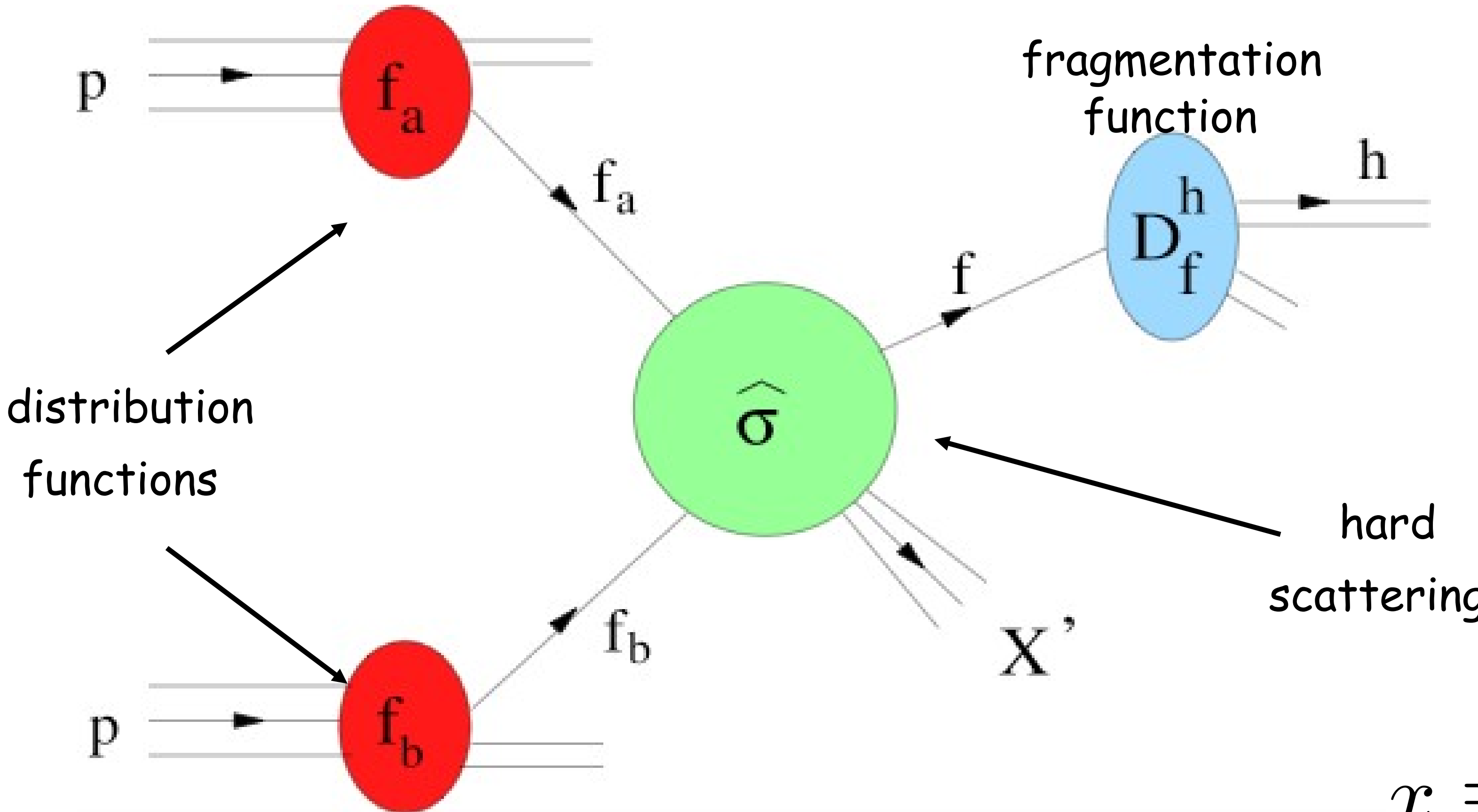
High energy heavy ion collisions (QGP)

Evidence from HERA/RHIC/LHC

Current/Future directions

pQCD in pp Collisions

collinear factorization: separation of soft (long distance) and hard (short distance)



$$x \equiv \frac{p}{P}$$

$$\frac{d\sigma^{pp\to h\,X}}{d^2p_t\,dy} \sim f_a(x_1) \otimes f_b(x_2) \otimes \hat{\sigma} \otimes D_f^h(z) + \cdots\cdots$$

power corrections